Solving Portfolio Optimization Problem by Clonal Selection Algorithm

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Abstract. Portfolio is a group of assets held by an institution or a private individual. Each asset has an investment share of the total investment. Therefore the investor tries to distribute the investment to these different assets. The main issue in portfolio is how to allocate them in way to reach the maximum return and minimum risk. These two objectives lead to multi objective portfolio optimization problem that must be solved. There are a lot of studies related to this issue. In this paper, we will use intelligence computational technique named Clonal Selection to optimize portfolio's return and risk. Then, we will show the results of the proposed solution through experiments are conducted using the stocks in Kingdom of Saudi Arabia stock exchange market (Tadawul).

Keywords: Affinity; Portfolio Optimization; Clonal Selection.

1 Introduction

Portfolio is "A grouping of financial assets such as stocks, funds or bonds and cash equivalents, as well as their mutual, exchange-traded and closed-fund counterparts. Portfolios are held directly by investors and/or managed by financial professionals" [1]. Each asset in portfolio has an investment share of the total investment on portfolio. The investment share is the percent of money invested in portfolio. Therefore the investor tries to distribute the investment in way to reach the maximum return and minimum risk. If we distribute the investment correctly, the portfolio will contain less risk and high return. These two objectives that must be dedicated in same time lead to multi objective portfolio optimization problem that must be solved. To solve the problem, we will use intelligence computational technique named Clonal Selection to optimize portfolio's return and risk. Then, we will show the results of the proposed solution through experiments are conducted using the stocks in (Tadawul)[18] based on historical data of stock prices for three years from 2009 to 2011. Moreover, up to our knowledge this research is the first portfolio optimization process applied on Tadawul and first application based on Mean-CvaR[2] portfolio optimization model.

Related Work: Many models have been developed to solve the problem of optimal portfolio selection. One of them is based on intelligent system techniques

[3].One of the common intelligent systems is Genetic Algorithm. Genetic Algorithm is developed as conventional [4] [7][9][10], adaptive [4], a novel [5] or with greedy coordinate [8]. Other intelligent systems that developed is Particle Swarm Algorithm [9][10], Artificial Bee Colony[11], Greedy Algorithm[13], and Artificial Immune System [14]. In this research, our goal here is to develop and implement our algorithm named Portfolio Optimization Based on Clonal Selection (POCS) for solving multi objective portfolio optimization problem. This research will be helpful to the investors and interested people in Saudi Arabia stock exchange market; no other research makes studies on this market. And this first attempt to apply CLONALG Algorithm on portfolio optimization. It offers helpful tool simplifies portfolio creation and management under the assumption that the portfolio returns subject to heavy tails.

The reminder of this paper is organized as follows: portfolio optimization model and clonal selection algorithm are defined in section 2 and 3. Our approach of applying Clonal selection to portfolio is described in Section 4. Experimental results are reported in Section 5. At the end, we concluded our work in section 6.

2 Portfolio Optimization Model

Modern Portfolio theory appears in 1952 by Markowitz [15]. He puts the cornerstone for solving multi objective portfolio optimization problem. Under the assumption that the portfolio return subjects to heavy tail, the authors of [2] proposed the optimal portfolio model that we are used in our research. To calculate the expected return on a portfolio, first, we need to calculate each day yield $y_{i,j}$ for each stock i:

$$y_{i,j} = \frac{p_{i,j} - p_{i,j-1}}{p_{i,j-1}}$$

Where $p_{i,j}$ is previous day's closing price, and $p_{i,j-1}$ is the day after. Then, take the mean of $y_{i,j}$ represented as m. Second, let $m^T = (m_1, m_2 \dots m_n)$ are n assets' mean return vector, $w^T = (w_1, w_2 \dots w_n)$ are their weights of the investment share.

Where w has two constraints:

$$\sum_{i=1}^{n} w_i = 1, \; 0 \le w_i \le 1$$

The investor's expected portfolio return, which is hoped to be maximized is:

$$\text{Max } E(R) = w^T m \qquad (1)$$

After calculate expected return, we are calculate the expected risk on a portfolio, first, we need to calculate portfolio loss function $f(w, y) = - w^T y$. Let a is the belief degree, t is trading days, and b is a parameter, Then $u_k = [f(w, y_k) - a]^+$. From the above the risk described as CVaR-Conditional Value at-Risk which is hoped to be minimized is:

$$\text{Min } a + \frac{1}{t(1-b)} \sum_{k=1}^{t} u_k \tag{2}$$

Subject to:

