Some Discrete Probability Distributions

DISCRETE UNIFORM DISTRIBUTION

Q1. Let the random variable X have a discrete uniform with parameter *k*=3 and with values 0,1, and 2. Then:

$$f(x) = \frac{1}{3} \; ; x = 0,1,2$$

(a) P(X=1) is

(A) 1.0 (B) 1/3 (C) 0.3 (D) 0.1 (E) None

(b) The mean of X is:

(A) 1.0 (B) 2.0 (C) 1.5 (D) 0.0 (E) None

$$\mu = \frac{\sum x_i}{k} = \frac{0+1+2}{3} = 1$$

(c) The variance of X is:

(A) 0/3=0.0 (B) 3/3=1.0 (C) 2/3=0.67 (D) 4/3=1.33 (E) None

$$\sigma^2 = \frac{\sum(x_i-\mu)^2}{k} = \frac{(0-1)^2+(1-1)^2+(2-1)^2}{3} = \frac{2}{3}$$

BINOMIAL DISTRIBUTION

Q1. Suppose that 4 out of 12 buildings in a certain city violate the building code (تنتهك حقوق البناء). A building engineer randomly inspects a sample of 3 new buildings in the city.

$$p = \frac{4}{12} = \frac{1}{3},\ n = 3$$

(a) Find the probability distribution function of the random variable X representing the number of buildings that violate the building code in the sample.

$$f(x) = \binom{3}{x}\left(\frac{1}{3}\right)^{x}\left(\frac{2}{3}\right)^{3-x};\ x = 0,1,2,3$$

(b) Find the probability that:

(i) none of the buildings in the sample violating the building code. $f(0) = \binom{3}{0}\left(\frac{1}{3}\right)^{0}\left(\frac{2}{3}\right)^{3} = 0.2963$

(ii) one building in the sample violating the building code. $f(1) = \binom{3}{1}\left(\frac{1}{3}\right)^{1}\left(\frac{2}{3}\right)^{2} = 0.4444$

(iii) at lease one building in the sample violating the building code. $P(X \geq 1) = 1 - f(0) = 1 - 0.2963 = 0.7037$

(c) Find the expected number of buildings in the sample that violate the building code (E(X)).

$E(X) = np = \frac{3}{3} = 1$

(d) Find $\sigma^2 = Var(X)$. $\sigma^2 = Var(X) = npq = 3\,\frac{1}{3}\frac{2}{3} = \frac{2}{3} = 0.6667$

Q4. Suppose that the percentage of females in a certain population is 50%. A sample of 3 people is selected randomly from this population. $p = 0.5\,,\ \ n = 3$

$f(x) = \binom{3}{x}\left(\frac{1}{2}\right)^{x}\left(\frac{1}{2}\right)^{3-x}; x = 0,1,2,3$

(a) The probability that no females are selected is $f(0) = \binom{3}{0}\left(\frac{1}{2}\right)^{3}$

(A) 0.000 (B) 0.500 (C) 0.375 (D) 0.125

(b) The probability that at most two females are selected is $P(X \leq 2) = 1 - f(3) = 1 - 0.125$

(A) 0.000 (B) 0.500 (C) 0.875 (D) 0.125

(c) The expected number of females in the sample is $E(X) = np = \frac{3}{2} = 1.5$

(A) 3.0 (B) 1.5 (C) 0.0 (D) 0.50

(d) The variance of the number of females in the sample is $\sigma^2 = Var(X) = npq = 3\,\frac{1}{2}\frac{1}{2} = 0.75$

(A) 3.75 (B) 2.75 (C) 1.75 (D) 0.75

Q7. From a box containing 4 black balls and 2 green balls, 3 balls are drawn independently in succession, each ball being replaced in the box before the next draw is made. The probability of drawing 2 green balls and 1 black ball is: (With replacement) $f(x) = \binom{3}{x}\left(\frac{2}{6}\right)^{x}\left(\frac{4}{6}\right)^{3-x}$; $x = 0,1,2,3$

X=number of a green balls.

$$f(2) = \binom{3}{2}\left(\frac{2}{6}\right)^{2}\left(\frac{4}{6}\right)^{1}$$

(A) 6/27 (B) 2/27 (C) 12/27 (D) 4/27

Q9. If X~Binomial(n,p), E(X)=1, and Var(X)=0.75, find P(X=1).

$E(X) = np = 1$

$Var(X) = npq = 0.75$

$q = 0.75 \Longrightarrow p = 0.25 \Longrightarrow n = 4$

$f(x) = \binom{4}{1}\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^3 = 0.4219$

H.W: Q2, Q3, Q5, Q6, Q8, Q11

Q10 Deleted

HYPERGEOMETRIC DISTRIBUTION

Q2. Suppose that a family has 5 children, 3 of them are girls and the rest are boys. A sample of 2 children is selected randomly and without replacement.

