

Random Variables, Distributions and Expectations

Discrete Distributions: Q1

(a) (b) $S = \{H,T\} \times \{H,T\} \times \{H,T\} = \{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$

$X = \text{number of heads} - \text{number of tails}$, $n(S) = 2 \times 2 \times 2 = 8$

$= -3, -1, 1, 3$

(c)

$P(X=-3) = P(\{TTT\}) = \frac{1}{8}$

$P(X=-1) = P(\{HTT, THT, TTH\}) = \frac{3}{8}$

$P(X=1) = P(\{HHT, HTH, THH\}) = \frac{3}{8}$

$P(X=3) = P(\{HHH\}) = \frac{1}{8}$

$F(x) = P(X \leq x) =$

- 0, $x < -3$
- $\frac{1}{8}$, $-3 \leq x < -1$
- $\frac{4}{8}$, $-1 \leq x < 1$
- $\frac{7}{8}$, $1 \leq x < 3$
- 1, $3 \leq x$

x	-3	-1	1	3	Σ
$P(X=x) = P(x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	1
$P(X \leq x) = F(x)$	$\frac{1}{8}$	$\frac{4}{8}$	$\frac{7}{8}$	1	
$2 P(x)$	$-\frac{3}{8}$	$-\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$E(X) = \Sigma x P(x) = 0 = \mu$
$x^2 P(x)$	$\frac{9}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	$E(x^2 P(x)) = E(X^2) = 3$

(d)

$P(X \leq 1) = P(1) + P(-1) + P(-3) = \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = \frac{7}{8}$

(e)

$P(X < 1) = P(-1) + P(-3) = \frac{3}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$

(f)

$E(X) = \mu = 0$

(g) $\text{Var}(X) = \sigma^2 = E(X^2) - [E(X)]^2 = 3 - 0 = 3$

Q2

$$(1) S = \{M, F\} \times \{M, F\} = \{MM, MF, FM, FF\}, \quad P(F) = .2, \quad P(M) = 1 - .2 = .8$$

X = number of Females in the committee.

$$= 0, 1, 2$$

$$(3) P(X=0) = P(\{MM\}) = P(\{M\})P(\{M\}) = (.8)(.8) = .64$$

$$P(X=1) = P(\{MF, FM\}) = (.8)(.2) + (.2)(.8) = .32$$

$$P(X=2) = P(\{FF\}) = (.2)(.2) = .04$$

$$F(x) = P(X \leq x)$$

$$\begin{aligned} 0 &\rightarrow x < 0 \\ .64 &\rightarrow 0 \leq x < 1 \\ .96 &\rightarrow 1 \leq x < 2 \\ 1 &\rightarrow 2 \leq x \end{aligned}$$

x	0	1	2	Σ
$P(X=x) = P(x)$	.64	.32	.04	1
$P(X \leq x) = F(x)$	.64	.96	1	
$x P(x)$	0	.32	.08	$E(X) = \Sigma x P(x) = .4$
$x^2 P(x)$	0	.32	.16	$E(X^2) = \Sigma x^2 P(x) = .48$

(4) $P(\text{at least one female in the committee})$

$$= P(X \geq 1) = P(X=1) + P(X=2) = P(1) + P(2) = .32 + .04 = .36$$

$$\text{or } 1 - P(X \leq 0) = 1 - \{P(X=0)\} = 1 - .64 = .36$$

(5) $P(\text{at most one female in the committee})$

$$= P(X \leq 1) = P(X=0) + P(X=1) = .64 + .32 = .96$$

$$(6) M = E(X) = \Sigma x P(x) = .4$$

$$(7) \text{Var}(X) = \sigma^2 = E(X^2) - [E(X)]^2 = .48 - (.4)^2 = .48 - (.16) = .32$$

Q3

40 of 5	2 cards with replacement, $S = \{5, 10\} \times \{5, 10\} = \{(5, 5), (5, 10), (10, 5), (10, 10)\}$
60 of 10	indep.
Total = 100	$P(5) = \frac{40}{100} = .4, \quad P(10) = \frac{60}{100} = .6$

X = the total sum of the two cards.

