Finite Impulse Response (FIR) Filter Design

Finite Impulse Response

❖ Frequency Response:

- $H(e^{j\Omega}) = e^{-j\omega_o}\left[H(e^{j\Omega})\right]$
- $\angle H(e^{j\Omega}) = -\omega_o - \frac{\pi}{2}\left[sign\left(H(e^{j\Omega})\right) - 1\right]$
- $sign(x) = \begin{cases} -1 & x < 0 \\ 0 & x = 0 \\ 1 & x > 0 \end{cases}$

❖ Fourier Transform Design Method:

- *Symmetric*

$$H(z) = h(M)z^{M} + \cdots + h(1)z^{1} + h(0) + h(1)z^{-1} + \cdots + h(M)z^{-M}$$

$$H(z) = b_o + b_1\, z^{-1} + \cdots + b_{2M}\, z^{-2M} \Longrightarrow 2M + 1 \text{ components or } \#\, of\, tap$$

$$b_n = h(n - M) \qquad for\; n = 0, 1, \ldots, 2M$$

Finite Impulse Response

$$\Omega_c = 2\pi f_c T_s = \frac{2\pi f_c}{f_s}$$

Filter Type	Ideal Impulse Response $h(n)$ (noncausal FIR coefficients)
Lowpass:	$h(n) = \begin{cases} \dfrac{\Omega_c}{\pi} & \text{for} \quad n = 0 \\ \dfrac{\sin(\Omega_c n)}{n\pi} & \text{for} \quad n \neq 0 \end{cases} \quad -M \le n \le M$
Highpass:	$h(n) = \begin{cases} \dfrac{\pi - \Omega_c}{\pi} & \text{for} \quad n = 0 \\ -\dfrac{\sin(\Omega_c n)}{n\pi} & \text{for} \quad n \neq 0 \end{cases} \quad -M \le n \le M$
Bandpass:	$h(n) = \begin{cases} \dfrac{\Omega_H - \Omega_L}{\pi} & \text{for} \quad n = 0 \\ \dfrac{\sin(\Omega_H n)}{n\pi} - \dfrac{\sin(\Omega_L n)}{n\pi} & \text{for} \quad n \neq 0 \end{cases} \quad -M \le n \le M$
Bandstop:	$h(n) = \begin{cases} \dfrac{\pi - \Omega_H + \Omega_L}{\pi} & \text{for} \quad n = 0 \\ -\dfrac{\sin(\Omega_H n)}{n\pi} + \dfrac{\sin(\Omega_L n)}{n\pi} & \text{for} \quad n \neq 0 \end{cases} \quad -M \le n \le M$

Finite Impulse Response

❖ **FIR Filtering with Window Method:**

$$h_w(n) = h(n).w(n)$$

1. Rectangular window:

$$w_{rec}(n) = 1, \ -M \leq n \leq M$$

2. Triangular (Bartlett) window:

$$w_{tri}(n) = 1 - \frac{|n|}{M}, \ -M \leq n \leq M$$

3. Hanning window:

$$w_{han}(n) = 0.5 + 0.5 \cos\left(\frac{n\pi}{M}\right), -M \leq n \leq M$$

4. Hamming window:

$$w_{ham}(n) = 0.54 + 0.46 \cos\left(\frac{n\pi}{M}\right), -M \leq n \leq M$$

5. Blackman window:

$$w_{black}(n) = 0.42 + 0.5 \cos\left(\frac{n\pi}{M}\right) + 0.08 \cos\left(\frac{2n\pi}{M}\right), \ -M \leq n \leq M$$

Finite Impulse Response

Exercise 1: Design a 5-tap FIR low-pass filter with a cut-off frequency of 100 Hz and a sampling rate of 1000 Hz using a Hamming window function. Determine the transfer function and difference equation of the designed FIR system, and compute and plot the magnitude frequency response for $\Omega = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi$:

Solution

$$2M + 1 = 5 \implies M = 2$$

$$\Omega_c = 2\pi\, f_c\, T_s = \frac{2\pi\, f_c}{f_s}$$

$$\Omega_c = \frac{2\pi\,(100)}{(1000)} = 0.6283\ radians$$

Low-pass Filter:

$$h(n) = \begin{cases} \frac{\Omega_c}{\pi} & for\ n = 0 \\ \frac{\sin(\Omega_c\, n)}{n\,\pi} & for\ n \neq 0 \end{cases} \qquad -M \leq n \leq M$$

$$h(n) \quad for\ \ -2 \leq n \leq 2$$

$$h(-2) = \frac{\sin(\Omega_c\, n)}{n\,\pi} = \frac{\sin\big(0.6283\,(-2)\big)}{(-2)\,\pi} = 0.1514$$

$$h(-1) = \frac{\sin(\Omega_c\, n)}{n\,\pi} = \frac{\sin\big(0.6283\,(-1)\big)}{(-1)\,\pi} = 0.1871$$

$$h(0) = \frac{\Omega_c}{\pi} = \frac{(0.6283)}{\pi} = 0.2$$

$$h(1) = \frac{\sin(\Omega_c\, n)}{n\,\pi} = \frac{\sin\big(0.6283\,(1)\big)}{(1)\,\pi} = 0.1871$$

$$h(2) = -\frac{\sin(\Omega_c\, n)}{n\,\pi} = \frac{\sin\big(0.6283\,(1)\big)}{(2)\,\pi} = 0.1514$$

Finite Impulse Response

Exercise 1: Design a 5-tap FIR low-pass filter with a cut-off frequency of 100 Hz and a sampling rate of 1000 Hz using a Hamming window function. Determine the transfer function and difference equation of the designed FIR system, and compute and plot the magnitude frequency response for $\Omega = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi$:

