10. Solutions of SAMPLING DISTRIBUTIONS

10.1. Single Mean:

Q1. A machine is producing metal pieces that are cylindrical in shape. A random sample of size 5 is taken and the diameters are 1.70, 2.11, 2.20, 2.31 and 2.28 centimeters. Then,

1) The sample mean is:

(A) 2.12 (B) 2.32 (C) 2.90 (D) 2.20 (E) 2.22

2) The sample variance is:

(A) 0.59757 (B) 0.28555 (C) 0.35633 (D) 0.06115 (E) 0.53400

Q2. The average life of a certain battery is 5 years, with a standard deviation of 1 year. Assume that the live of the battery approximately follows a normal distribution.

Solution :

$\mu = 5$, $\sigma = 1$

1) The sample mean $\bar{X}$ of a random sample of 5 batteries selected from this product has a mean

$E(\bar{X}) = \mu_{\bar{X}}$ equal to: [**$E(\bar{x}) = \mu = 5$**]

(A) 0.2 **(B) 5** (C) 3 (D) None of these

2) The variance $Var(\bar{X}) = \sigma_{\bar{X}}^2$ of the sample mean $\bar{X}$ of a random sample of 5 batteries selected from this product is equal to: [**$Var(\bar{x}) = \frac{\sigma^2}{n} = \frac{1}{5} = 0.2$**]

(A) 0.2 (B) 5 (C) 3 (D) None of these

3) The probability that the average life of a random sample of size 16 of such batteries will be between 4.5 and 5.4 years is:

Solution :

$$Z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{\bar{x}-5}{1/\sqrt{16}} = \frac{\bar{x}-5}{1/4}$$

$P(4.5 < \bar{x} < 5.4) = P(\frac{4.5-5}{\frac{1}{4}} < Z < \frac{5.4-5}{\frac{1}{4}}) =$

$$P(-2 < Z < 1.6) = 0.9452 - 0.0228 = 0.9224$$

(A) 0.1039 (B) 0.2135 (C) 0.7865 (D) 0.9224

4) The probability that the average life of a random sample of size 16 of such batteries will be less than 5.5 years is:

Solution : $P(\bar{x} < 5.5) = P(Z < \frac{5.5-5}{\frac{1}{4}}) = P(Z < 2) = 0.9772$

(A) 0.9772 (B) 0.0228 (C) 0.9223 (D) None of these

5) The probability that the average life of a random sample of size 16 of such batteries will be more than 4.75 years is:

Solution : $P(\bar{x} > 4.75) = P(Z > \frac{4.75-5}{\frac{1}{4}}) = P(Z > -1) = 0.8413$

(A) 0.8413 (B) 0.1587 (C) 0.9452 (D) None of these

6) ملغي

(H.W)Q3. The random variable X, representing the lifespan of a certain light bulb, is distributed normally with a mean of 400 hours and a standard deviation of 10 hours.

1. What is the probability that a particular light bulb will last for more than 380 hours?
2. Light bulbs with lifespan less than 380 hours are rejected. Find the percentage of light bulbs that will be rejected.
3. If 9 light bulbs are selected randomly, find the probability that their average lifespan will be less than 405.

Q4. Suppose that you take a random sample of size n=64 from a distribution with mean μ=55 and standard deviation σ=10. Let $\bar{X}=\frac{1}{n}\sum_{i=1}^{n}X_i$ be the sample mean.

(a) What is the approximated sampling distribution of $\bar{X}$?

$\bar{x} \approx N(\mu, \sigma^2)$

(b) What is the mean of $\bar{X}$?

[E($\bar{x}$) =μ =55]

(c) What is the standard error (standard deviation) of $\bar{X}$?

$[\mathrm{Var}(\bar{x}) = \sqrt{\mathrm{Var}(\bar{x})} = \sqrt{\frac{\sigma^2}{n}} = \sqrt{\frac{100}{64}} = \frac{10}{8} = 1.25\,]$

(d) Find the probability that the sample mean $\bar{X}$ exceeds 52.

