

١٠ برهان

الاختبار الشهري الثاني للمقرر 111 رياض للفصل الأول 1437-1438 هـ	كلية العلوم - قسم الرياضيات جامعة الملك سعود King Saud University
الزمن: ساعة ونصف. الدرجة:	الإسم: الرقم الجامعي: أستاذ المقرر:

ملاحظات: 1. عدد الورقات 4
2. ممنوع استخدام الآلة الحاسبة.

السؤال الأول (4 درجات): احسب $\frac{dy}{dx}$ فيما يلي:

(درجتان) $x > 0, y = \sinh\left(\frac{1}{x}\right) + \tanh(\sqrt{x})$ (1)

$$x > 0; \frac{dy}{dx} = \left(\cosh\left(\frac{1}{x}\right)\right)\left(-\frac{1}{x^2}\right) + \operatorname{sech}^2(\sqrt{x})\left(\frac{1}{2\sqrt{x}}\right)$$

(درجتان)

$x > 0, y = (\ln x) \cdot \sinh^{-1}(x)$ (2)

$$\frac{dy}{dx} = \left(\frac{1}{x}\right) \sinh^{-1}(x) + (\ln x) \cdot \left(\frac{1}{\sqrt{1+x^2}}\right)$$

السؤال الثاني (21 درجة): احسب التكاملات التالية:

(درجتان)

$$\int \frac{\tanh(\ln x)}{x} dx \quad (1)$$

$$du = \frac{dx}{x} \quad \text{فإن} \quad u = \ln x$$

$$\int \frac{\tanh(\ln x)}{x} dx = \int \tanh(u) du = \int \frac{\sinh(u)}{\cosh(u)} du$$
$$= \ln(\cosh u) + \text{const} = \ln(\cosh(\ln x)) + \text{const}$$

(درجتان)

$$\int \frac{x}{\sqrt{x^4-16}} dx \quad (2)$$

$$\int \frac{x}{\sqrt{x^4-16}} dx = \int \frac{x}{\sqrt{(x^2)^2-4^2}} dx$$

فان $u = x^2$ $du = 2x dx$ $2x dx = du$

$$\int \frac{x}{\sqrt{x^4-16}} dx = \frac{1}{2} \int \frac{du}{\sqrt{u^2-4^2}} = \frac{1}{2} \cosh^{-1}\left(\frac{u}{4}\right) + \text{const}$$

$$\int \frac{x}{\sqrt{x^4-16}} dx = \frac{1}{2} \cosh^{-1}\left(\frac{x^2}{4}\right) + \text{const}, \quad \text{const} \in \mathbb{R}$$
$$= \frac{1}{2} \ln \left| \frac{x^2 + \sqrt{x^4-16}}{4} \right| + \text{const}$$

(درجتان)

$$\int \ln(1+x^2) dx \quad (3)$$

با استفاده از روش جداسازی اجزای $u = \ln(1+x^2)$ $u' = \frac{2x}{1+x^2}$ $v' = 1$ $v = x$

$$\int \ln(1+x^2) dx = x \ln(1+x^2) - \int \frac{2x^2}{1+x^2} dx$$

$$\int \ln(1+x^2) dx = x \ln(1+x^2) - 2 \int \frac{x^2+1-1}{1+x^2} dx$$

$$\int \ln(1+x^2) dx = x \ln(1+x^2) - 2 \int \left[1 - \frac{1}{1+x^2} \right] dx$$

$$= x \ln(1+x^2) - 2(x - \tan^{-1}x) + \text{const}$$
$$= x \ln(1+x^2) - 2x + 2 \tan^{-1}x + \text{const}$$

(درجتان)

$$\int \cos^3 x \sin^2 x dx \quad (4)$$

$$\int \cos^3 x \sin^2 x dx = \int \cos^2 x \sin^2 x \cos x dx$$

$$= \int (1 - \sin^2 x) \sin^2 x \cos x dx$$

با استفاده از $u = \sin x$ $du = \cos x dx$

$$\int \cos^3 x \sin^2 x dx = \int (1-u^2) u^2 du = \int [u^2 - u^4] du$$

$$= \frac{u^3}{3} - \frac{u^5}{5} + \text{const} = \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + \text{const}$$