Solution

$$h_w(n) = h(n).w(n)$$

Hamming Window:

$$w_{ham}(n) = 0.54 + 0.46\cos\left(\frac{n\,\pi}{M}\right) \qquad -M \le n \le M$$

$$w_{ham}(n) \quad for \ -2 \le n \le 2$$

$$w_{ham}(-2) = 0.54 + 0.46\cos\left(\frac{(-2)\,\pi}{2}\right) = 0.08$$

$$w_{ham}(-1) = 0.54 + 0.46\cos\left(\frac{(-1)\,\pi}{2}\right) = 0.54$$

$$w_{ham}(0) = 0.54 + 0.46\cos\left(\frac{(0)\,\pi}{2}\right) = 1$$

$$w_{ham}(1) = 0.54 + 0.46\cos\left(\frac{(1)\,\pi}{2}\right) = 0.54$$

$$w_{ham}(2) = 0.54 + 0.46\cos\left(\frac{(2)\,\pi}{2}\right) = 0.08$$

$$h_w(n) = h(n).w(n)$$

$$b_n = h_w(n - M) \qquad for\ n = 0, 1, \ldots, 2M$$

Finite Impulse Response

Exercise 1: Design a 5-tap FIR low-pass filter with a cut-off frequency of 100 Hz and a sampling rate of 1000 Hz using a Hamming window function. Determine the transfer function and difference equation of the designed FIR system, and compute and plot the magnitude frequency response for $\Omega = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi$:

Solution

$h = 0.1514$	0.1871	0.2	0.1871	0.1514
$w_{ham} = 0.08$	0.54	1	0.54	0.08
$h_w = 0.0121$	0.101	0.2	0.101	0.0121
b_0	b_1	b_2	b_3	b_4

$$H(z) = b_o + b_1\, z^{-1} + b_2\, z^{-2} + b_3\, z^{-3} + b_4\, z^{-4}$$

$$H(z) = 0.0121 + 0.101\, z^{-1} + 0.2\, z^{-2} + 0.101\, z^{-3} + 0.0121\, z^{-4}$$

Transfer Function

Finite Impulse Response

Exercise 1: Design a 5-tap FIR low-pass filter with a cut-off frequency of 100 Hz and a sampling rate of 1000 Hz using a Hamming window function. Determine the transfer function and difference equation of the designed FIR system, and compute and plot the magnitude frequency response for $\Omega = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi$:

Solution

$$H(z) = 0.0121 + 0.101\, z^{-1} + 0.2\, z^{-2} + 0.101\, z^{-3} + 0.0121\, z^{-4}$$

$$H(z) = \frac{Y(z)}{X(z)} = 0.0121 + 0.101\, z^{-1} + 0.2\, z^{-2} + 0.101\, z^{-3} + 0.0121\, z^{-4}$$

$$Y(z) = 0.0121\, X(z) + 0.101\, z^{-1}\, X(z) + 0.2\, z^{-2}\, X(z) + 0.101\, z^{-3}\, X(z) + 0.0121\, z^{-4}\, X(z)$$

$$y(n) = 0.0121\, x(n) + 0.101\, x(n-1) + 0.2\, x(n-2) + 0.101\, x(n-3) + 0.0121\, x(n-4)$$

$Difference\ Equation$

Finite Impulse Response

Exercise 1: Design a 5-tap FIR low-pass filter with a cut-off frequency of 100 Hz and a sampling rate of 1000 Hz using a Hamming window function. Determine the transfer function and difference equation of the designed FIR system, and compute and plot the magnitude frequency response for $\Omega = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi$:

Solution

$$H(z) = 0.0121 + 0.101\, z^{-1} + 0.2\, z^{-2} + 0.101\, z^{-3} + 0.0121\, z^{-4}$$

❖ $H(e^{j\Omega}) = H(z)|_{z=e^{j\Omega}}$

$$H(e^{j\Omega}) = 0.0121 + 0.101\, e^{-j\Omega} + 0.2\, e^{-j2\Omega} + 0.101\, e^{-j3\Omega} + 0.0121\, e^{-j4\Omega}$$

$$H(e^{j\Omega}) = e^{-j2\Omega}\left[0.0121\, e^{j2\Omega} + 0.101\, e^{j\Omega} + 0.2 + 0.101\, e^{-j\Omega} + 0.0121\, e^{-j2\Omega}\right]$$

$$|H(e^{j\Omega})| = |0.0121\, e^{j2\Omega} + 0.101\, e^{j\Omega} + 0.2 + 0.101\, e^{-j\Omega} + 0.0121\, e^{-j2\Omega}|$$

$$|H(e^{j\Omega})| = |0.2 + 0.0242\cos(2\Omega) + 0.202\cos(\Omega)|$$

❖ $\angle H(e^{j\Omega}) = -\omega_o - \frac{\pi}{2}\left[sign\left(H(e^{j\Omega})\right) - 1\right]$

$$\angle H(e^{j\Omega}) = -2\Omega - \frac{\pi}{2}[sign(0.2 + 0.0242\cos(2\Omega) + 0.202\cos(\Omega)) - 1]$$

Finite Impulse Response

Exercise 1: Design a 5-tap FIR low-pass filter with a cut-off frequency of 100 Hz and a sampling rate of 1000 Hz using a Hamming window function. Determine the transfer function and difference equation of the designed FIR system, and compute and plot the magnitude frequency response for $\Omega = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi$:

$$\left|H(e^{j\Omega})\right| = |0.2 + 0.0242\cos(2\Omega) + 0.202\cos(\Omega)|$$

$$\angle H(e^{j\Omega}) = -2\Omega - \frac{\pi}{2}[sign(0.2 + 0.0242\cos(2\Omega) + 0.202\cos(\Omega)) - 1]$$

Ω	$f = \frac{\Omega}{2\pi} f_s \ (Hz)$	$\left\|H(e^{j\Omega})\right\|$	$\angle H(e^{j\Omega})$
0	0	0.4262	$0\,°$
$0.25\,\pi$	125	0.3428	$-90\,°$
$0.50\,\pi$	250	0.1758	$-180\,°$
$0.75\,\pi$	375	0.0571	$-270\,°$
$1.00\,\pi$	500	0.0222	$-360\,°$