$P(\bar{x} > 52) = P(Z > \frac{52-55}{10/8}) = P(Z > -2.4) = 0.9918$

(H.W)Q5. The amount of time that customers using ATM (Automatic Teller Machine) is a random variable with the mean 3.0 minutes and the standard deviation of 1.4 minutes. If a random sample of 49 customers is observed, then

(1) the probability that their mean time will be at least 2.8 minutes is

(A) 1.0 **(B) 0.8413** (C) 0.3274 (D) 0.4468

(2) the probability that their mean time will be between 2.7 and 3.2 minutes is

(A) 0.7745 (B) 0.2784 (C) 0.9973 (D) 0.0236

(H.W)Q6. The average life of an industrial machine is 6 years, with a standard deviation of 1 year. Assume the life of such machines follows approximately a normal distribution. A random sample of 4 of such machines is selected. The sample mean life of the machines in the sample is $\bar{X}$.

(1) The sample mean has a mean $\mu_{\bar{X}} = E(\bar{X})$ equals to:

(A) 5 **(B) 6** (C) 7 (D) 8

(2) The sample mean has a variance $\sigma^2_{\bar{X}} = Var(\bar{X})$ equals to:

(A) 1 **(B) 0.5** **(C) 0.25** (D) 0.75

(3) $P(\bar{X} < 5.5) =$

(A) 0.4602 (B) 0.8413 **(C) 0.1587** (D) 0.5398

(4) If $P(\bar{X} > a) = 0.1492$, then the numerical value of a is: **((ملغي**

(A) 0.8508 (B) 1.04 (C) 6.52 (D) 0.2

ملغي

8.22 The heights of 1000 students are approximately normally distributed with a mean of 174.5 centimeters and a standard deviation of 6.9 centimeters. Suppose 200 random samples of size 25 are drawn from this population and the means recorded to the nearest tenth of a centimeter. Determine

(a) the mean and standard deviation of the sampling distribution of $\bar{X}$;

(b) the number of sample means that fall between 172.5 and 175.8 centimeters inclusive;

(c) the number of sample means falling below 172.0 centimeters.

10.2. Two Means:

Q1. A random sample of size $n_1 = 36$ is taken from a normal population with a mean $\mu_1 = 70$ and a standard deviation $\sigma_1 = 4$. A second independent random sample of size $n_2 = 49$ is taken from a normal population with a mean $\mu_2 = 85$ and a standard deviation $\sigma_2 = 5$. Let $\bar{X}_1$ and $\bar{X}_2$ be the averages of the first and second samples, respectively.

a) Find $E(\bar{X}_1)$ and $Var(\bar{X}_1)$.

$E(\bar{X}_1)=\mu_1=70$, $Var(\bar{X}_1)=\sigma_1^2/n$ =16/36 = 0.444

b) Find $E(\bar{X}_1-\bar{X}_2)$ and $Var(\bar{X}_1-\bar{X}_2)$

$E(\bar{X}_1-\bar{X}_2) = \mu_1 - \mu_2 = 70-85 = -15$

$$Var\left(\bar{X}_1-\bar{X}_2\right) = \frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2} = \frac{16}{36}+\frac{25}{49} = 0.5465$$

c) Find $P(70 < \bar{X}_1 < 71)$. $= P\left(\frac{70-70}{0.444} < Z < \frac{71-70}{0.444}\right) = P(0<Z<2.25)$

= 0.9878-0.500 =0.4878

d) Find $P(\bar{X}_1 - \bar{X}_2 > -16)$.= $P\left(Z > \frac{-16-(-15)}{\sqrt{0.5465}}\right) = P(Z > -1.35) = 0.9115$

Q2. A random sample of size 25 is taken from a normal population (first population) having a mean of 100 and a standard deviation of 6. A second random sample of size 36 is taken from a different normal population (second population) having a mean of 97 and a standard deviation of 5. Assume that these two samples are independent.

(1) the probability that the sample mean of the first sample will exceed the sample mean of the second sample by at least 6 is

mean($\bar{X}_1$-$\bar{X}_2$)=$\mu_1 - \mu_2 = 100 - 97 = 3$

Var($\bar{X}_1$-$\bar{X}_2$)=$\frac{36}{25} + \frac{25}{36}$ = 2.134

P($\bar{X}_1$-$\bar{X}_2 \geq$ 1) = P($Z \geq \frac{1-3}{\sqrt{2.134}}$)= P($Z \geq$ -1.37) =0.9147

(A) 0.0013 **<u>(B)</u>** **<u>0.9147</u>** (C) 0.0202 (D) 0.9832

(2) the probability that the difference between the two sample means will be less than 2 is

