Exercise 1

Given the difference equation with the input $x(n) = (0.5)^n u(n)$, calculate the system response $y(n)$

$$y(n) = x(n-1) - 0.75y(n-1) - 0.125y(n-2)$$

for $n = 0, 1, 2, 3, 4$ and initial condition $x(-1) = -1$, $y(-2) = 2, y(-1) = 1$

Solution

$$y(0) = -2,\ y(1) = 2.3750,\ y(2) = -1.0312,\ y(3) = 0.7266,\ y(4) = -0.2910$$

Exercise 2

For the difference equations find H(z)

a) $y(n) = 0.5x(n) + 0.5x(n-1)$

b) $y(n) = x(n) - 0.25x(n-2) - 1.1y(n-1) - 0.28y(n-2)$

Solution

a) Applying Z Transform: $Y(z) = 0.5\,X(z) + 0.5\,z^{-1}X(z)$ → $Y(z) = [0.5 + 0.5\,z^{-1}]X(z)$

→ $\frac{Y(z)}{X(z)} = H(z) = 0.5 + 0.5\,z^{-1}$ → $H(z) = 0.5 + 0.5z^{-1}$

b) Applying Z Transform: $Y(z) = X(z) + 0.25\,z^{-2}X(z) - 1.1z^{-1}Y(z) - 0.28\,z^{-2}Y(z)$

→ $Y(z)[1 + 1.1z^{-1}Y(z) + 0.28\,z^{-2}Y(z)] = [1 + 0.25\,z^{-2}]X(z)$ → $H(z) = \frac{1 - 0.25z^{-2}}{1 + 1.1z^{-1} + 0.28z^{-2}}$

Exercise 3

Convert each of the following transfer functions into difference equations:

a) $H(z) = \dfrac{z^2 - 0.25}{z^2 + 1.1z + 0.18}$

b) $H(z) = \dfrac{z^2 - 0.1z + 0.3}{z^3}$

Solution

a) $y(n) = x(n) - 0.25x(n-2) - 1.1y(n-1) - 0.28y(n-2)$

b) $y(n) = x(n-1) - 0.1x(n-2) + 0.3x(n-3)$

Exercise 4

Convert the following transfer function into pole-zero form:

$$H(z) = \frac{1 - 0.16z^{-2}}{1 + 0.7z^{-1} + 0.1z^{-2}}$$

Solution

$$H(z) = \frac{(z+0.4)(z-0.4)}{(z+0.2)(z+0.5)}$$

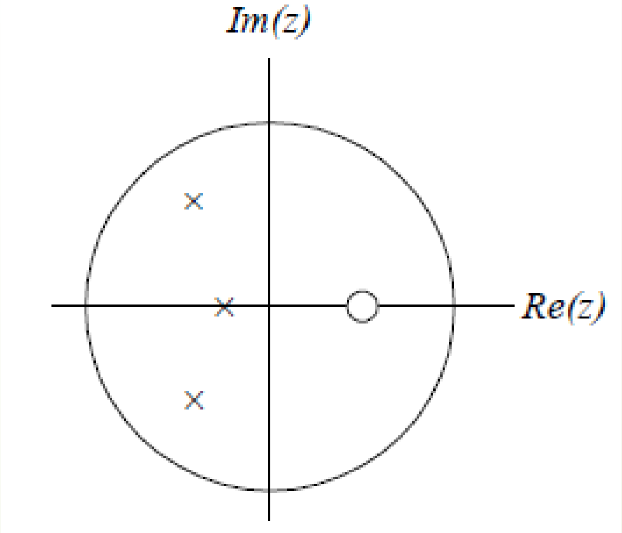
Exercise 5

Given the following transfer function that describe digital system, sketch the z-plane pole-zero plot and determine the stability of the digital system.

$$H(z) = \frac{z - 0.5}{(z + 0.25)(z^2 + z + 0.8)}$$

Solution

zero: $z = 0.5$, poles: $z = -0.25$ $(|z| = 0.25)$, $z = -0.5 \pm 0.7416j$ $(|z| = 0.8944)$, stable



Exercise 6

Given the digital system, with a sampling rate of 8,000 Hz,

$$y(n) = 0.5x(n) + 0.5x(n-2)$$

Determine and plot the magnitude and phase frequency responses. Determine the filter type.

