5.29 A homeowner plants 6 bulbs selected at random from a box containing 5 tulip bulbs شتلة الخزامى and 4 daffodil bulbs شتلة النرجس البري. What is the probability that he planted 2 daffodil bulbs and 4 tulip bulbs?

$$f(x) = \frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}} \quad , \quad x = \max(0, n-(N-k)) \ldots \min(n,k)$$

X: number of tulip bulbs.
X~H(N=9, n=6, k=5)

$$f(x) = \frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}} = \frac{\binom{5}{x}\binom{4}{6-x}}{\binom{9}{6}} \quad , \quad x = 2,3,4,5.$$

$$f(4) = P(x=4) = \frac{\binom{5}{4}\binom{4}{2}}{\binom{9}{6}} = \frac{30}{84} = 0.35$$

5.31 A random committee of size 3 is selected from 4 doctors and 2 nurses. Write a formula for the probability distribution of the random variable X representing the number of doctors on the committee. Find P(2 ≤ X ≤ 3).

X: number of doctors on the committee. N=4+2=6
X~H(N=6, n=3, k=4)

$$f(x) = \frac{\binom{4}{x}\binom{6-4}{3-x}}{\binom{6}{3}} \quad , \quad x = 1,2,3.$$

$$p(2 \le X \le 3) = f(2) + f(3) = \frac{\binom{4}{2}\binom{2}{1}}{\binom{6}{3}} + \frac{\binom{4}{3}\binom{2}{0}}{\binom{6}{3}} = 0.6 + 0.2 = 0.8$$

5.32 From a lot of 10 missiles, 4 are selected at random and fired. If the lot contains 3 defective missiles that will not fire, what is the probability that
X: number of defective missiles.
X~H(N=10, n=4, k=3)

$$f(x) = \frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}} = \frac{\binom{4}{x}\binom{7}{4-x}}{\binom{10}{4}} \quad , \quad x = 0,1,2,3$$

(a) all 4 will fire?

$$f(0) = \frac{\binom{3}{0}\binom{7}{4}}{\binom{10}{4}} = 0.1667$$

(b) at most 2 will not fire?

$$p(X \le 2) = f(0) + f(1) + f(2) = 1 - p(x > 2) = 1 - f(3) = 0.9667$$

5.40 It is estimated that 4000 of the 10,000 voting residents of a town are against a new sales tax. If 15 eligible voters are selected at random and asked their opinion, what is the probability that at most 7 favor the new tax?

NOTE: If **n** is small compared to **N**, then a binomial distribution **B(n; p = K/N)** can be used to approximate the hypergeometric distribution **h(N;K; n).** The approximation is good when **n/N ≤ 0.05.**

Since $\frac{n}{N} = \frac{15}{10000} = 0.0015 \leq 0.05$

We can use Binomial as approximation to Hypergeometric.

$$P = \frac{k}{N} = \frac{6000}{10000} = 0.6$$

X: number of voters who favor the new tax.

$$f(x) = \binom{15}{x}(0.6)^x(0.4)^{15-x} \quad ; x = 0,1,2,\ldots,15$$

$$p(X \leq 7) = 0.213$$

5.56 On average, 3 traffic accidents per month occur at a certain intersection. What is the probability that in any given month at this intersection

$$\boldsymbol{f(x) = \frac{(\lambda t)^x e^{-\lambda t}}{x!}; x = 0, 1, 2, \ldots}$$

$\lambda = 3,\ t{:}\,One\ Month,$ X: number of accident per month.

X~ Poisson (3)

(a) exactly 5 accidents will occur?

$$P(X = 5) = f(5) = \frac{(3)^5 e^{-3}}{5!} = 0.1008$$

(b) fewer than 3 accidents will occur?

$$P(X < 3) = f(0) + f(1) + f(2) = e^{-3}\left[\frac{(3)^0}{0!} + \frac{(3)^1}{1!} + \frac{(3)^2}{2!}\right]$$

$$= e^{-3}\ [8.5] = 0.4232$$

(c) at least 2 accidents will occur?

$$P(X \geq 2) = 1 - p(X < 2) = 1 - f(0) + f(1) = 0.8009$$

5.67 The number of customers arriving per hour at a certain automobile service facility is assumed to follow a Poisson distribution with mean **λ = 7.**

$\lambda = 7,\ t{:}\,One\ hour,$ X: number of customers arriving per hour.