P($\bar{X}_1$-$\bar{X}_2 <$ 2) = P($Z < \frac{2-3}{\sqrt{2.134}}$)= P($Z <$ -0.68) = 0.2483

(A) 0.099 **<u>(B)</u>** **<u>0.2483</u>** (C) 0.8499 (D) 0.9499

(H.W)8.28 A random sample of size 25 is taken from a normal population having a mean of 80 and a standard deviation of 5. A second random sample of size 36 is taken from a different normal population having a mean of 75 and a standard deviation of 3. Find the probability that the sample mean computed from the 25 measurements will exceed the sample mean computed from the 36 measurements by at least 3.4 but less than 5.9. Assume the difference of the means to be measured to the nearest tenth.

8.29 The distribution of heights of a certain breed of terrier has a mean of 72 centimeters and a standard deviation of 10 centimeters, whereas the distribution of heights of a certain breed of poodle has a mean of 28 centimeters with a standard deviation of 5 centimeters. Assuming that the sample means can be measured to any degree of accuracy, find the probability that the sample mean for a random sample of heights of 64 terriers exceeds the sample mean for a random sample of heights of 100 poodles by at most 44.2 centimeter

Homework : 8.26 , 8.32

<u>10.3. Single Proportion:</u>

Q1. Suppose that 20% of the students in a certain university smoke cigarettes. A random sample of 5 students is taken from this university. Let $\hat{p}$ be the proportion of smokers in the sample.

(1) Find $E(\hat{p}) = \mu_{\hat{p}}$, the mean $\hat{p}$.

$E(\hat{p}) = \mu_{\hat{p}}$ **= P =0.2**

(2) Find $Var(\hat{p})=\sigma^2_{\hat{p}}$, the variance of $\hat{p}$.

$$Var(\hat{p})=\sigma^2_{\hat{p}}=\frac{pq}{n}=\frac{0.2x0.8}{5}=0.032$$

(3) Find an approximate distribution of $\hat{p}$.

$\hat{p}$~N(P, $\frac{pq}{n}$)

(4) Find P($\hat{p}$>0.25).= $P(Z>\frac{0.25-0.2}{\sqrt{0.032}})=P(Z>0.28)=0.3897$

(H.W) Q2: Suppose that you take a random sample of size n=100 from a binomial population with parameter p=0.25 (proportion of successes). Let $\hat{p}=X/n$ be the sample proportion of successes, where X is the number of successes in the sample.

(a) What is the approximated sampling distribution of $\hat{p}$?
(b) What is the mean of $\hat{p}$?
(c) What is the standard error (standard deviation) of $\hat{p}$?
(d) Find the probability that the sample proportion $\hat{p}$ is less than 0.2.

(ملغى)10.4. Two Proportions:

Q1. Suppose that 25% of the male students and 20% of the female students in a certain university smoke cigarettes. A random sample of 5 male students is taken. Another random sample of 10 female students is independently taken from this university. Let $\hat{p}_1$ and $\hat{p}_2$ be the proportions of smokers in the two samples, respectively.

(1) Find $E(\hat{p}_1-\hat{p}_2)=\mu_{\hat{p}_1-\hat{p}_2}$, the mean of $\hat{p}_1-\hat{p}_2$.

(2) Find $Var(\hat{p}_1-\hat{p}_2)=\sigma^2_{\hat{p}_1-\hat{p}_2}$, the variance of $\hat{p}_1-\hat{p}_2$.

(3) Find an approximate distribution of $\hat{p}_1-\hat{p}_2$.

(4) Find P(0.10< $\hat{p}_1-\hat{p}_2$<0.20).

10.5 t-distribution:

Q1. Using t-table with degrees of freedom df=14, find $t_{0.05}$, $t_{0.975}$

$t_{0.05} \gg \; t=-t_{0.95}=1.761$

$t_{0.975} \gg t=-t_{0.025}=-2.145$

Q2. From the table of t-distribution with degrees of freedom v=15 , the value of

$t_{0.025}$ equals to $t_{0.025} \gg t=t_{0.975}=2.131$

(A) 2.131 (B) 1.753 (C) 3.268 (D) 0.0

H.W

Q3. The random variable X, representing the lifespan of a certain light bulb, is distributed normally with a mean of 400 hours and a standard deviation of 10 hours.