1) $w^T y + a + u_k \geq 0, \quad u_k \geq 0$

2) $\sum_{i=1}^{n} w_i = 1, \quad 0 \leq w_i \leq 1$

From the two formulas (1) and (2) result the multi objective optimization problem. To solve it we used linear weighted sum method [19] to formulate the objective function:

$$\text{Max } 0.5(w^T m) + 0.5\left(a + \frac{1}{t(1-b)} \sum_{k=1}^{t} u_k\right) \tag{3}$$

3 Clonal Selection Algorithm

Clonal Selection Algorithms (CSAs) are class of Immune algorithms (IA) which are inspired by the Clonal Selection Principle [17]. "The clonal selection principle is used to explain the basic features of an adaptive immune response to an antigenic stimulus"[16]. When antigens attack the human body, the immune system activates some antibodies and begins the cloning process to create large quantity of antibodies that bind powerfully to a specific antigen. The cloned antibodies' mutation process is in reverse proportional to affinity to the antigens, i.e., the highest affinity antibodies (the highest fitness function value [17]) trial the lowest mutation rates and vice versa [14]. CSA was introduced by Leandro and Zuben in 2002 [16] and can be defined as an evolutionary algorithm aims to find a good solution for solving optimization problems in a reasonable time of execution by using techniques inspired by evolutionary biology such as cloning and hyper mutation.

4 Portfolio Optimization Using Clonal Selection Algorithm

This section describes our Portfolio Optimization based on Clonal Selection (POCS) for optimizing portfolio. In this algorithm, we applied one of the common Clonal Selection algorithms named CLONALG[17] to portfolio.

4.1 Antibody

As we know that the antibody (Ab) represents a solution. Based on that, the Ab in (POCS) represents a portfolio. The size of Ab in POCS is determined based number of stocks in the portfolio and each weight corresponds to one stock. The following Table1 illustrates the Ab structure.

Table1. Antibody structure

Weight	Mean Return	Expected return	Expected Risk	Fitness Function Value

4.2 Clonal Selection Operator

CSA uses two clonal selection operators to generate the children of the existing population. The cloning operator is applied on each Ab in the current population by copying each Ab without any changes to it. In hyper mutation operator every cloned Ab is submitted to maturation process inversely proportional to the fitness function value; the higher the fitness function value, the smaller the mutation rate. The mutation rate formula = (1.0 / mutation factor) * Exp(-1 * normalized Fitness)[16].

4.3 Fitness Function

The amount of the binding between the Ab and the antigen represents the fitness or the quality of the solution [14]. The fitness function is used to measure the performance of the antibodies. We will use equation (3) to calculate the fitness function and we suppose that b=0.5. According to our experience we set a=0.05.

4.4 Pseudo code for POCS

The pseudo code of POCS is shown in Fig.1. First, population size, maximum number of generations, maximum number of cloning and number of random antibodies to insert at the end of each generation will be taken as inputs.

Input:
population size: N
Maximum number of generations: Ngen
Maximum number of cloning: Nclo
Number of random antibodies to insert at the end of each generation: d
Antibody as shown in Table.1 represents a portfolio with weight and mean return for each stock, expected return, expected risk and fitness function value for the portfolio.
Output:
Antibody population P(t)
Algorithm:

```
1.  P(0) ← Generate  random Ab population of size N
2.  For t = 1 to Ngen
3.  For each Ab i ∈ P(t)
    3.1. Calculate  fitness function value (i)
4.  For each Ab i ∈ P(t)
    4.1. For j=1 to Nclo
         4.1.1. P(clo)← clone(i)
5.  For each Ab i ∈ P(clo)
    5.1. P(hyp)← hyper mutate(i)
6.  For each Ab i ∈ P(hyp)
    6.1. Calculate fitness function value ( i)
7.  For each Ab  i ∈ P(t)
    7.1. Ab* ← Select highest fitness function value  from P(hyp)
    7.2. If (Ab* > i)
         7.2.1. Replace (i , Ab* )
8.  Abd ← Generate  d  random Ab
    8.1. Replace (d lowest  fitness function value  from P(t), Abd)
9.  Go to step2
```

Fig. 1. Pseudo code for POCS algorithm

5 Results and Discussion

This section reports the results of the experiments conducted to solving the portfolio optimization problem. All the experiments were performed on 2.53 GHz Intel(R) Core(TM) 2 Duo CPU PC machine with 2.72 GB RAM, running Microsoft Windows 7. The algorithm is written with C# in Visual Studio.Net 2008 environment. The dataset is a daily adjusted close price of some assets from Saudi Arabia stock exchange market (Tadawul) from 2009 to 2011. It has about 747 trading days. The output of this experiment is the top portfolio that has maximum fitness function with the investment share for each stock in the portfolio. We applied the algorithm on five different stock companies belong to five different market sectors. From Banks and Financial Services sector, Al Rajhi bank was selected, SABIC from Petrochemical

