

i. JOINT, MARGINAL, AND CONDITIONAL DISTRIBUTIONS**ii. COVARIANCE, CORRELATION, INDEPENDENCE OF VARIABLES (STOCHASTIC INDEPENDENCE)**

- Q1)** The joint probability function of two discrete random variables X and Y is given by $f(x, y) = cxy$ for $x = 1, 2, 3$ and $y = 1, 2, 3$ and equals zero otherwise. Find:
- The constant c .
 - $P(X = 2, Y = 3)$.
 - $P(1 \leq X \leq 2, Y \leq 2)$.
 - $P(X \geq 2)$.
 - $P(Y < 2)$.
 - $P(X = 1)$.
 - $P(Y = 3)$.

- Q2)** For the random variables of Problem 1, find the marginal probability function of X and Y . Determine whether X and Y are independent.

- Q3)** Let X and Y be continuous random variables having joint density function

$$f(x, y) = \begin{cases} c(x^2 + y^2) & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Determine:

- The constant c .
 - $P\left(X < \frac{1}{2}, Y > \frac{1}{2}\right)$.
 - $P\left(\frac{1}{4} < X < \frac{3}{4}\right)$.
 - $P\left(Y < \frac{1}{2}\right)$.
 - Whether X and Y are independent.
- Q4)** For the random variables of Problem 3, find the marginal probability function of X and Y .
- Q5)** For the distribution of Problem 1, find the conditional probability function of X given Y , Y given X .
- Q6)** Let $f(x, y) = \begin{cases} x + y & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$
Find the conditional probability function of X given Y , Y given X .

- Q7)** For the distribution of Problem 3, find the conditional probability function of X given Y , Y given X .

- Q8)** Let $f(x, y) = \begin{cases} e^{-(x+y)} & x \geq 0, y \geq 0 \\ 0 & \text{otherwise} \end{cases}$

be the joint density function of X and Y . Find the conditional probability function of X given Y , Y given X .

Q9) Let X and Y be random variables having joint density function

$$f(x, y) = \begin{cases} c(2x + y) & 0 < x < 1, 0 < y < 2 \\ 0 & \text{otherwise} \end{cases}$$

Find

- The constant c .
- $P\left(X > \frac{1}{2}, Y < \frac{3}{2}\right)$.
- The (marginal) density function of X .
- The (marginal) density function of Y .

Q10) The joint probability function for the random variables X and Y is given in following table, then find

X \ Y	0	1	2
0	1/18	1/9	1/6
1	1/9	1/18	1/9
2	1/6	1/6	1/18

- The marginal probability functions of X and Y .
- $P(1 \leq X < 3, Y \geq 1)$.
- Determine whether X and Y are independent.

Q11) Let X and Y be random variables having joint density function

$$f(x, y) = \begin{cases} x + y & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find:

- $Var(X)$.
- $Var(Y)$.
- σ_X .
- σ_Y .
- $\sigma_{X,Y} = Cov(X, Y)$.
- ρ .

Q12) Work Problem 11 if the joint density function is

$$f(x, y) = \begin{cases} e^{-(x+y)} & x \geq 0, y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Q13) Find

- The covariance.
 - The correlation coefficient of two random variables X and Y .
- If $E(X) = 2, E(Y) = 3, E(XY) = 10, E(X^2) = 9, E(Y^2) = 16$.

Q14) The correlation coefficient of two random variables X and Y is $-1/4$ while their variances are 3 and 5. Find the covariance.

Q15) Let $f(x, y) = \begin{cases} \frac{xy}{36} & x = 1, 2, 3 \text{ and } y = 1, 2, 3 \\ 0 & \text{otherwise} \end{cases}$

be the joint density function of X and Y . Find the conditional probability function of X given Y , Y given X .

Q16) The joint probability function of two discrete random variables X and Y is given by $f(x, y) = c(2x + y)$, where x and y can assume all integers such that $0 \leq x \leq 2$, $0 \leq y \leq 3$, and $f(x, y) = 0$ otherwise.

Find:

- The value of the constant c .
- $P(X = 2, Y = 1)$.
- $P(X \geq 1, Y \leq 2)$

Q17) For the Problem 16, find:

- $E(X)$.
- $E(Y)$.
- $E(XY)$.
- $E(X^2)$.
- $E(Y^2)$.
- $Var(X)$.
- $Var(Y)$.
- $Cov(X, Y)$.
- ρ .

Q18) The joint density function of X and Y is given by

$$f(x, y) = \begin{cases} 8xy & 0 \leq x \leq 1, 0 \leq y \leq x \\ 0 & \text{otherwise} \end{cases}$$

Find:

- The marginal density of X .
- The marginal density of Y .
- The conditional density of X .
- The conditional density of Y .

Q19) Find the conditional expectation of X given Y and Y given X in Problem 18.

Q20) Find the conditional variance of Y given X for Problem 18.