CE 430
Transportation Systems

Tutorial #2

(Ch. 2: Equations of motion and human factors)

2.2 Equation of motion

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Ex.5/P.96

A car collided with a telephone pole and left a 20 ft skid marks on the dry pavement. Based on the damages sustained, an engineer estimated that the speed at collision was 15 mph. if the roadway had a +3% grade, calculate the speed of the car at the onset of skidding.

$$v_f = 15mph = 15 \times \frac{5280}{3600} = 22ft/s$$

$$D_b = \frac{v_\circ^2 - v_f^2}{2g(f \mp G)}$$, where $f_{dry} = 0.6$

$$v_\circ^2 = 2D_b g(f \mp G) + v^2$$

$$v_\circ^2 = 20 \times 2 \times 32.2 \times (0.6 + 0.03) + 22^2 = 1295.44$$

$$v_\circ = 35.99ft/s = 35.99 \times \frac{3600}{5280} = 24.5mph$$

Conversion factors

$$unit\ conversion: \frac{1km}{1hr} = \frac{1000\ m}{3600\ sec} \rightarrow 1\text{km} / \text{hr} = \frac{1}{3.6}\ \text{m} / \text{sec}$$

$$unit\ conversion: \frac{1mi}{1hr} = \frac{5280\ ft}{3600\ sec} \rightarrow 1\text{mph} = 1.467\ \text{ft} / \text{sec}$$

A vehicle crashed into an abutment wall of a bridge leaving a skid mark on the road (d=110m, f=0.6) followed by skid mark on the side slope (21 m, f=0.3) all the way to the wall. The road has an uphill slope of 2% and an equivalent of -5% on the side slope. The crash velocity was estimated to e 30 km/hr. Was the driver obeying the speed limit of 120 km/hr before applying the breaks?

Side Slope.

$v_f = 30\ km/hr = 30 \times \frac{1000}{3600} = 8.33 m/s$

$v_{0(side\ slope)}$: $D_b = \dfrac{v^2_{o(side\ slope)} - v_f^2}{[2g(f \pm G)]}$

$v_\circ^2{}_{(slid\ slope)} = D_b 2g(f \mp G) + v_f^2$

$v_\circ^2 = 21 \times 2 \times 9.8 \times (0.3 - 0.05) + 8.33^2$
$= 172.34$

$v_\circ = 13.13 m/s = 13.13 \times \frac{3600}{1000}$
$= 47.26 km/hr$

Road: v_o (at side slope) = v (at road)

$v_\circ^2 = D_b 2g(f \mp G) + v^2$

$v_\circ^2 = 110 \times 2 \times 9.8 \times (0.6 + 0.02) + 13.13^2$
$= 1509.1$

$v_\circ = 38.86\ m/s = 140\ km/hr$

Thus, the driver did not obey the speed limit

Check for safety against sliding and overturning on a curve with radios R=200 m, super elevation e=3% and fs=0.2. the posted speed limit is 80 km/hr and vehicle's center of mass is at: X=1.1 m, Y = 1.5 m

Sliding

$$e + f = \frac{v^2}{gR} \Rightarrow v_{max} = \sqrt{gR(e+f)}$$

$$v_{max} = \sqrt{(0.03 + 0.2) \times 9.8 \times 200} = 21.23 m/s$$

$$v_{max} = 21.23 \times \frac{3600}{1000} = 76.4 km/hr$$

< 80 km/hr ➔ not safe

Over turning

$$\frac{X + Ye}{Y - Xe} = \frac{v^2}{gR} \Rightarrow v_{max} = \sqrt{gR\,\frac{X + Ye}{Y - Xe}}$$

$$v_{max} = \sqrt{9.81 * 200\left(\frac{1.1+(1.5*0.03)}{1.5-(1.1*0.03)}\right)} = 39.13 m/s = 140.90\ km/hr$$

The curve is not safe because the maximum speed before sliding is less than the posted speed limit.

What should be the speed for a 1,000 ft curve with super elevation of 2% ensuring no slidding or overturning on wet conditions (f=0.15). The vehicles have a center of mass at X=4.5 ft and Y=5.5 ft.

Sliding

$$e + f = \frac{v^2}{gR} \rightarrow v_{max} = \sqrt{gR(e+f)}$$

$$v_{max} = \sqrt{(0.02\ +\ 0.15) \times 32.2 \times 1000}$$

$$v_{max} = 73.99\,ft/s = 50.44\ mi/hr$$

Over turning

$$\frac{X + Ye}{Y - Xe} = \frac{v^2}{gR} \rightarrow v_{max} = \sqrt{gR\,\frac{X + Ye}{Y - Xe}}$$

$$v_{max} = \sqrt{32.2 * 1000\left(\frac{4.5+(5.5*0.02)}{5.5-(4.5-0.02)}\right)}$$

$$v_{max} = 165.65\,ft/s = 112.9\ mi/hr$$

The proposed speed limit must be less than both results.
Thus, an appropriate speed limit is **v = 45 mph**