(درجات) $\sin a \sin b = \frac{1}{2} [\cos(a-b) - \cos(a+b)]$: مع العلم ان $\int \sin(7x) \sin(4x) dx$ (5)

بمع $b=4x$ و $a=7x$ فان

$$\sin(7x) \sin(4x) = \frac{1}{2} [\cos(3x) - \cos(11x)]$$

بذناطر طرفي المتكافئة

$$\int \sin(7x) \sin(4x) dx = \frac{1}{2} \int [\cos 3x - \cos(11x)] dx$$

$$= \frac{1}{2} \left[\frac{\sin(3x)}{3} - \frac{\sin(11x)}{11} \right] + \text{const}$$

(3 درجات)

$$\int x^3 \sqrt{x^2+1} dx \quad (6)$$

طريقة الاولى: $x^2 = u - 1 \Leftrightarrow u = x^2 + 1$
 $du = 2x dx$

$$\int x^3 \sqrt{x^2+1} dx = \int x^2 \cdot x \sqrt{x^2+1} dx$$

$$= \frac{1}{2} \int (u-1) \sqrt{u} du$$

$$= \frac{1}{2} \int [u^{3/2} - u^{1/2}] du$$

$$= \frac{1}{2} \left[\frac{2}{5} u^{5/2} - \frac{2}{3} u^{3/2} \right] + \text{const}$$

$$= \frac{1}{5} (x^2+1)^{5/2} - \frac{1}{3} (x^2+1)^{3/2} + \text{const}$$

طريقة ثانية: بضع $x = \tan \theta$ فان $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$
 $dx = \sec^2 \theta d\theta$
 $\sqrt{x^2+1} = \sqrt{1+\tan^2 \theta} = \sqrt{\sec^2 \theta} = \sec \theta$

$$\int x^3 \sqrt{x^2+1} dx = \int \tan^3 \theta \sec^3 \theta d\theta$$

$$\int x^3 \sqrt{x^2+1} dx = \int \tan^2 \theta \sec^2 \theta \sec \theta \tan \theta d\theta$$

$$\int x^3 \sqrt{x^2+1} dx = \int (\sec^2 \theta - 1) \sec^2 \theta \sec \theta \tan \theta d\theta$$

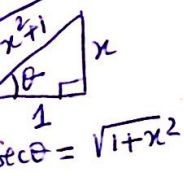
$v = \sec \theta$

$$= \int (v^2 - 1) v^2 dv$$

$$= \int (v^4 - v^2) dv = \frac{v^5}{5} - \frac{v^3}{3} + \text{const}$$

$$= \frac{\sec^5 \theta}{5} - \frac{\sec^3 \theta}{3} + \text{const} = \frac{(x^2+1)^{5/2}}{5} - \frac{(x^2+1)^{3/2}}{3} + \text{const}$$

(3 درجات)



$$\int \sqrt{x^2-4x} dx \quad (7)$$

اولا: يكمل المربع: $x^2 - 4x = (x-2)^2 - 4$

$$\int \sqrt{x^2-4x} dx = \int \sqrt{(x-2)^2 - 2^2} dx$$

بضع $x-2 = 2 \sec \theta$ فان $dx = 2 \sec \theta \tan \theta d\theta$

$$\sqrt{(x-2)^2 - 2^2} = \sqrt{(2 \sec \theta)^2 - 2^2} = 2 \tan \theta$$

$$\int \sqrt{x^2-4x} dx = 4 \int \sec \theta \tan^2 \theta d\theta$$

$$\int \sqrt{x^2-4x} dx = 4 \int \sec \theta (\sec^2 \theta - 1) d\theta$$

$$= 4 \int (\sec^3 \theta - \sec \theta) d\theta$$

$$= 2 \sec \theta \tan \theta - 2 \ln |\sec \theta + \tan \theta| + \text{const}$$