$$= 10, 15, 20$$

$$(iii) P(X=10) = P(\{(5, 5)\}) = (.4)(.4) = .16$$

$$P(X=15) = P(\{(5, 10), (10, 5)\}) = P(\{(5, 10)\}) + P(\{(10, 5)\})$$

$$= (.4)(.6) + (.4)(.6) = (.2)(.4)(.6) = .48$$

$$P(X=20) = P(\{(10, 10)\}) = (.6)(.6) = .36$$

x	10	15	20	Σ
$P(X=x) = P(x)$	.16	.48	.36	1
$x P(x)$	1.6	7.2	7.2	$E(X) = 16$
$x^2 P(x)$	16	108	144	$E(X^2) = 268$

②

$$\text{iv)} P(X=0) = 0$$

$$\text{v)} P(X > 10) = P(X=15) + P(X=20) = .48 + .36 = .84$$

$$\text{vi)} \mu = E(X) = 16$$

$$\text{vii)} \sigma^2 = \text{Var}(X) = E(X^2) - [E(X)]^2 = 268 - (16)^2 = 12$$

Q4

$$\text{1)} \mu = E(X) = (-3)(.1) + (6)(.5) + (9)(.4) = 6.3$$

$$\text{2)} E(X^2) = (-3)^2(.1) + (6)^2(.5) + (9)^2(.4) = 51.3$$

$$\text{3)} \text{Var}(X) = \sigma_X^2 = E(X^2) - [E(X)]^2 = (51.3) - (6.3)^2 = 11.61$$

$$\text{4)} \mu_{2X+1} = E(2X+1) = E(2X) + E(1) = 2E(X) + 1 = 2(6.3) + 1 = 13.6$$

5)

$$\sigma_{2X+1}^2 = \text{Var}(2X+1) = \text{Var}(2X) + \text{Var}(1) = 4\text{Var}(X) + 0 = 4(11.61) = 46.44$$

Q5

$$\text{A)} P(x) = \frac{x+1}{10} \Rightarrow P(0) = \frac{1}{10} < 1$$

$$P(1) = \frac{2}{10} < 1$$

$$P(2) = \frac{3}{10} < 1$$

$$P(3) = \frac{4}{10} < 1$$

$$P(4) = \frac{5}{10} < 1$$

$$\text{and } \sum P(x) = \frac{1+2+3+4+5}{10} = \frac{15}{10} \neq 1$$

$\therefore$ it is not probability

$$\text{B)} P(x) = \frac{x-1}{10} \Rightarrow P(0) = \frac{-1}{10} < 0$$

$\therefore$ it is not probability

$$\text{C)} P(x) = \frac{1}{5} \Rightarrow P(0) = P(1) = P(2) = P(3) = P(4) = \frac{1}{5} < 1$$

$$\text{and } \sum P(x) = \frac{1+1+1+1+1}{5} = \frac{5}{5} = 1$$

$\therefore$ it is probability

$$\text{6)} \text{1)} E(X) = \mu = \sum x \cdot P(x) = (0)\left(\frac{1}{3}\right) + (1)\left(\frac{1}{3}\right) + (2)\left(\frac{1}{3}\right) = \frac{1+2}{3} = \frac{3}{3} = 1$$

$$\text{2)} E(X^2) = \sum x^2 P(x) = (0)^2\left(\frac{1}{3}\right) + (1)^2\left(\frac{1}{3}\right) + (2)^2\left(\frac{1}{3}\right) = \frac{1+4}{3} = \frac{5}{3} = 1.67$$

$$\therefore \text{Var}(X) = E(X^2) - [E(X)]^2 = 1.67 - 1 = 0.67$$

Q6 $P(x) = kx, x=1,2,3$

$$\text{1)} \text{ We know that } \sum P(x) = 1 \Rightarrow k + 2k + 3k = 1 \Rightarrow 6k = 1 \Rightarrow k = \frac{1}{6}$$

$$\therefore P(x) = \frac{x}{6}, x=1,2,3$$

$$\text{2)} F(1) = P(X \leq 1) = P(X=1) = P(1) = \frac{1}{6}$$

$$F(2) = P(X \leq 2) = P(X=2) + P(X=1) = \frac{3}{6}$$

$$F(3) = P(X \leq 3) = P(X=3) + P(X=2) + P(X=1) = 1$$

3

x	1	2	3
P(x)	1/6	2/6	3/6
F(x)	1/6	3/6	1

$$\therefore F(x) = P(X \leq x) = \begin{cases} 0 & x < 1 \\ 1/6 & 1 \leq x < 2 \\ 3/6 = 1/2 & 2 \leq x < 3 \\ 1 & 3 \leq x \end{cases}$$