Finite Impulse Response

Exercise 1: Design a 5-tap FIR low-pass filter with a cut-off frequency of 100 Hz and a sampling rate of 1000 Hz using a Hamming window function. Determine the transfer function and difference equation of the designed FIR system, and compute and plot the magnitude frequency response for $\Omega = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi$:

$$|H(e^{j\Omega})| = |0.2 + 0.0242\cos(2\Omega) + 0.202\cos(\Omega)|$$

- MODE > 7(TABLE)
- **f(X)=|0.2+0.0242 cos(2X)+0.202 cos(X) |**
- Start? **0**
- End? **180**
- Step? **45**

$\lvert H(e^{j\Omega})\rvert$
0.4262
0.3428
0.1758
0.0571
0.0222

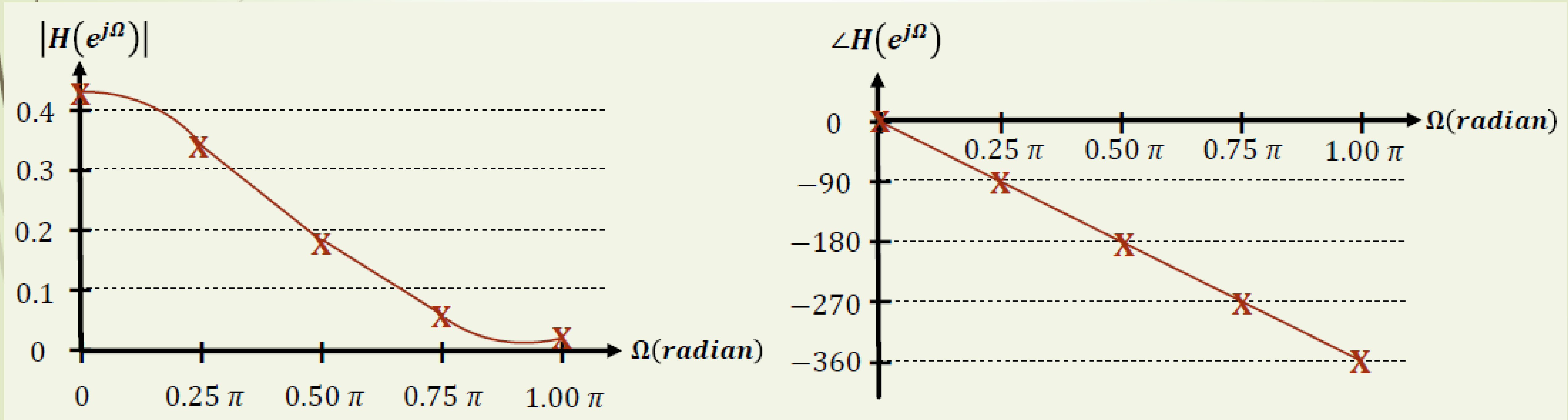
$180°$

*Also you can calculate the **phase** using the **same way***

$$\angle H(e^{j\Omega}) = -2\Omega - \frac{\pi}{2}[sign(0.2 + 0.0242\cos(2\Omega) + 0.202\cos(\Omega)) - 1]$$

Finite Impulse Response

Exercise 1: Design a 5-tap FIR low-pass filter with a cut-off frequency of 100 Hz and a sampling rate of 1000 Hz using a Hamming window function. Determine the transfer function and difference equation of the designed FIR system, and compute and plot the magnitude frequency response for $\Omega = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi$:



Finite Impulse Response

❖ Frequency Sampling Design Method:

$$H_k \ at \ \Omega_k = \frac{2\pi k}{(2M+1)} \qquad for \ k = 0, 1, \ldots, M$$

$[0\,,\pi]$

$2M+1$ *components*
or
# of tap

$$h(n) = \frac{1}{2M+1}\left\{H_0 + 2\sum_{k=1}^{M} H_k \cos\left(\frac{2\pi k\,(n-M)}{2M+1}\right)\right\} \qquad for \ n = 0, 1, \ldots, 2M$$

- *Symmetric*

$$h(n) = h(2M-n) \qquad for \ n = M+1, \ldots, 2M$$

Finite Impulse Response

Exercise 2: Design a 7-tap FIR low-pass filter with a cut-off frequency of $\Omega_c = 0.4\pi$ radians using the frequency sampling method.

Solution

$$2M + 1 = 7 \implies M = 3$$

$$\Omega_k = \frac{2\pi k}{(2M+1)} \qquad for \;\; k = 0, 1, \ldots, M$$

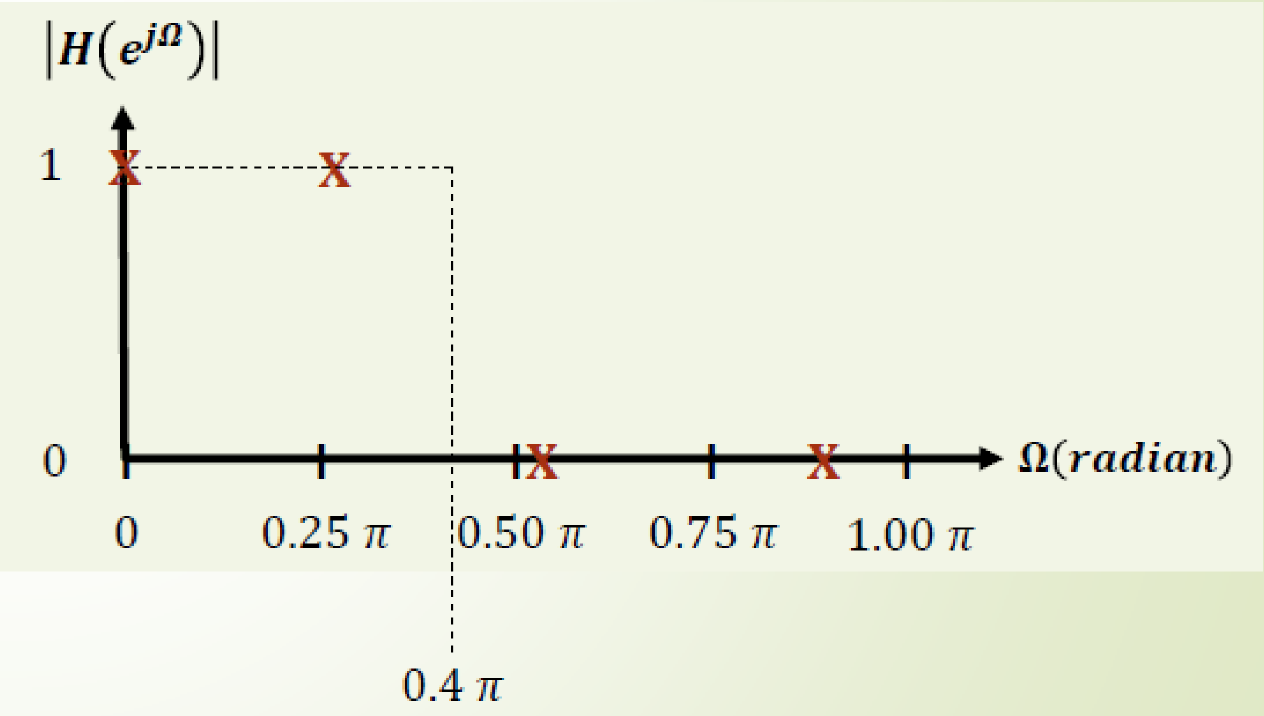
$$\Omega_k = \frac{2\pi}{7} k \qquad for \;\; k = 0, 1, 2, 3$$

