# Tutorial-1

3.3 Let **W** be a random variable giving the number of **heads** minus the number of **tails** in three tosses of a coin. List the elements of the sample space **S** for the three tosses of the coin and to each sample point assign a value w of W.

Answer:

**W= #H - #T**

S={HHH, HHT, HTH, THH,HTT, THT ,TTH, TTT}

W= -3 , -1 , 1, 3

**3.8** Find the probability distribution of the random variable *W* in Exercise 3.3, assuming that the coin is biased so that a head is twice as likely to occur as a tail.

Answer:

P(H)=2P(T) ,   we know P(H)+P(T)=1

Thus, 2P(T)+P(T)=1  >>  3P(T)=1  >>  P(T)=1/3  >>  P(H)=2/3

P(W=3)=P(HHH)=(2/3)( 2/3)(2/3) =8/27

P(W=1)=3P(HHT)=3(2/3)(2/3)(1/3)= 12/27=4/9

P(W=-1)=3P(HTT)=3(2/3)(1/3)(1/3)= 6/27=2/9

P(W=-3)=P(TTT)=(1/3)(1/3)(1/3)=1/27

| W | -3 | -1 | 1 | 3 | $\sum f(w)$ |
|---|---|---|---|---|---|
| f(w) | 1/27 | 6/27 | 12/27 | 8/27 | 1 |

3.5 Determine the value c so that each of the following functions can serve as a probability distribution of the discrete random variable X:

a) $f(x) = c(x^2 + 4),\ for\ x = 0,1,2,3$

b) $f(x) = c\binom{2}{x}\binom{3}{3-x},\ for\ x = 0,1,2.$

Answer:

**a)** $\sum_{x=0}^{3} f(x) = 1$ thus,

C[4+5+8+13]=1 >> 30C=1 >> C=1/30 =0.033

**b)** $\sum_{x=0}^{2} f(x) = 1$ thus,

$$C\left[\binom{2}{0}\binom{3}{3} + \binom{2}{1}\binom{3}{2} + \binom{2}{2}\binom{3}{1}\right] = C[1 + 2(3) + 3] = 10C$$

$10C = 1$ >> $C = \frac{1}{10} = 0.1$

**4.4** A coin is biased such that a head is <u>three</u> times as likely to occur as a tail. Find the expected number of tails when this coin is tossed twice.

Answer:

P(H)=3P(T)

3P(T)+P(T)=1 >> 4P(T)=1 >> P(T)=1/4 >> P(H)=3/4

X: number of tail , n=2

S={}

f(0)= P(HH)=(3/4) (3/4)=9/16

f(1)= 2P(HT)=2(1/4) (3/4)= 6/16= 3/8

f(2)=P(TT)=(1/4)(1/4)=1/16

| X | 0 | 1 | 2 |
|---|---|---|---|
| f(x) | 9/16 | 6/16 | 1/16 |

$$E(X) = \sum_{x=0}^{2} x\, f(x) = (0)\left(\frac{9}{16}\right) + (1)\left(\frac{6}{16}\right) + (2)\left(\frac{1}{16}\right) = \frac{1}{2}$$

$$E(X^2) = \sum_{x=0}^{2} x^2\, f(x) = (0)^2\left(\frac{9}{16}\right) + (1)^2\left(\frac{6}{16}\right) + (2)^2\left(\frac{1}{16}\right) = \frac{5}{8}$$

$$V(X) = E(X^2) - \left(E(X)\right)^2 = \frac{5}{8} - \left(\frac{1}{2}\right)^2 = 3/8$$

**4.7** By investing in a particular stock, a person can make a profit in one year of \$4000 with probability 0.3 or take a loss of \$1000 with probability 0.7. What is this person's expected gain?

Answer:

(4000) (0.3) –(1000) (0.7)= 5001

**4.17** Let *X* be a random variable with the following probability distribution:

| X | -3 | 6 | 9 |
|---|---|---|---|
| f(x) | 1/6 | 1/2 | 1/3 |

Find $\mu_{g(X)}$, where $g(X) = (2X + 1)^2$.

Answer:

| X | -3 | 6 | 9 |
|---|---|---|---|
| f(x) | 1/6 | 1/2 | 1/3 |
| $(2x+1)^2$ | 25 | 169 | 361 |

$$E\big(g(X)\big) = \sum g(x)f(x)$$

$E((2x+1)^2) = \sum(2x+1)^2 f(x)$=25(1/6) +169(1/2) + 361(1/3)= 209

**Or**

$$E((2X+1)^2) = E(4x^2 + 4x + 1) = 4E(x^2) + 4E(x) + 1 = 209$$

**4.34** **(H.W)** Let *X* be a random variable with the following probability distribution:

| X | -2 | 3 | 5 |
|---|---|---|---|
| f(x) | 0.3 | 0.2 | 0.5 |

Find the standard deviation of *X*.

Answer:

$$\boldsymbol{X \sim Binomial\ (n, p)}$$
$$\boldsymbol{f(x) = \binom{n}{x} (p)^x (q)^{n-x}; x = 0, 1, 2, 3, \ldots, n}$$

**5.5** According to *Chemical Engineering Progress* (November 1990), approximately 30% of all pipework failures in chemical plants are caused by operator error.

(a) What is the probability that out of the next 20 pipework failures at least 10 are due to operator error?

(b) What is the probability that no more than 4 out of 20 such failures are due to operator error?

(c) Suppose, for a particular plant, that out of the random sample of 20 such failures, exactly 5 are due to operator error. Do you feel that the 30% figure stated above applies to this plant? Comment.

Answer:

X: number of failures due to operator error.

P=0.3 , n=20 .

$$\boldsymbol{X \sim Binomial\ (20, 0.3)}$$
$$\boldsymbol{f(x) = \binom{20}{x} (0.3)^x (0.7)^{20-x} \quad ; x = 0, 1, 2, \ldots, 20}$$

**a)** $P(X \geq 10) = 1 - P(X < 9) = 0.048$
**b)** $P(X \leq 4) = f(1) + f(2) + f(3) + f(4) = 0.2375$
**c)** $P(X = 5) = 0.1789 \ \approx 18\%$ , NO

Tha 30% is an incorrect percentage of operator errors. It's just that in this sample, the number of actual errors was less than the number of expected errors (which is 30% x 20, or 6 errors).

**5.15** It is known that 60% of mice inoculated with a serum are protected from a certain disease. If 5 mice are inoculated, find the probability that
(a) none contracts the disease;
(b) fewer than 2 contract the disease;

(c) more than 3 contract the disease.

Answer:

q=0.6 ,  p=0.4  ,n=5

X: number of mice contract the disease.

$$\boldsymbol{X \sim Binomial\ (5, 0.4)}$$

$$\boldsymbol{f(x) = \binom{5}{x} (0.4)^{x} (0.6)^{5-x} \quad ; x = 0, 1, 2, 3, 4, 5.}$$

a) $P(X = 0) = 0.0778$
b) P(X< 2)= 0.337
c) P(X >3)=0.087