$$\begin{aligned} \text{[3]} \quad P(.5 \leq X \leq 2.5) &= P(X \leq 2.5) - P(X < .5) = F(2.5) - F(.5) \\ &= \frac{1}{2} - 0 = \frac{1}{2} \end{aligned}$$

$$\begin{aligned} \text{or by } P(x): \quad P(.5 \leq X \leq 2.5) &= P(X=1) + P(X=2) \\ &= \frac{1}{6} + \frac{2}{6} = \frac{3}{6} = \frac{1}{2} \end{aligned}$$

Q7

$$\text{[1]} \quad \text{we have } F(x) = \begin{cases} 0 & x < 0 \\ .25 & 0 \leq x < 1 \\ .6 & 1 \leq x < 2 \\ 1 & 2 \leq x \end{cases}$$

$F(0) = .25 - 0 = .25$
 $F(1) = .6 - .25 = .35$
 $F(2) = 1 - .6 = .4$

$$\therefore P(x) = \begin{cases} .25 & x=0 \\ .35 & x=1 \\ .4 & x=2 \end{cases}$$

[2] $\begin{cases} 0 \end{cases}$ other way

$$P(1 \leq X < 2) = \begin{cases} P(X=1) = .35 = P(1) & \text{, by used } P(x) \\ F(2-1) - F(1-1) = F(1) - F(0) = .6 - .25 = .35 & \text{, by used } F(x) \end{cases}$$

$$\begin{aligned} \text{[3]} \quad P(X > 2) &= 1 - P(X \leq 2) = \begin{cases} 1 - (P(0) + P(1) + P(2)) = 0 & \text{, by used } P(x) \\ 1 - F(2) = 1 - 1 = 0 & \text{, by used } F(x) \end{cases} \end{aligned}$$

Q8

we know that $\sum P(x) = 1$

$$\Rightarrow (.4) + (.3) + (.1) = 1 \Rightarrow .8 + C = 1 \Rightarrow C = 1 - .8 = .2$$

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$$\text{Var}(X) = E(X^2) - [E(X)]^2 = 0.6$$

$$\sum_{i=1}^3 x_i^2 P(x_i) = 4.6$$

$$\sum_{i=1}^3 x_i P(x_i) = 2$$

$$\text{Var}(Y) = E(Y^2) - [E(Y)]^2 = 1.6$$

$$\sum_{i=1}^5 y_i^2 P(y_i) = 5.6$$

$$\sum_{i=1}^5 y_i P(y_i) = 2$$

$$\text{So, } \text{Var}(X) = 0.6 < \text{Var}(Y) = 1.6$$

5

6

Continuous distributions:

Q1 $P(X=16) = 0$ (because X is continuous random)

Q2 $\square$ we know $\int_{-\infty}^{\infty} f(x) dx = 1$

$\Rightarrow \int_0^1 k \sqrt{x} dx = 1 \Rightarrow k \left[\frac{x^{3/2}}{3/2} \right]_0^1 = 1 \Rightarrow \frac{2}{3} k = 1 \Rightarrow k = \frac{3}{2} = 1.5$

$$f(x) = \begin{cases} \frac{3}{2} \sqrt{x}, & 0 < x < 1 \\ 0, & \text{o.w.} \end{cases}$$

$\square$ $P(.3 < X \leq .6) = \int_{.3}^{.6} 1.5 \sqrt{x} dx = 1.5 \left[\frac{x^{3/2}}{3/2} \right]_{.3}^{.6} = .3004$

$f(x)$

$F(x) = \int_{-\infty}^x f(t) dt = \int_0^x \frac{3}{2} \sqrt{t} dt = x^{3/2}, 0 \leq x < 1$

$F(x) = 0$

$F(x) = 1$

$\square$ $E(X) = \int_0^1 x f(x) dx = \int_0^1 x (1.5 \sqrt{x}) dx = 1.5 \int_0^1 x^{3/2} dx = 1.5 \left[\frac{x^{5/2}}{5/2} \right]_0^1 = .6$

Q3 $\square$ $P(0 < X < 2) = F(2) - F(0) = \frac{2}{2+1} - \frac{0}{0+1} = \frac{2}{3} = .667$