$$\Omega_0 = 0 \implies H_0 = 1$$

$$\Omega_1 = \frac{2}{7}\pi \approx 0.28\,\pi \implies H_1 = 1$$

$$\Omega_2 = \frac{4}{7}\pi \approx 0.57\,\pi \implies H_2 = 0$$

$$\Omega_3 = \frac{6}{7}\pi \approx 0.85\,\pi \implies H_3 = 0$$



Finite Impulse Response

Exercise 2: Design a 7-tap FIR low-pass filter with a cut-off frequency of $\Omega_c = 0.4\pi$ radians using the frequency sampling method.

Solution

$M = 3$

$$H_0 = 1$$
$$H_1 = 1$$
$$H_2 = 0$$
$$H_3 = 0$$

$$h(n) = \frac{1}{2M+1}\left\{H_0 + 2\sum_{k=1}^{M} H_k \cos\left(\frac{2\pi k\,(n-M)}{2M+1}\right)\right\} \quad for\ \ n = 0, 1, \ldots, 2M$$

$$h(n) = \frac{1}{7}\left\{1 + 2\sum_{k=1}^{3} H_k \cos\left(\frac{2\pi k\,(n-3)}{7}\right)\right\} \quad for\ \ n = 0, 1, \ldots, 6$$

$$h(n) = \frac{1}{7}\left\{1 + 2\cos\left(\frac{2\pi\,(n-3)}{7}\right)\right\} \quad for\ \ n = 0, 1, \ldots, 6$$

$$h(0) = \frac{1}{7}\left\{1 + 2\cos\left(\frac{2\pi\,(0-3)}{7}\right)\right\} = -0.1145$$

$$h(1) = \frac{1}{7}\left\{1 + 2\cos\left(\frac{2\pi\,(1-3)}{7}\right)\right\} = 0.07927$$

$$h(2) = \frac{1}{7}\left\{1 + 2\cos\left(\frac{2\pi\,(2-3)}{7}\right)\right\} = 0.3209$$

$$h(3) = \frac{1}{7}\left\{1 + 2\cos\left(\frac{2\pi\,(3-3)}{7}\right)\right\} = 0.4285$$