X~ Poisson (7)

(a) Compute the probability that more than 10 customers will arrive in a 2-hour period.

X: number of customers arriving in two hours.

$$\lambda t = 2(7) = 14, \qquad X \sim poisson(14)$$

$$P(X > 10) = 1 - p(X \le 10) = 1 - \frac{(14)^0 e^{-14}}{0!} - \cdots - \frac{(14)^{10} e^{-14}}{10!}$$

$$= 0.8243$$

(b) What is the mean number of arrivals during a 2-hour period?

$$E(X) = \mu = \lambda t = 14$$

3.6 The shelf life, in days, for bottles of a certain prescribed medicine is a random variable having the density function

$$f(x) = \begin{cases} \dfrac{20{,}000}{(x+100)^3}, & 0 < x \\ 0, & O.W \end{cases}$$

Find the probability that a bottle of this medicine will have a shell life of

(a) at least 200 days;

$$P(X>200) = \int_{200}^{\infty} \frac{20{,}000}{(x+100)^3}\, dx = \int_{200}^{\infty} 20000\,(x+100)^{-3}\, dx$$

let u=x+100 , x=u-100 , dx=du , 200 <x < ∞ >> 300 < u < ∞

$$P(X>200) = \int_{300}^{\infty} 20000\,(u)^{-3}\, du = 20000\left[\frac{u^{-2}}{-2}\right]_{300}^{\infty} = 20000\left[\frac{-1}{2u^2}\right]_{300}^{\infty}$$

$$= 20000\left[\frac{-1}{\infty} - \frac{-1}{2(300^2)}\right] = \frac{1}{9}$$

(a) anywhere from 80 to 120 days.

$$P(80<X<120) = \int_{80}^{120} \frac{20{,}000}{(x+100)^3}\, dx = 20000\left[\frac{-1}{2(x+100)^2}\right]_{80}^{120} = 0.1020$$

3.7 The total number of hours, measured in units of 100 hours, that a family runs a vacuum cleaner over a period of one year is a continuous random variable **X** that has the density function

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 \le x < 2 \\ 0, & elsewhere \end{cases}$$

Find the probability that over a period of one year, a family runs their vacuum cleaner

(a) less than 120 hours;

$$P(X < 1.20) = \int_0^1 x\,dx + \int_1^{1.2} (2-x)\,dx = \left[\frac{x^2}{2}\right]_0^1 - \left[\frac{(2-x)^2}{2}\right]_1^{1.2} = 0.68$$

(b) between 50 and 100 hours.

$$P(0.5 < X < 1) = \int_{0.5}^1 x\,dx = \left[\frac{x^2}{2}\right]_{0.5}^1 = \frac{3}{8} = 0.375$$

3.9 The proportion of people who respond to a certain mail-order solicitation is a continuous random variable **X** that has the density function

$$f(x) = \begin{cases} \dfrac{2(x+2)}{5}, & 0 < x < 1 \\ 0, & elsewhere \end{cases}$$

(a) *Show that P(0 < X < 1) = 1.*

$$P(0 < X < 1) = \int_0^1 \frac{2}{5}(x+2)\,dx = \frac{2}{5}\left[\frac{(x+2)^2}{2}\right]_0^1 = \frac{1}{5}[9-4] = 1$$

(b) Find the probability that more than 1/4 but fewer than 1/2 of the people contacted will respond to this type of solicitation.

$$p\left(\frac{1}{4} < x < \frac{1}{2}\right) = \int_{0.25}^{0.5} \frac{2}{5}(x+2)\,dx = \frac{2}{5}\left[\frac{(x+2)^2}{2}\right]_{0.25}^{0.5}$$
$$= \frac{1}{5}\left[\left(\frac{5}{2}\right)^2 - \left(\frac{9}{4}\right)^2\right] = 0.2375$$

3.12 An investment firm offers its customers municipal bonds that mature after varying numbers of years.
Given that the cumulative distribution function of T, the number of years to maturity for a randomly selected bond, is

$$F(t) = \begin{cases} 0, & t < 1 \\ \frac{1}{4}, & 1 \le t < 3 \\ \frac{1}{2}, & 3 \le t < 5 \\ \frac{3}{4}, & 5 \le t < 7 \\ 1, & 7 \le t \end{cases}$$