$\square$ $P(X \leq k) = .5 \Rightarrow F(k) = .5 \Rightarrow \frac{k}{k+1} = .5 \Rightarrow k = .5k + .5 \Rightarrow .5k = .5 \Rightarrow k = 1$

if $f(x) = F'(x) = \frac{(x+1) - x}{(x+1)^2} = \frac{1}{(1+x)^2}, 0 \leq x$. Then,

$P(0 < X < 2) = \int_0^2 \frac{1}{(1+x)^2} dx = \int_1^3 \frac{1}{u^2} du$

$= \int_1^3 u^{-2} du = \frac{1}{-1} u^{-2+1} \Big|_1^3 = -\frac{1}{u} \Big|_1^3 = -\left[\frac{1}{3} - \frac{1}{1}\right] = -\left[\frac{1-3}{3}\right] = \frac{2}{3}$

$u = 1+x$
 $du = dx$
 $x=0 \Rightarrow u=1$
 $x=2 \Rightarrow u=3$

$P(X \leq k) = .5 \Rightarrow \int_0^k f(x) dx = .5 \Rightarrow \int_0^k \frac{1}{(1+x)^2} dx = .5 \Rightarrow \frac{k}{k+1} = .5 \Rightarrow k = 1$

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$$(1) \int_a^b e^{cx} dx = \frac{1}{c} e^{cx} \Big|_a^b, \quad c \in \mathbb{R}$$

$$(2) \int_a^b x^n e^{kx} dx = \left[\frac{x^n}{k} - \frac{\frac{d}{dx} x^n}{k^2} + \frac{\frac{d^2}{dx^2} x^n}{k^3} - \dots \right. \\ \left. \dots + (-1)^n \frac{\frac{d^n}{dx^n} x^n}{k^{n+1}} \right] e^{kx} \Big|_a^b$$

$$\int_a^b x^n e^{-kx} dx = - \left[\frac{x^n}{k} + \frac{\frac{d}{dx} x^n}{k^2} + \frac{\frac{d^2}{dx^2} x^n}{k^3} + \dots + \frac{\frac{d^n}{dx^n} x^n}{k^{n+1}} \right] e^{-kx} \Big|_a^b$$

where $n \in \mathbb{N}$, $k \in \mathbb{R}^+$ and $a, b \in \mathbb{R}$ which can take $-\infty$ and b can also take ∞ .

$$(3) \int_0^\infty x^{n-1} e^{-kx} dx = \frac{\Gamma(n)}{k^n}, \quad n \text{ and } k \in \mathbb{R}^+$$

where $\Gamma(n) = (n-1)!$ if $n \in \mathbb{N}$

$$(4) \frac{1}{\sqrt{2\pi}} \int_{-\infty}^\infty e^{-\frac{1}{2}x^2} dx = 1$$

$$(5) \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi} \quad \text{and} \quad \Gamma\left(\frac{3}{2}\right) = \frac{1}{2} \sqrt{\pi}$$

a) $f_1(x) = C(2x - x^3)$, $0 < x < \frac{5}{2}$

we have that $0 < 2x < 5$ and $-\frac{5^3}{2^3} < -x^3 < 0$

so, $-\frac{5^3}{2^3} < 2x - x^3 < 5$

we can see that it is impossible to multiply by C to make

$f_1(x) \geq 0$ on $0 < x < \frac{5}{2}$

c) $f_3(x) = C(2x^2 - 4x)$, $0 < x < 3$

we have that $0 < 2x^2 < 18$ and $-12 < -4x < 0$

so, $-12 < 2x^2 - 4x < 8$

we can see that it is impossible to multiply by C to make

$f_3(x) \geq 0$ on $0 < x < 3$

8

1

Q5

$$a) \int_{-1}^1 f(x) dx = 1 \Leftrightarrow \int_{-1}^1 c(1-x^2) dx = 1$$

$$\Leftrightarrow c \left[x - \frac{x^3}{3} \right]_{-1}^1 = c \left[\left(1 - \frac{1}{3}\right) - \left(-1 + \frac{1}{3}\right) \right] = 1 \Leftrightarrow c \left[\frac{2}{3} + \frac{2}{3} \right] = \frac{4}{3} c = 1$$

