Frequently Used Continouse Probability Distributions

Q1) Find the moment generating function for the general normal distribution.

Q2) Show that the moment generating function of the random variable X which is Chi square distribution with v degree of freedom is $M(t) = (1 - 2t)^{-v/2}$.

Q3) If X_1 and X_2 be independent r.v. that are chi-square dis. with v_1 and v_2 degrees of freedom, respectively.
   a. Show that the moment generating function of the random variable $Z = X_1 + X_2$ is $M(t) = (1 - 2t)^{-(v1+v2)/2}$
   b. What you can say about the distribution of the random variable Z.

Q4) Show that the mean and variance of gamma distribution are given by
   (a) $\mu = \alpha\beta$
   (b) $\sigma^2 = \alpha\beta^2$.

Q5) The graph of chi-square distribution with 5 degrees of freedom is shown below. Find the values of x_1^2, x_2^2 for which
   a. The shaded area on the right $= 0.05$,
   b. The total shaded area $= 0.05$,
   c. The shaded area on the left $= 0.10$,
   d. The shaded area on the right $= 0.01$.
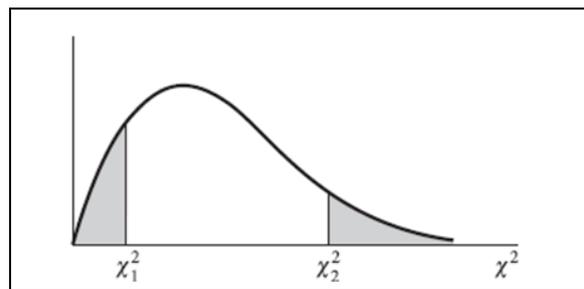
Q6) The graph of t- distribution with 9 degrees of freedom is shown below. Find the values of t_1, t_2 for which
   a. The shaded area on the right $= 0.05$,
   b. The total shaded area $= 0.05$,
   c. The total unshaded area $= 0.99$,
   d. The shaded area on the left $= 0.01$,
   e. The area on the left of $t_1 = 0.90$.
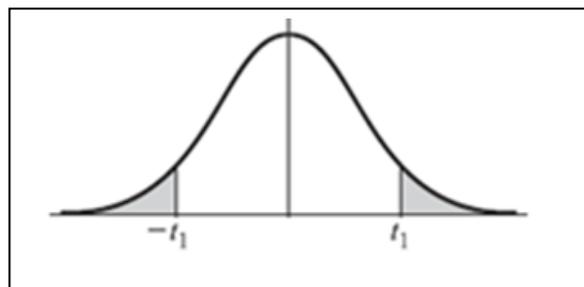
Q7) Let X be an exponential random variable with parameter $\theta = ln(3)$. Compute the following probability: $P(2 \le X \le 4)$.

Q8) Suppose the random variable X has an exponential distribution with parameter $\theta = 1$.
   (a) Find $F(x)$.
   (b) Using $F(x)$, compute $P(X > 2)$.

Q9) What is the probability that a random variable X   is less than its expected value, if X has an exponential distribution with parameter θ ?

Q10) Identify the distribution of the r.v. from the moment generating function:

(a) $M_x(t) = \frac{1}{1-2t}, t < 1/2$.

(b) $M_x(t) = e^{3t+2t^2}$.

(c) X, Y independent, $M_{X+Y}(t) = \left(\frac{2}{2-t}\right)^3, t < \frac{1}{2}, Y \sim Exp(1/2)$.

Q11) X, Y independent, $M_{X+Y}(t) = \frac{e^{2t}-1}{2t-t^2}$, $X \sim Exp(1/2)$, what is the distribution of Y?