Tutorial #1

$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$ (1.1)

Question 1:

Grade point average. The director of admissions of a small college selected 120 students at random from the new freshman class in a study to determine whether a student's grade point average (OPA) at the end of the freshman year (Y) can be predicted from the ACT test score *(X)*. The results of the study follow. Assume that first-order regression model (1.1) is appropriate.

a. Obtain the least squares estimates of $\beta_0$and β_1, and state the estimated regression function.

b. Plot the estimated regression function and the data."Does the estimated regression function appear to fit the data well?

c. Obtain a point estimate of the mean freshman OPA for students with ACT test score *X* = 30.

d. What is the point estimate of the change in the mean response when the entrance test score increases by one point?

Question 2:

Copier maintenance. The Tri-City Office Equipment Corporation sells an imported copier on a franchise basis and performs preventive maintenance and repair service on this copier. The data below have been collected from 45 recent calls on users to perform routine preventive maintenance service; for each call, X is the number of copiers serviced" عدد الناسخات الخدمات" and *Y* is the total number of minutes spent by the service person" العدد الإجمالي للدقائق التي يقضيها الشخص للخدمة". Assume that first-order regression model (1.1) is appropriate.

a. **Obtain the estimated regression function.**
b. **b. Plot the estimated regression function and the data. How well does the estimated regression function fit the data?**
c. **c. Interpret $b_0$in your estimated regression function. Does $b_0$provide any relevant information here? Explain.**
d. **d. Obtain a point estimate of the mean service time when X = 5 copiers are serviced.**
e. **e. Obtain the residuals e_i and the sum of the squared residuals $\sum e_i^2$. What is the relation between the sum of the squared residuals here and the quantity $Q = \sum(Y_i - b_0 - Xb_1)^2$?**
f. **f. Obtain point estimates of σ^2 and. In what units is σ expressed?**

Question 3:

Airfreight breakage. A substance used in biological and medical research is shipped by airfreight to users in cartons of 1,000 ampules. The data below, involving 10 shipments, were collected on the number of times the carton was transferred from one aircraft to another

over the shipment route (X) and the number of ampules found to be broken upon arrival (Y). Assume that first-order regression model (1.1) is appropriate.

a. **Plot the estimated regression function and the data. Does a linear regression function appear to give a good fit here?**
b. **Obtain the estimated regression function?**
c. **Obtain a point estimate of the expected number of broken ampules when X = 1 transfer is made.**
d. **Estimate the increase in the expected number of ampules broken when there are 2 transfers as compared to 1 transfer.**
e. **Verify that your fitted regression line goes through the point** $(\bar{X}, \bar{Y})$.
f. **Obtain the residual for the first case. What is its relation to** e_1**?**
g. **Compute** $\sum e_i^2$ **and MSE. What is estimated by MSE?**

Question 4:

Plastic hardness. Refer to Problems 1.3 and 1.14. Sixteen batches of the plastic were made, and from each batch one test item was molded. Each test item was randomly assigned to one of the four predetermined time levels, and the hardness was measured after the assigned elapsed time. The results are shown below; X is the elapsed time in hours? and Y **is hardness in Brinell units. Assume that first-order regression model (1.1) is appropria'te.**

a. Obtain the estimated regression function. Plot the estimated regression function and the data. Does a linear regression function appear to give a good fit here?
b. Obtain a point estimate of the mean hardness when X = 40 hours.
c. Obtain a point estimate of the change in mean hardness when X increases by 1 hour.

Question 5:

Suppose that you are given observations y_1 **and** y_2 **such that :** $y_1 = \propto + \beta + \epsilon_1$, $y_2 = -\propto + \beta + \epsilon_2$ **The random variables** ϵ_i **for i = 1,2 are independent and normally distributed with mean 0 and variance** σ^2

a. **find the least squares estimators of the parameters** $\propto$ *and* β **, also verify that : they are unbiased estimators .(Hint : obtain the minimum of the sum of the** ϵ_i^2 **using the least squares technique.)**
b. **find variance of** $\hat{\propto}$ **.**

Question 6:

An investigation , conducted by a mail-order company, into the relation between the sales revenues (y_i , $in\ millions\ of\ dollars$) and the price per gallon of gasoline (x_i , in cents) over a period of 10 months yields :

$\sum_{i=1}^{10} y_i = 527$, $\sum_{i=1}^{10} x_i = 6509$, $\sum_{i=1}^{10} x_i^2 = 4909311$, $\sum_{i=1}^{10} x_i\, y_i = 325243$.

Estimate the parameters β_0 and β_1 in the regression model $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$, where the ϵ_i are uncorrelated with a mean of zero and a common variance of σ^2 for i=1,2,…,10 .

Question 7:

Let X and ϵ be two independent random variables , and E(ϵ) = 0 . Let Y = $\beta_0 + \beta_1 X + \epsilon$.

Show that : $\beta_1 = \frac{cov(X,Y)}{V(X)} = Corr(X,Y)\sqrt{\frac{V(Y)}{V(X)}}$