$X: live\ of\ the\ battery \quad ; \quad X \sim N(\mu = 400\ , \sigma^2 = 10^2)$

1. What is the probability that a particular light bulb will last for more than 380 hours?

$p(\mathrm{X} > 380) = p\left(\mathrm{z} > \frac{380-400}{10} = -2\right) = 1 - p(\mathrm{z} < -2) = 1 - 0.0228 = 0.9772$

2. Light bulbs with lifespan less than 380 hours are rejected. Find the percentage of light bulbs that will be rejected. $p(\mathrm{X} < 380) = p\left(Z < \frac{380-400}{10}\right) = p(Z < -2) = 0.0228$

3. If 9 light bulbs are selected randomly, find the probability that their average lifespan will be less than 405.

$$n = 9 \quad ; \quad \bar{X} \sim N\left(\mu = 400\ , \sigma^2 = \frac{10^2}{9} = (\frac{10}{3})^2\right)$$

$$p(\bar{\mathrm{X}} < 405) = p\left(Z < \frac{405 - 400}{10/3} = 1.5\right) = p(Z < -2) = 0.9332$$

Q5. The amount of time that customers using ATM (Automatic Teller Machine) is a random variable with the mean 3.0 minutes and the standard deviation of 1.4 minutes. If a random sample of 49 customers is observed, then X: The amount of time that customers using ATM

$$X \sim N(\mu = 3\ , \sigma^2 = (1.4)^2)$$

$$n = 49 \quad ; \quad \bar{X} \sim N\left(\mu = 3\ , \frac{\sigma^2}{n} = \frac{1.4^2}{49} = (\frac{1.4}{7})^2\right)$$

(1) the probability that their mean time will be at least 2.8 minutes is

$$p(\bar{X} \geq 2.8) = p\left(Z > \frac{2.8 - 3}{1.4/7} = -1\right) = 1 - p(Z < -1) = 1 - 0.1587$$
$$= 0.8413$$

(A) 1.0 (B) 0.8413 (C) 0.3274 (D) 0.4468

(2) the probability that their mean time will be between 2.7 and 3.2 minutes is

$$p(2.7 < \bar{X} < 3.2) = p\left(\frac{2.7 - 3}{1.4/7} < \frac{\bar{X} - \mu}{\sigma} < \frac{3.2 - 3}{1.4/7}\right) = p(-1.5 < Z < 1)$$

$$= p(Z < 1) - p(Z < -1.5) = 0.8413 - 0.0668 = 0.7745$$

8.28 A random sample of size 25 is taken from a normal population having a mean of 80 and a standard deviation of 5. A second random sample of size 36 is taken from a different normal population having a mean of 75 and a standard deviation of 3. Find the probability that the sample mean computed from the 25 measurements will exceed the sample mean computed from the 36 measurements by at least 3.4 but less than 5.9. Assume the difference of the means to be measured to the nearest tenth.

8.29 The distribution of heights of a certain breed of terrier has a mean of 72 centimeters and a standard deviation of 10 centimeters, whereas the distribution of heights of a certain breed of poodle has a mean of 28 centimeters with a standard deviation of 5 centimeters. Assuming that the sample means can be measured to any degree of accuracy, find the probability that the sample mean for a random sample of heights of 64 terriers exceeds the sample mean for a random sample of heights of 100 poodles by at most 44.2 centimeter

Q2: Suppose that you take a random sample of size n=100 from a binomial population with parameter p=0.25 (proportion of successes). Let $\hat{p}$ □□X / n be the sample proportion of successes, where X is the number of successes in the sample.

n=100 , p=0.25

(a) What is the approximated sampling distribution of $\hat{p}$?

$$as \quad n = 100 \geq 30 \; then \; \hat{p} \sim N\left(\mu_{\hat{p}} = 0.25 \,, \sigma^2{}_{\hat{p}} = \frac{pq}{n} = \frac{0.25(0.65)}{100} = \frac{3}{1600}\right)$$

(b) What is the mean of $\hat{p}$?

$$\mu_{\hat{p}} = E(\hat{p}) = 0.25$$

(c) What is the standard error (standard deviation) of $\hat{p}$?

$$\sigma_{\hat{p}} = \sqrt{\frac{3}{1600}} = 0.0433$$

(d) Find the probability that the sample proportion $\hat{p}$ is less than 0.2.

$$p(\hat{p} < 0.2) = p\left(Z < \frac{0.2 - 0.25}{0.0433} = 0.12\right) = 0.1251$$