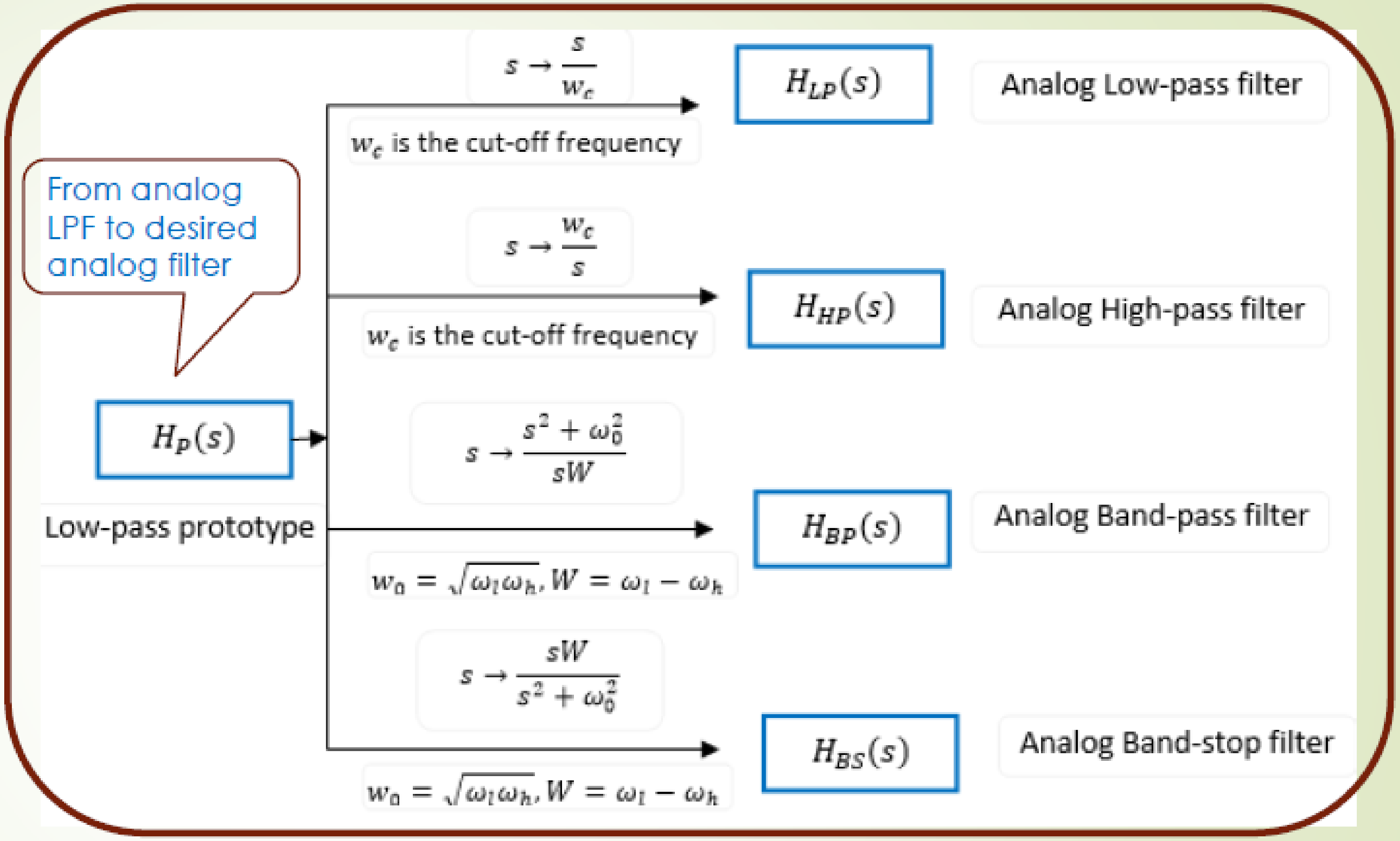
$$h(4) = h(2) = 0.3209$$

$$h(5) = h(1) = 0.07927$$

$$h(6) = h(0) = -0.1145$$

Infinite Impulse Response (IIR) Filter Design

IIR Filter



❖ Bilinear Method:

$$\omega_d = 2\pi f_c \qquad T = \frac{1}{f_s}$$

❖ Frequency Warping:

- *For LPF and HPF*

$$\omega_a = \frac{2}{T} \tan\left(\frac{\omega_d T}{2}\right)$$

- *For BPF and BSF*

$$\omega_{al} = \frac{2}{T} \tan\left(\frac{\omega_l T}{2}\right) \qquad \omega_{ah} = \frac{2}{T} \tan\left(\frac{\omega_h T}{2}\right)$$

❖ *BPF & BSF*

- $\omega_0 = \sqrt{\omega_{ah} . \omega_{al}}$
- $w = \omega_{ah} - \omega_{al}$

❖ Digital Filter Transfer Function:

$$H(z) = H(s)\Big|_{s=\frac{2}{T}\frac{z-1}{z+1}}$$

Exercise 1: Given an analog filter with the transfer function:

$$H(s) = \frac{1000}{s + 1000}$$

convert it to the digital filter ***transfer function and difference equation*** using the BLT if the DSP system has a sampling period of $T = 0.001\ second.$

IIR Filter

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Exercise 1: Given an analog filter with the transfer function:

$$\boldsymbol{H(s) = \frac{1000}{s+1000}}$$

convert it to the digital filter *transfer function and difference equation* using the BLT if the DSP system has a sampling period of $T = 0.001\ second.$

Solution

$$H(z) = H(s)\Big|_{s=\frac{2}{T}\frac{z-1}{z+1}}$$

$\frac{2}{T} \rightarrow 2000$

$$H(z) = \frac{1000}{2000\left(\frac{z-1}{z+1}\right) + 1000} * \frac{z+1}{z+1}$$

$$H(z) = \frac{1000\,(z+1)}{2000\,(z-1) + 1000\,(z+1)}$$

$$H(z) = \frac{1000\,z + 1000}{2000\,z - 2000 + 1000\,z + 1000}$$

$$H(z) = \frac{1000\,z + 1000}{3000\,z - 1000}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + \cdots + b_M z^{-M}}{1 + a_1 z^{-1} + \cdots + a_N z^{-N}}$$

$$H(z) = \frac{1000\,z + 1000}{3000\,z - 1000} * \frac{z^{-1}}{z^{-1}}$$

$$H(z) = \frac{1000 + 1000\,z^{-1}}{3000 - 1000\,z^{-1}} * \frac{\frac{1}{3000}}{\frac{1}{3000}}$$

$$H(z) = \frac{0.3333 + 0.3333\,z^{-1}}{1 - 0.3333\,z^{-1}}$$

IIR Filter

Exercise 1: Given an analog filter with the transfer function:

$$\boldsymbol{H(s) = \frac{1000}{s + 1000}}$$

convert it to the digital filter *<u>transfer function and difference equation</u>* using the BLT if the DSP system has a sampling period of $T = 0.001\ second$.

Solution

$$H(z) = \frac{0.3333 + 0.3333\, z^{-1}}{1 - 0.3333\, z^{-1}} = \frac{Y(z)}{X(z)}$$

$$Y(Z) - 0.3333\, z^{-1}\, Y(z) = 0.3333\, X(z) + 0.3333\, z^{-1}\, X(z)$$

$$y(n) - 0.3333\, y(n-1) = 0.3333\, x(n) + 0.3333\, x(n-1)$$

$$y(n) = 0.3333\, x(n) + 0.3333\, x(n-1) + 0.3333\, y(n-1)$$

Exercise 2: The **low pass filter** with a cutoff frequency of 1 rad/sec is given as

$$H_p(s) = \frac{1}{s+1}$$

Use $H_p(s)$ and the **BLT** to obtain a corresponding **IIR digital low pass filter** with a **cutoff frequency of 30 Hz**, assuming a **sampling rate of 200 Hz**.

IIR Filter

Exercise 2: The **low pass filter** with a cutoff frequency of 1 rad/sec is given as

$$H_p(s) = \frac{1}{s+1}$$

Use $H_p(s)$ and the **BLT** to obtain a corresponding **IIR digital low pass filter** with a **cutoff frequency of 30 Hz**, assuming a **sampling rate of 200 Hz**.