X=number of girls., k=3, N=5, n=2

$f(x) = \frac{{}_3C_x \; {}_2C_{2-x}}{{}_5C_2}; x = 0,1,2$

(a) The probability that no girls are selected is $f(0) = \frac{{}_3C_0 \; {}_2C_2}{{}_5C_2}$

(A) 0.0 (B) 0.3 (C) 0.6 (D) 0.1

(b) The probability that at most one girls are selected is $P(X \leq 1) = f(0) + f(1)$

$= \frac{{}_3C_0 \; {}_2C_2}{{}_5C_2} + \frac{{}_3C_1 \; {}_2C_1}{{}_5C_2}$

(A) 0.7 (B) 0.3 (C) 0.6 (D) 0.1

(c) The expected number of girls in the sample is $E(X) = \frac{n*k}{N} = \frac{2*3}{5}$

(A) 2.2 (B) 1.2 (C) 0.2 (D) 3.2

(d) The variance of the number of girls in the sample is

$$V(X) = \frac{n*k(N-k)(N-n)}{N^2(N-1)} = \frac{2*3(5-3)(5-2)}{5^2(5-1)}$$

(A) 36.0 (B) 3.6 (C) 0.36 (D) 0.63

Q5. From a lot of 8 missiles, 3 are selected at random and fired. The lot contains 2 defective missiles that will not fire. Let X be a random variable giving the number of defective missiles selected.

X= number of defective missiles, k=2, N=8, n=3

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1. Find the probability distribution function of X.

$$f(x) = \frac{{}_2C_x \, {}_6C_{3-x}}{{}_8C_3}; x = 0,1,2 = \min(k, n)$$

2. What is the probability that at most one missile will not fire?

$$P(X \leq 1) = f(0) + f(1) = \frac{{}_2C_0 \, {}_6C_3}{{}_8C_3} + \frac{{}_2C_1 \, {}_6C_2}{{}_8C_3} = 0.8928$$

3. Find E(X) and Var(X).

$$E(X) = \frac{n*k}{N} = \frac{3*2}{8} = 0.75$$

$$V(X) = n\frac{k}{N}\left(1 - \frac{k}{N}\right)\frac{N-n}{N-1} = 3\frac{2}{8}\left(1 - \frac{2}{8}\right)\frac{8-3}{8-1} = 0.4018$$

H.W: Q4

POISSON DISTRIBUTION

Q8. The number of faults in a fiber optic cable follows a Poisson distribution with an average of 0.6 per 100 feet.

$f(x) = \frac{(0.6t)^x}{x!} e^{-0.6t}; x = 0,1,2, \ldots$

(1) The probability of 2 faults per 100 feet of such cable is: $f(2) = \frac{(0.6)^2}{2!} e^{-0.6}$

(A) 0.0988 (B) 0.9012 (C) 0.3210 (D) 0.5

(2) The probability of less than 2 faults per 100 feet of such cable is:

$P(X < 2) = P(X \leq 1) = \sum_{x=0}^{1} p(x, 0.6) = \frac{(0.6)^1}{1!} e^{-0.6} + \frac{(0.6)^0}{0!} e^{-0.6}$

(A) 0.2351 (B) 0.9769 (C) 0.8781 (D) 0.8601

(3) The probability of 4 faults per 200 feet of such cable is: $f(4) = \frac{(0.6*2)^4}{4!} e^{-0.6*2}$

(A) 0.02602 (B) 0.1976 (C) 0.8024 (D) 0.9739

Table A.2 Poisson Probability Sums $\sum_{x=0}^{r} p(x;\mu)$

	μ								
r	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
0	0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493	0.4066
1	0.9953	0.9825	0.9631	0.9384	0.9098	0.8781	0.8442	0.8088	0.7725
2	0.9998	0.9989	0.9964	0.9921	0.9856	0.9769	0.9659	0.9526	0.9371
3	1.0000	0.9999	0.9997	0.9992	0.9982	0.9966	0.9942	0.9909	0.9865
4		1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9986	0.9977
5				1.0000	1.0000	1.0000	0.9999	0.9998	0.9997
6							1.0000	1.0000	1.0000

	μ								
r	**1.0**	**1.5**	**2.0**	**2.5**	**3.0**	**3.5**	**4.0**	**4.5**	**5.0**
0	0.3679	0.2231	0.1353	0.0821	0.0498	0.0302	0.0183	0.0111	0.0067
1	0.7358	0.5578	0.4060	0.2873	0.1991	0.1359	0.0916	0.0611	0.0404
2	0.9197	0.8088	0.6767	0.5438	0.4232	0.3208	0.2381	0.1736	0.1247
3	0.9810	0.9344	0.8571	0.7576	0.6472	0.5366	0.4335	0.3423	0.2650
4	0.9963	0.9814	0.9473	0.8912	0.8153	0.7254	0.6288	0.5321	0.4405
5	0.9994	0.9955	0.9834	0.9580	0.9161	0.8576	0.7851	0.7029	0.6160
6	0.9999	0.9991	0.9955	0.9858	0.9665	0.9347	0.8893	0.8311	0.7622
7	1.0000	0.9998	0.9989	0.9958	0.9881	0.9733	0.9489	0.9134	0.8666
8		1.0000	0.9998	0.9989	0.9962	0.9901	0.9786	0.9597	0.9319
9			1.0000	0.9997	0.9989	0.9967	0.9919	0.9829	0.9682
10				0.9999	0.9997	0.9990	0.9972	0.9933	0.9863
11				1.0000	0.9999	0.9997	0.9991	0.9976	0.9945
12					1.0000	0.9999	0.9997	0.9992	0.9980
13						1.0000	0.9999	0.9997	0.9993
14							1.0000	0.9999	0.9998
15								1.0000	0.9999
16									1.0000