$$\Leftrightarrow c = \frac{3}{4}$$

$$\therefore f(x) = \begin{cases} \frac{3}{4} (1-x^2), & -1 < x < 1 \\ 0, & \text{o.w.} \end{cases}$$

b)

$$i) P(X < 0) = \frac{3}{4} \int_{-1}^0 (1-x^2) dx$$



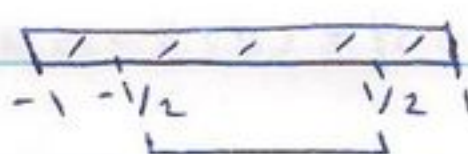
$$= \frac{3}{4} \left[x - \frac{x^3}{3} \right]_{-1}^0 = \frac{3}{4} \left[0 - \left(-1 + \frac{1}{3}\right) \right] = \frac{1}{2}$$

$$ii) P(X \geq \frac{1}{2}) = \frac{3}{4} \int_{1/2}^1 (1-x^2) dx$$



$$= \frac{3}{4} \left[x - \frac{x^3}{3} \right]_{1/2}^1 = \frac{3}{4} \left[\left(1 - \frac{1}{3}\right) - \left(\frac{1}{2} - \frac{1}{24}\right) \right] = \frac{5}{32}$$

$$iii) P(-\frac{1}{2} \leq x < \frac{1}{2}) = \frac{3}{4} \left[x - \frac{x^3}{3} \right]_{-1/2}^{1/2}$$



$$= \frac{3}{4} \left[\left(\frac{1}{2} - \frac{1}{24}\right) - \left(-\frac{1}{2} + \frac{1}{24}\right) \right] = \frac{11}{16}$$

d)

$$F(x) = P(X \leq x) = \begin{cases} 0, & x < -1 \\ \int_{-1}^x \frac{3}{4} (1-t^2) dt = \frac{3}{4} \left(t - \frac{1}{3} t^3 \right) \Big|_{-1}^x = \frac{3}{4} x - \frac{1}{4} x^3 + \frac{1}{2}, & -1 \leq x < 1 \\ 1, & +1 \leq x \end{cases}$$

$$e) P(X < 0) = F(0) = \frac{3}{4}(0) - \frac{1}{4}(0)^3 + \frac{1}{2} = \frac{1}{2}$$

$$P(X \geq \frac{1}{2}) = 1 - P(X < \frac{1}{2}) = 1 - F(\frac{1}{2}) = 1 - \left[\frac{3}{4}(\frac{1}{2}) - \frac{1}{4}(\frac{1}{2})^3 + \frac{1}{2} \right] = 1 - \frac{5}{32} = \frac{27}{32}$$

$$P(-\frac{1}{2} < X \leq \frac{1}{2}) = F(\frac{1}{2}) - F(-\frac{1}{2}) = \left[\frac{3}{4}(\frac{1}{2}) - \frac{1}{4}(\frac{1}{2})^3 + \frac{1}{2} \right] - \frac{5}{32} = \frac{27}{32} - \frac{5}{32} = \frac{22}{32} = \frac{11}{16}$$

$$P(X > 1) = 1 - P(X \leq 1) = 1 - F(1) = 1 - 1 = 0$$

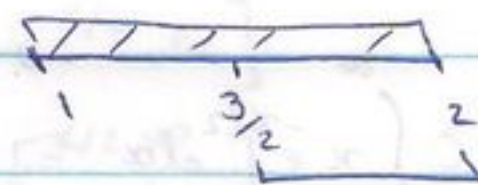
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Q 6

$$\textcircled{a} 1 = \int_1^2 c x^2 dx = c \left(\frac{1}{3} x^3 \Big|_1^2 \right) = c \left(\frac{8}{3} - \frac{1}{3} \right) = \frac{7}{3} c \Rightarrow c = \frac{3}{7}$$

$$\therefore f(x) = \begin{cases} \frac{3}{7} x^2, & 1 < x < 2 \\ 0, & \text{o.w.} \end{cases}$$

$$\textcircled{b} P\left(X > \frac{3}{2}\right) = \int_{3/2}^2 \frac{3}{7} x^2 dx = \frac{1}{7} x^3 \Big|_{3/2}^2 = \frac{8}{7} - \frac{27}{56} = \frac{37}{56}$$



$$\textcircled{c} F(x) = P(X \leq x) = \begin{cases} 0, & x < 1 \\ \int_1^x \frac{3}{7} t^2 dt = \frac{1}{7} t^3 \Big|_1^x = \frac{1}{7} x^3 - \frac{1}{7}, & 1 \leq x < 2 \\ 1, & 2 \leq x \end{cases}$$

$$\textcircled{d} P\left(X > \frac{3}{2}\right) = 1 - P\left(X \leq \frac{3}{2}\right) = 1 - F\left(\frac{3}{2}\right) = 1 - \left(\frac{1}{7} \cdot \frac{27}{8} - \frac{1}{7}\right) = \frac{37}{56}$$