Solution $f_c = 30\ Hz$ $f_s = 200\ Hz$

$$\omega_d = 2\pi\,(30) = 60\,\pi$$

$$T = \frac{1}{f_s} = \frac{1}{200}\ seconds$$

$$\omega_a = \frac{2}{T}\tan\left(\frac{\omega_d\,T}{2}\right) = \frac{2}{\frac{1}{200}}\tan\left(\frac{60\pi\left(\frac{1}{200}\right)}{2}\right)$$

$\pi \rightarrow 180$

$$\omega_a = 203.81$$

❖ *LPF*

$$H(s) = H_p(s)\Big|_{s=\frac{s}{\omega_a}}$$

$$H(s) = \frac{1}{\frac{s}{\omega_a}+1} = \frac{\omega_a}{s\ +\ \omega_a}$$

$$H(s) = \frac{203.81}{s + 203.81}$$

$$H(z) = H(s)\Big|_{s=\frac{2}{T}\frac{z-1}{z+1}}$$

$\frac{2}{T} \rightarrow 400$

$$H(z) = \frac{203.81}{400\left(\frac{z-1}{z+1}\right) + 203.81} * \frac{z+1}{z+1}$$

$$H(z) = \frac{203.81\,(z+1)}{400\,(z-1) + 203.81\,(z+1)}$$

IIR Filter

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Exercise 2: The **low pass filter** with a cutoff frequency of 1 rad/sec is given as

$$H_p(s) = \frac{1}{s+1}$$

Use $H_p(s)$ and the **BLT** to obtain a corresponding **IIR digital low pass filter** with a **cutoff frequency of 30 Hz**, assuming a **sampling rate of 200 Hz**.

Solution

$$H(z) = \frac{203.81\,z + 203.81}{400\,z - 400 + 203.81\,z + 203.81}$$

$$H(z) = \frac{203.81\,z + 203.81}{603.81\,z - 196.19}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + \cdots + b_M z^{-M}}{1 + a_1 z^{-1} + \cdots + a_N z^{-N}}$$

$$H(z) = \frac{203.81\,z + 203.81}{603.81\,z - 196.19} * \frac{z^{-1}}{z^{-1}}$$

$$H(z) = \frac{203.81 + 203.81\,z^{-1}}{603.81 - 196.19\,z^{-1}} * \frac{\frac{1}{603.81}}{\frac{1}{603.81}}$$

$$H(z) = \frac{0.3375 + 0.3375\,z^{-1}}{1 - 0.3249\,z^{-1}}$$

$$y(n) - 0.3249\,y(n-1) = 0.3375\,x(n) + 0.3375\,x(n-1)$$

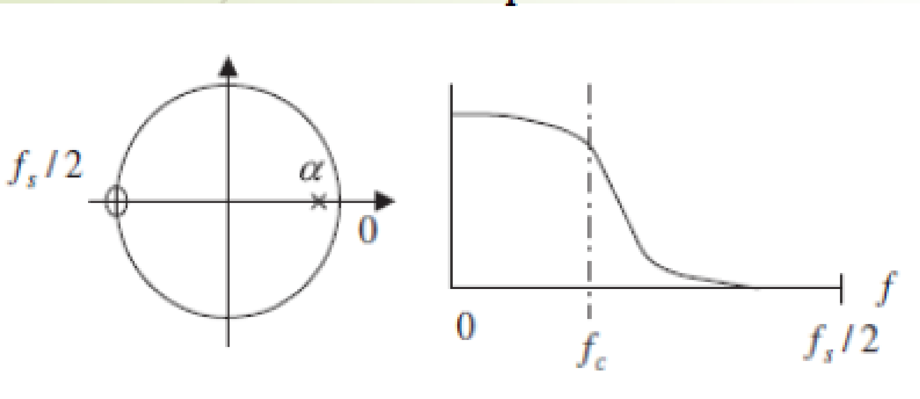
$$y(n) = 0.3375\,x(n) + 0.3375\,x(n-1) + 0.3249\,y(n-1)$$

IIR Filter

❖ Pole-Zero Placement Method:

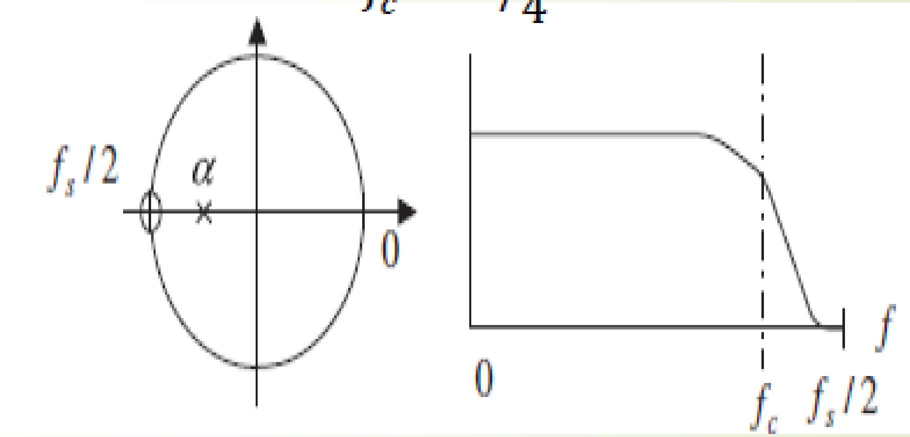
- **First-Order LPF**

$f_c < {f_s}/{4}$



$\alpha \approx 1 - 2 \times (f_c/f_s) \times \pi, \quad \text{good for} \quad 0.9 \le r < 1$

$f_c > {f_s}/{4}$



$\alpha \approx -(1 - \pi + 2 \times (f_c/f_s) \times \pi), \quad \text{good for} \quad -1 < r \le -0.9$

$$K = \frac{(1-\alpha)}{2}$$

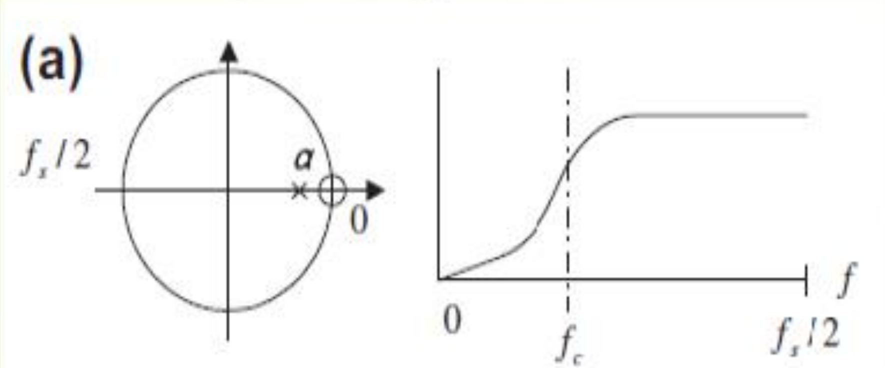
$$H(z) = \frac{K(z+1)}{(z-\alpha)}$$

IIR Filter

❖ Pole-Zero Placement Method:

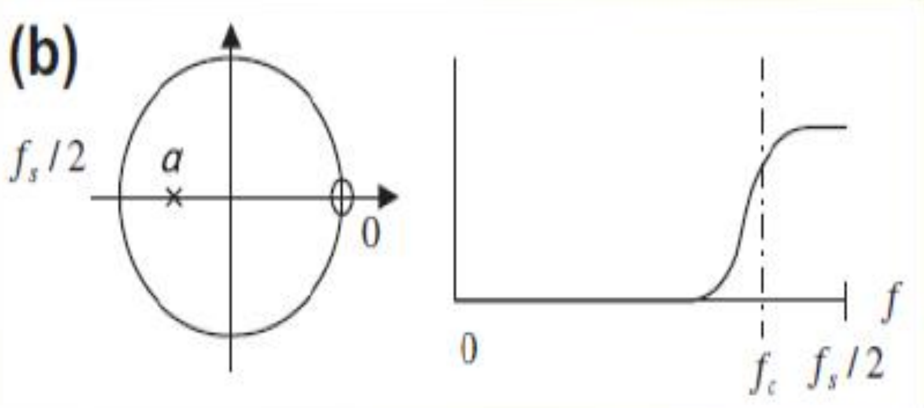
- **First-Order HPF**

$f_c < {f_s}/{4}$



$\alpha \approx 1 - 2 \times (f_c/f_s) \times \pi, \quad$ good for $\quad 0.9 \le r < 1$

$f_c > {f_s}/{4}$



$\alpha \approx -(1 - \pi + 2 \times (f_c/f_s) \times \pi), \quad$ good for $\quad -1 < r \le -0.9$

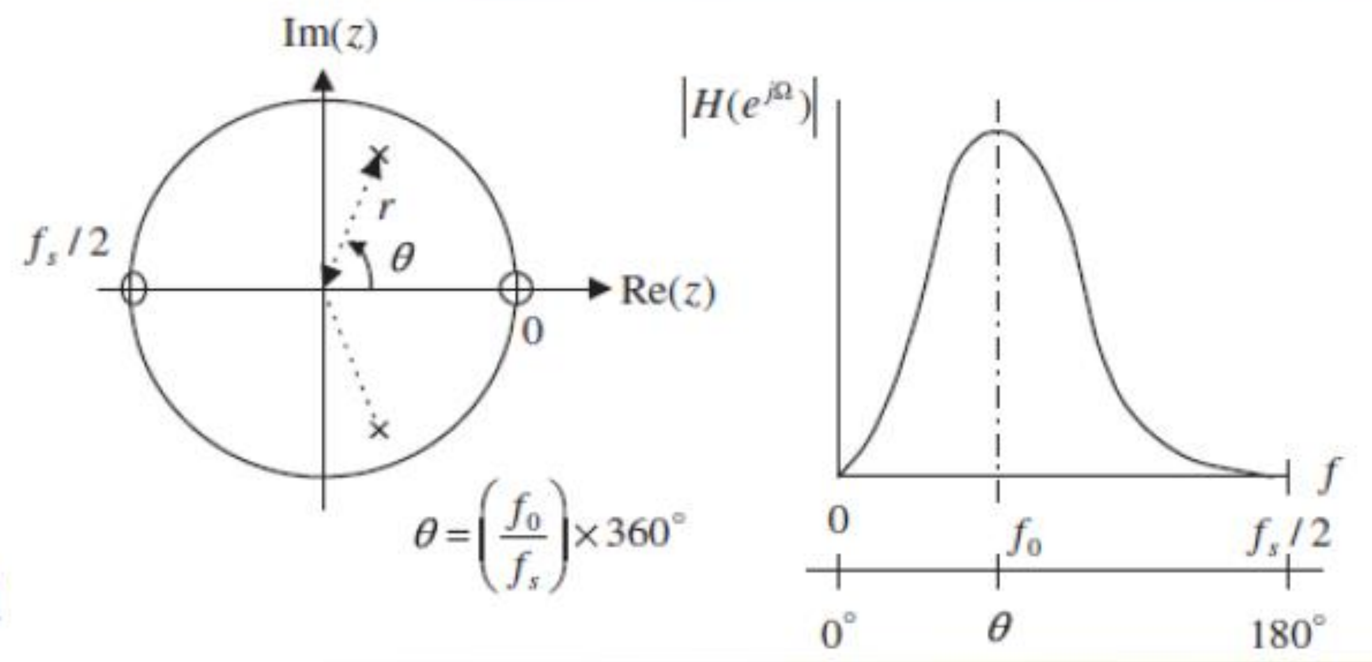
$$K = \frac{(1+\alpha)}{2}$$

$$H(z) = \frac{K(z-1)}{(z-\alpha)}$$

IIR Filter

❖ Pole-Zero Placement Method:

- **Second-Order BPF**



$$r \approx 1 - (BW_{3dB}/f_s) \times \pi, \text{ good for } 0.9 \le r < 1$$

$$\theta = \left(\frac{f_0}{f_s}\right) \times 360^\circ$$

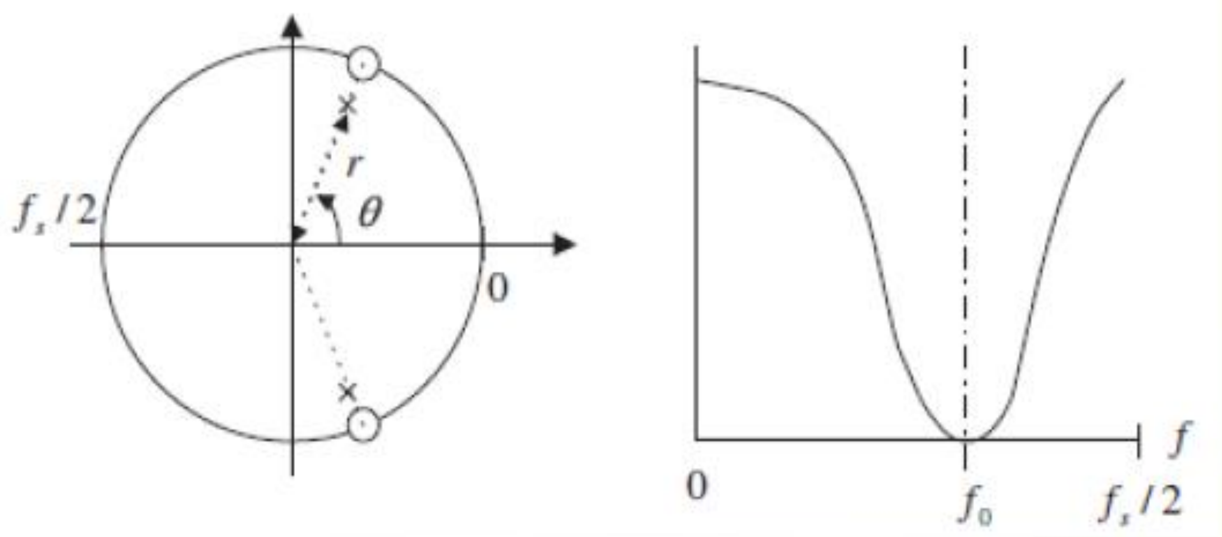
$$K = \frac{(1-r)\sqrt{1 - 2r\cos 2\theta + r^2}}{2|\sin\theta|}$$

$$H(z) = \frac{K(z-1)(z+1)}{(z - re^{j\theta})(z - re^{-j\theta})} = \frac{K(z^2-1)}{(z^2 - 2rz\cos\theta + r^2)}$$

IIR Filter

❖ Pole-Zero Placement Method:

- **Second-Order BSF (Notch)**



$$r \approx 1-(BW_{3dB}/f_s)\times\pi,\ \text{good for } 0.9 \le r < 1$$

$$\theta = \left(\frac{f_0}{f_s}\right) \times 360^\circ$$

$$K = \frac{(1-2r\cos\theta+r^2)}{(2-2\cos\theta)}$$

$$H(z) = \frac{K\left(z-e^{j\theta}\right)\left(z+e^{-j\theta}\right)}{\left(z-re^{j\theta}\right)\left(z-re^{-j\theta}\right)} = \frac{K\left(z^2-2z\cos\theta+1\right)}{\left(z^2-2rz\cos\theta+r^2\right)}$$

IIR Filter

Exercise 3: A **second-order bandpass filter** is required to satisfy the following specifications:

- Sampling rate $= 8,000\ Hz$
- $3\ dB$ bandwidth: $BW = 100\ Hz$
- Narrow passband centered at $f_0 = 2,000\ Hz$
- Zero gain at $0\ Hz$ and $4,000\ Hz$

Find the transfer function and difference equation by the pole-zero placement method.

- $r \approx 1 - \left(\frac{BW_{3dB}}{f_s}\right) \times \pi, \quad good\ for\ 0.9 \le r < 1$
- $\theta = \left(\frac{f_0}{f_s}\right) \times 360°$
- $K = \frac{(1-r)\sqrt{1-2\,r\cos(2\theta)+r^2}}{2|\sin(\theta)|}$

$$r \approx 1 - \left(\frac{100}{8000}\right) \times \pi = 0.9607$$

($\pi \rightarrow 3.14$)

$$\theta = \left(\frac{2000}{8000}\right) \times 360° = 90°$$

$$K = \frac{(1-0.9607)\sqrt{1-2(0.9607)\cos\left(2(90°)\right)+(0.9607)^2}}{2|\sin(90°)|} = 0.03853$$

IIR Filter

Exercise 3: A **second-order bandpass filter** is required to satisfy the following specifications:

- Sampling rate $= 8,000\ Hz$
- $3\ dB$ bandwidth: $BW = 100\ Hz$
- Narrow passband centered at $f_0 = 2,000\ Hz$
- Zero gain at $0\ Hz$ and $4,000\ Hz$

Find the transfer function and difference equation by the pole-zero placement method.

$$H(z) = \frac{K\,(z-1)(z+1)}{(z-re^{j\theta})(z-re^{-j\theta})} = \frac{K(z^2-1)}{(z^2-2rz\cos(\theta)+r^2)}$$

$r = 0.9607$

$\theta = 90°$

$K = 0.03853$

$$H(z) = \frac{(0.03853)(z^2-1)}{(z^2-2(0.9607)z\cos(90°)+(0.9607)^2)} = \frac{0.03853\,z^2 - 0.03853}{z^2 - 0.9229}$$

($\cos(90°) \rightarrow 0$)

IIR Filter

Exercise 3: A **second-order bandpass filter** is required to satisfy the following specifications:

- Sampling rate $= 8,000\ Hz$
- $3\ dB$ bandwidth: $BW = 100\ Hz$
- Narrow passband centered at $f_0 = 2,000\ Hz$
- Zero gain at $0\ Hz$ and $4,000\ Hz$

Find the **transfer function** and **difference equation** by the **pole-zero placement method.**

$Transfer\ Function$

$$H(z) = \frac{0.03853\, z^2 - 0.03853}{z^2 - 0.9229} * \left[\frac{z^{-2}}{z^{-2}}\right] \Longrightarrow H(z) = \frac{0.03853 - 0.03853\, z^{-2}}{1 - 0.9229\, z^{-2}}$$

$$y(n) - 0.9229\, y(n-2) = 0.03853\, x(n) - 0.03853\, x(n-2)$$

$$y(n) = 0.03853\, x(n) - 0.03853\, x(n-2) + 0.9229\, y(n-2)$$

$Difference\ Equation$