Find

(a) P(T = 5);

$$F(t) = p(T \le t)$$

$$p(T = 5) = f(5) = F(5) - F(4) = \frac{3}{4} - \frac{1}{2} = \frac{1}{4}$$

(b) P(T > 3);

$$p(T > 3) = 1 - p(T \le 3) = 1 - F(3) = 1 - \frac{1}{2} = \frac{1}{2}$$

(c) P(1.4 < T < 6);

$$p(1.4 < T < 6) = F(5) - F(2) = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$$

(d) P(T ≤ 5 | T ≥ 2).

$$p(T \le 5 \mid T \ge 2) = \frac{p(2 \le T \le 5)}{p(T \ge 2)} = \frac{F(5) - F(1)}{1 - P(T \le 1)} = \frac{\frac{3}{4} - \frac{1}{4}}{1 - \frac{1}{4}} =$$

#Note:

Discrete random variables

$p(a < X \le b) = F(b) - F(a)$

$p(a \le X < b) = F(b-1) - F(a-1)$

$p(a \le X \le b) = F(b) - F(a-1)$

$p(a < X < b) = F(b-1) - F(a)$

Continuous random variables

$p(a < X \le b) = p(a \le X < b) = p(a \le X \le b) = p(a < X < b) = F(b) - F(a)$

3.14 The waiting time, in hours, between successive speeders spotted by a radar unit is a continuous random variable with cumulative distribution function:

$$F(x) = \begin{cases} 0, & x < 0 \\ 1 - e^{-8x}, & x \geq 0 \end{cases}$$

Find the probability of waiting less than 12 minutes between successive speeders:

To convert from minute to the hours: 12/60 =1/5 =0.2 h , P(x< 0.2)

(a) using the cumulative distribution function of X;

$$P(X < 0.2) = P(X \leq 0.2) = F(0.2) = 1 - e^{-8(0.2)} = 0.7981$$

(b) using the probability density function of X.

$$f(x) = \frac{dF(x)}{dx} = \frac{d}{dx}\left(1 - e^{-8(x)}\right) = 8\,e^{-8(x)}, \quad x \geq 0$$

$$p(X < 0.2) = \int_0^{0.2} 8\,e^{-8(x)}\,dx = \left[-e^{-8(x)}\right]_0^{0.2} = 1 - e^{-8(0.2)}$$

3.17 A continuous random variable X that can assume values between **x = 1** and **x = 3** has a density function given by ***f(x) = 1/2.***

(a) Show that the area under the curve is equal to 1.

$$f(x) = \frac{1}{2}, \quad 1 < x < 3$$

we know that $\int_{-\infty}^{\infty} f(x)\,dx = 1$.

$$\int_1^3 \frac{1}{2}\,dx = \frac{1}{2}\,[x]_1^3 = 1$$

(c) Find P(2 < X < 2.5).

$$p(2 < X < 2.5) = \int_2^{2.5} \frac{1}{2}\,dx = \frac{1}{2}\,[x]_2^{2.5} = \frac{2.5-2}{2} = \frac{1}{4}$$

(d) Find P(X ≤ 1.6).

$$p(X \leq 1.6) = \int_1^{1.6} \frac{1}{2}\,dx = \frac{1.6-1}{2} = 0.3$$

4.50 For a laboratory assignment, if the equipment is working, the density function of the observed outcome *X* is

$$f(x) = \begin{cases} 2(1-x), & 0 < x < 1 \\ 0\,, & elsewhere \end{cases}$$

Find the variance and standard deviation of X.

$$E(X) = \int_{-\infty}^{\infty} x f(x)\, dx = \int_{0}^{1} 2x(1-x)\, dx = 2\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \frac{1}{3}$$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx = \int_{0}^{1} 2x^2(1-x)\, dx = 2\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1 = \frac{2}{3} - \frac{1}{3}$$

$$= \frac{1}{6}$$

$$v(X) = \sigma^2 = E(X^2) - [E(X)]^2 = \frac{1}{6} - \left[\frac{1}{3}\right]^2 = 0.056$$

$$\sigma = \sqrt{v(x)} = 0.2357$$