$$\int \sqrt{x^2-4x} dx = 2 \left(\frac{x-2}{2} \right) \frac{\sqrt{x^2-4x}}{2} - 2 \ln \left| \frac{x-2}{2} + \frac{\sqrt{x^2-4x}}{2} \right| + \text{const}$$

$$\int \sqrt{x^2-4x} dx = \frac{1}{2} (x-2) \sqrt{x^2-4x} - 2 \ln \left| \frac{(x-2) + \sqrt{x^2-4x}}{2} \right| + \text{const}$$

$$I = \int \sec^3 \theta d\theta = \int \sec \theta \sec^2 \theta d\theta$$

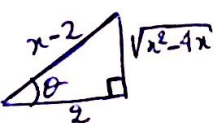
$u = \sec \theta \Rightarrow u' = \sec \theta \tan \theta$
 $v = \tan \theta$

$$= \sec \theta \tan \theta - \int \sec \theta \tan^2 \theta d\theta$$

$$= \sec \theta \tan \theta - \int \sec \theta (\sec^2 \theta - 1) d\theta$$

$$= \sec \theta \tan \theta - I + \ln |\sec \theta + \tan \theta|$$

$$= \frac{1}{2} [\sec \theta \tan \theta + \ln |\sec \theta + \tan \theta|] + \text{const}$$



(3 درجات)

(تكامل بالة كسرية) $\int \frac{x-2}{x^3+x} dx$ (8)

$$x \neq 0, f(x) = \frac{x-2}{x^3+x} = \frac{x-2}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

حيث A, B, C ثوابت حقيقية.

$$A = \lim_{x \rightarrow 0} x f(x) = \lim_{x \rightarrow 0} \frac{x-2}{x^2+1} = -2, \boxed{A=-2}$$

$$0 = \lim_{x \rightarrow \infty} x f(x) = A+B \Rightarrow \boxed{B=2}$$

$$f(1) = \frac{-1}{2} = -2 + \frac{2+C}{2} = \frac{C-2}{2} \Rightarrow \boxed{C=1}$$

$$\int \frac{x-2}{x^3+x} dx = -2 \int \frac{1}{x} dx + \int \frac{2x+1}{x^2+1} dx$$

$$\int \frac{x-2}{x^3+x} dx = -2 \ln|x| + \int \frac{2x}{x^2+1} dx + \int \frac{1}{x^2+1} dx$$

$$\int \frac{x-2}{x^3+x} dx = -2 \ln|x| + \ln(x^2+1) + \tan^{-1}(x) + cst$$

(درجتان)

$\int \frac{dx}{x^{1/2}+x^{1/6}}$ (9)

نضع $u = x^{1/6}$ و بالتالي $u^6 = x$ فان $6u^5 du = dx$

$$x^{1/2} = (u^6)^{1/2} = u^3$$

$$\int \frac{dx}{x^{1/2}+x^{1/6}} = \int \frac{6u^5}{u^3+u} du = 6 \int \frac{u^5}{u(u^2+1)} du$$

$$= 6 \int \frac{u^4}{u^2+1} du. \text{ (تكامل دالة كسرية)}$$

$$\begin{array}{r} u^4 \mid u^2+1 \\ -u^4-u^2 \mid u^2-1 \\ \hline -u^2 \\ u^2+1 \\ \hline 1 \end{array}$$

$$u \in \mathbb{R} \int \frac{u^4}{u^2+1} = u^2-1 + \frac{1}{1+u^2}$$

$$\int \frac{u^4}{u^2+1} du = \int \left[u^2-1 + \frac{1}{1+u^2} \right] du$$

$$= \frac{u^3}{3} - u + \tan^{-1}(u) + cst$$

$$\int \frac{dx}{x^{1/2}+x^{1/6}} = 2\sqrt{x} - 6x^{1/6} + 6\tan^{-1}(x^{1/6}) + c$$