Industries, Arab Cement from Cement sector, Etihad Etisalat from Telecommunication & Information Technology, and Taiba from Real Estate Development. In this experiment there are four parameters that must be determined: population size, clone number, number of generation and the maturation factor. We have conducted two experiments in order to test the speed, fitness and the portfolio return and risk of (POCS). First experiment set the population to 20 and the generation of [10...100]. The second, set the generation to 10 and the population of [10...100]. In all experiments mutation factor is set to 80 and clone number to 20. Both experiments shows the result related to increasing the generations and population size. In the first experiment, as we can see that Fig.2 shows the time, in seconds, that is spent by the (POCS) when the generation is increased. In addition, the second experiment as shown in Fig.3 shows the average fitness of the final results of (POCS) when increasing population size. The fitness value in (3) is calculated: where m for each stock as follow: AlRajhi=0.0004, SABIC=0.001, ArabCement=0.0006, Etihad Etisalat=0.0008, Taiba=0.0002. t=747 trading days**.**

Our tests illustrate how the time will increase according to the increase of the generation or population. Consequently, comparing the two experiments, as shown in Fig.2, (POCS) takes less time when increasing the population rather than when the generation is increased. The same situation will occur in the average of fitness, so when the generations increase the average of fitness will increase more than when increasing the population. Form the results given in Fig.2 and Fig.3; we can conclude that the size of population and generation should be compromised in order to get better results. As we now the investment allocation to different stocks in the portfolio will completely affect the return of the portfolio as well as the risk. Table.2 shows the best way of distributing the investment to portfolio stocks. About 31% of portfolio's value is invested in SABIC and Arab Cement has the second value.

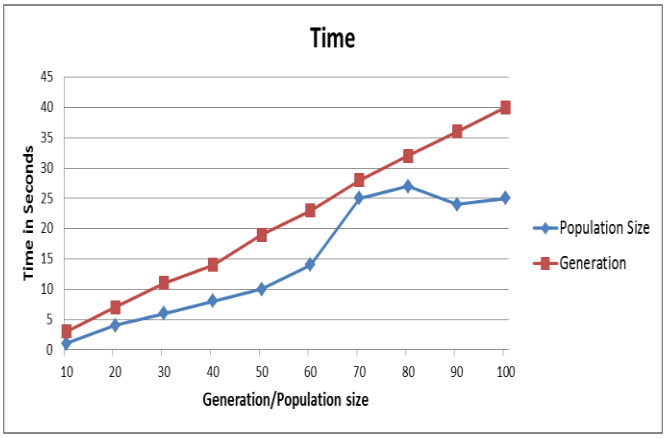

Fig.2. The effect on time by altering generation/population size.

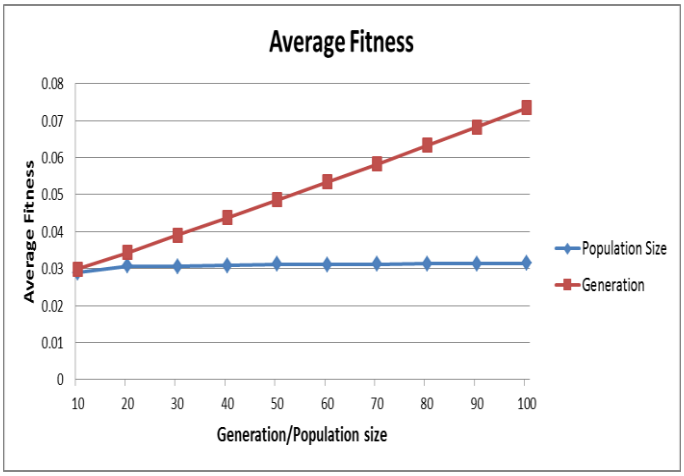

Fig.3. The effect on average fitness by altering generation/population size.

Table 2. Sample shows the best way of distributing the investment to portfolio stocks.

Company Name	AlRajhi bank	SABIC	Arab Cement	Etihad Etisalat	Taiba
% Of Portfolio	18.09	30.62	26.07	17.79	07.43

6 Conclusion

In this paper, we applied Portfolio Optimization based on Clonal Selection to solve portfolio optimization problem and find best portfolio allocation. The experiments are conducted using the stocks in (Tadawul). We applied the algorithm on five different stock companies belong to five different market sectors. As a result, the algorithm is functioning well and the fitness function value is increased. Also, the basic idea of the stock selecting has been realized by this Clonal Selection model. The results show that when the generation increases, it generates best fitness and the return of the portfolio is increased too.

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