

# CE 430

# Transportation Systems

## **Tutorial #1**

## **(Ch.5: Transportation modes)**

## EX.2 / page 258

The cost function of a large railroad corporation is  $Y = 10^7 + 0.5T$ , where  $Y$  is the total cost of shipping in US dollars and  $T$  is the tons shipped. Last year the company charged on average 88¢ for each ton of freight. Their annual shipments total 48 million tons. This year they are considering geographical expansion through the purchase of a smaller railroad corporation that last year shipped total of 21 million tons. Economists estimated that the total cost of function (for the merged corporation) will be  $Y = 10.5^7 + 0.3T$ , while 10% more freight should be expected due to the better geographic coverage, at a price discounted by 8¢.

Show that the large and the merged railroad realize substantial economies of scale (EOS). Which railroad realizes greater EOS? Use a numerical example or a graphic for proof. Show numerically that the large railroad should merge with the smaller one.

	Large railroad corp.	Merged corp.
Cost function (Y)	$Y=10^7+0.5 T$	$Y=10.5^7+0.3 T$
Unit revenue	88 ¢/ton	= 88 - 8 = 80 ¢/ton
Total Shipment (T)	$48 \times 10^6$ ton	$1.1 \times 10^6(48+21)$ $= 75.9 \times 10^6$ ton
Total cost	$Y=10^7+(0.5)(48 \times 10^6)$ $= \$ 34 \times 10^6$	$Y=10.5^7+0.03(75.9 \times 10.5^6)$ $= 36.841 \times 10^6$
Unit cost (u)	$u= 34 \times 10^6 / 48 \times 10^6$ $= 71 \text{ ¢/ton}$	$u = 36.841 \times 10^6 / 75.9 \times 10$ $49 \text{ ¢/ton}$

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Total Shipment (T)	$48 \times 10^6$ ton	$75.9 \times 10^6$ ton
Total cost	\$ $34 \times 10^6$	\$ $36.841 \times 10^6$
Unit cost (u)	71 ¢/ton	49 ¢/ton

	Large railroad corp.	Merged corp.
Unit cost for $20 \times 10^6$ tons shipped ( $U_{20}$ ) $U_{20} = Y_{20}/T_{20}$	$= \frac{10^7 + (0.5)(20 \times 10^6)}{20 \times 10^6}$ = \$ 1/ton	$= \frac{10.5^7 + (0.3)(20 \times 10^6)}{20 \times 10^6}$ = \$ 1.003 /ton
Unit cost for $10 \times 10^6$ tons shipped ( $U_{10}$ ) $U_{10} = Y_{10}/T_{10}$	$= \frac{10^7 + (0.5)(10 \times 10^6)}{10 \times 10^6}$ = \$ 1.5 /ton	$= \frac{10.5^7 + (0.3)(10 \times 10^6)}{10 \times 10^6}$ = \$ 1.707 /ton
% of cost reduction $\frac{U_{10} - U_{20}}{U_{10}}$	$\frac{1.5 - 1}{1.5} = 33.3\%$	$\frac{1.707 - 1.003}{1.707} = 41.2\%$
EOS?	Yes	Yes

Thus, both corporations have EOS and it is greater in the merged corporation

## Ex.9/page 260

A businessman residing in Chicago considers his options for a trip to Detroit. His options are private car, rental car, bus, or airplane. Given the following data, suggest the best mode for his travel.

Distance between cities (one way) = 425 km.

Estimated access travel at origin and destination = 38 km at each city; the access distance and access trips are the same for all modes.

### Costs:

1/ private auto: 20¢/km (all costs combined); no access mode required;

2/ rental car (2 days): \$50/day plus \$12/day for insurance and tax; 11 l/100 km fuel efficiency and gas price is 40¢/l; no access mode required;

3/ bus: round-trip fare \$55; access mode required;

4/ air: round-trip fare \$100; access mode required.

### Access modes and costs:

In Chicago taxies charge \$2 plus 20¢/km, buses charge \$1 per ride (assume two rides);

In Detroit taxies charge \$2.40 plus 15¢/km, buses charge \$1.5 per ride (assume two rides).

The door-to-door travel times by mode are as follows:

	Private auto	Rental car	Bus	Air
Best	4.5	5.0	6.0	1.5
Worst	6.0 <sub>a</sub>	6.5 <sub>a</sub>	8.0 <sub>b</sub>	3.0 <sub>b</sub>

a Accounts for potentially congested conditions.

b Use of bus for access.

In order to make his selection, the businessman assumed a disutility function (a measure of discomfort due to the cost and travel time encountered):

Disutility = (total trip cost)/5 + 8 (one-way travel time)

**Which mode did the businessman select? (Round out all the cost estimates to the nearest integer)**

	Privet car	Rental car	Air plan (Taxi access mode)	Air plan (Bus access mode)
One-way Distance	$=38+425+38$ $= 501 \text{ km}$ (access+ intercity distance+ access)	501 km	501 km	501 km
Total distance	$= (2)(501)$ $= 1002 \text{ km}$	1002 km	1002 km	1002 km
Cost	$= (0.2)(1002)$ $= \$ 200.4$	$= 2(50+12)+0.4$ $(11 * \frac{1002}{100})$ $= \$ 168.1$	$= 100 + 2(2 + 0.2 * 38)$ $+ 2(2.4 + 0.15 * 38)$ $= \$ 135.4$	$= 100 + 2(1 * 2) + 2(1.5 * 2)$ $= \$ 110$
One-way time	4.5 hr	5 hr	1.5 hr	3 hr
Disutility	$= (\frac{200.4}{5}) + (8)(4.5)$ $= 76.08$	$= (\frac{168.1}{5}) + (8)(5)$ $= 73.62$	$= (\frac{135.4}{5}) + (8)(1.5)$ $= 39.08$	$= (\frac{110}{5}) + (8)(3)$ $= 46$

Among these modes, businessman selected to travel by air plan using taxi access mode