Solution

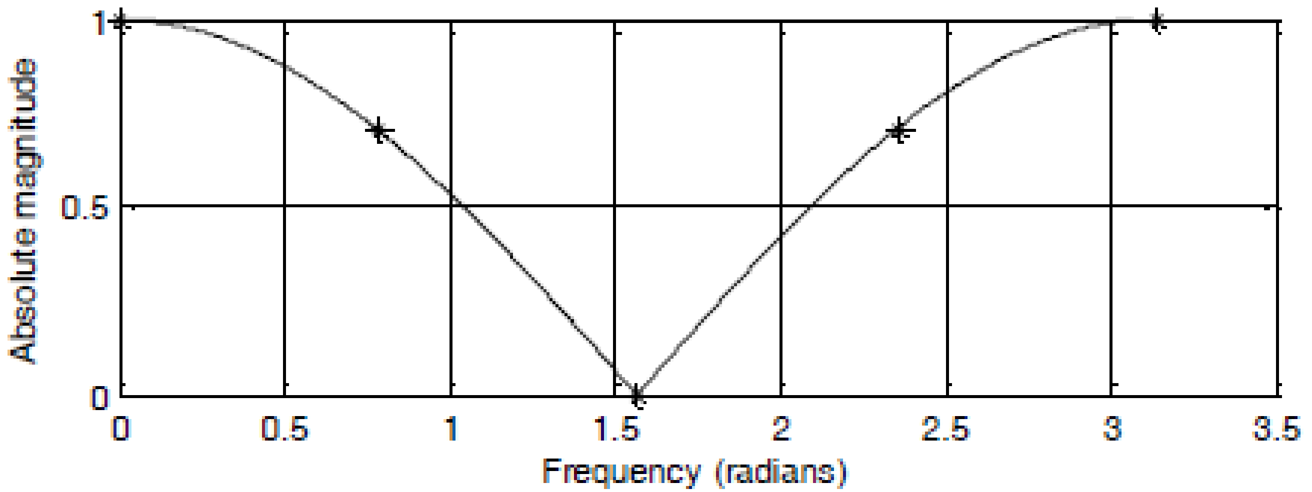
$$Y(z) = 0.5\,X(z) + 0.5z^{-2}X(z) \Rightarrow Y(z) = [0.5 + 0.5\,z^{-2}]\,X(z)$$

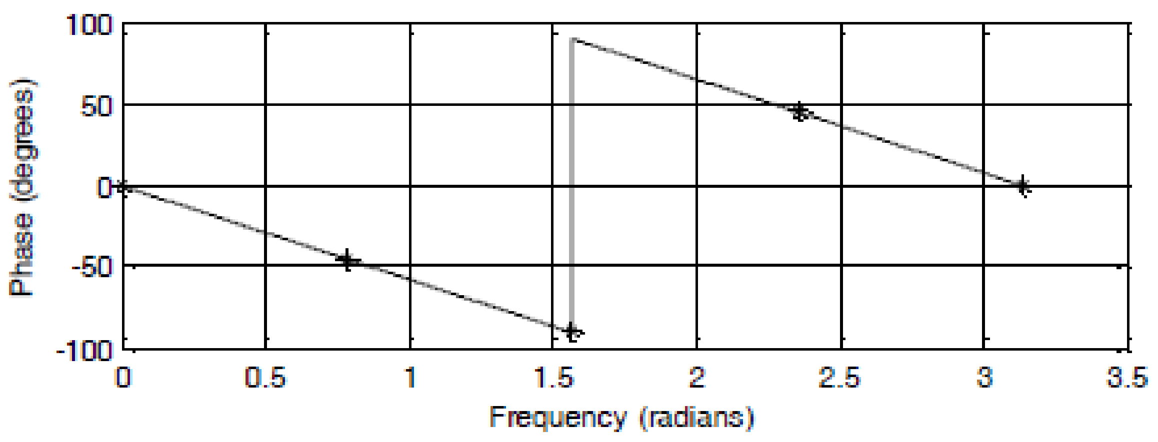
$$\Rightarrow \frac{Y(z)}{X(z)} = H(z) = 0.5 + 0.5z^{-2},\ H(e^{j\Omega}) = 0.5 + 0.5e^{-j2\Omega}$$

$$\Rightarrow H(e^{j\Omega}) = 0.5 + 0.5\big(\cos(2\Omega) - j\sin(2\Omega)\big) = 0.5(1+\cos(2\Omega)) - j0.5\,\sin(2\Omega)$$

$$\left|H(e^{j\Omega})\right| = 0.5\sqrt{(1+\cos 2\Omega)^2 + (\sin 2\Omega)^2},\ \angle H(e^{j\Omega}) = \tan^{-1}\left(\frac{-\sin 2\Omega}{1+\cos 2\Omega}\right)$$

Ω (radians)	$f = \frac{\Omega}{2\pi} f_s$ (Hz)	$\lvert H(e^{j\Omega}) \rvert$	$\lvert H(e^{j\Omega}) \rvert_{dB}$	$\angle H(e^{j\Omega})$
0	0	1.000	0 dB	0.00^0
0.25π	1000	0.707	-3.0102 dB	-45.00^0
0.50π	2000	0.0	$-\infty$ dB	-90.00^0
0.75π	3000	0.707	-3.0102 dB	45.00^0
1.00π	4000	1.000	0 dB	0.00^0





Band-stop filter.