Q 7

$$P(a \leq X \leq b) = \int_a^b f(x) dx = \int_{-\infty}^b f(x) dx - \int_{-\infty}^a f(x) dx = F(b) - F(a)$$



where $F(x) = P(X \leq x) = \int_{-\infty}^x f(x) dx$ and we know that

$$\int_a^b f(x) dx = P(a < X < b) = P(a \leq X < b) = P(a < X \leq b) = P(a \leq X \leq b).$$

Q 8

$$P(X \geq c) = 1 - P(X \leq c) = 1 - F(c)$$

$$\left(\int_c^{\infty} f(x) dx = \int_{-\infty}^{\infty} f(x) dx - \int_{-\infty}^c f(x) dx = 1 - P(X \leq c) = 1 - F(c) \right)$$



Q9

(a) $P(X \geq 5) = C \int_5^{\infty} x e^{-x/2} dx$

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$$= -C \left[\frac{x}{1/2} + \frac{1}{(1/2)^2} \right] e^{-x/2} \Big|_5^{\infty}$$

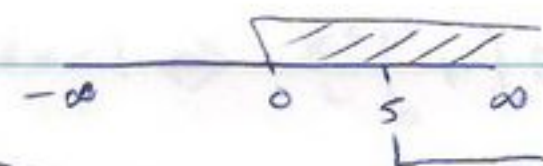
$$= -C [2x + 4] e^{-x/2} \Big|_5^{\infty}$$

$$= -C [2x e^{-x/2} + 4 e^{-x/2}] \Big|_5^{\infty}$$

$$= 0 - (-C) [10 e^{-5/2} + 4 e^{-5/2}]$$

$$= C 14 e^{-5/2} = 1.149 C$$

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$u = x \Rightarrow du = dx, dv = e^{-x/2} dx \Rightarrow v = -2 e^{-x/2}$

$$= C [-2x e^{-x/2} \Big|_5^{\infty} + 2 \int_5^{\infty} e^{-x/2} dx]$$

$$= C [-2 [0 - 5 e^{-5/2}] - 4 e^{-x/2} \Big|_5^{\infty}]$$

$$= C [10 e^{-5/2} - 4 (0 - e^{-5/2})] = 10 e^{-5/2} + 4 e^{-5/2}$$

$$= C [14 e^{-5/2}] = 1.149 C$$

(b) $P(3 < X < 6) = C \int_3^6 x e^{-x/2} dx$

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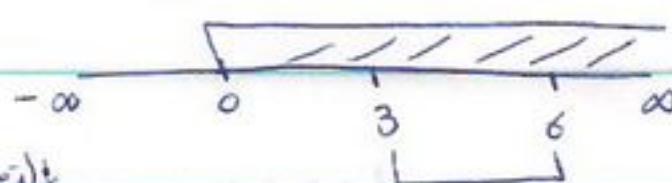
$$= -C \left[\frac{x}{1/2} + \frac{1}{(1/2)^2} \right] e^{-x/2} \Big|_3^6$$

$$= -C [2x + 4] e^{-x/2} \Big|_3^6$$

$$= C [10 e^{-3/2} - 16 e^{-3}]$$

$$= 1.435 C$$

بالتعويض



$u = x \Rightarrow du = dx, dv = e^{-x/2} dx \Rightarrow v = -2 e^{-x/2}$

$$= C [-2x e^{-x/2} \Big|_3^6 + 2 \int_3^6 e^{-x/2} dx]$$

$$= C [-12 e^{-3} + 6 e^{-3/2} - 4 e^{-x/2} \Big|_3^6]$$

$$= C [-12 e^{-3} + 6 e^{-3/2} - 4 e^{-3} + 4 e^{-3/2}]$$

$$= C [10 e^{-3/2} - 16 e^{-3}] = 1.435 C$$

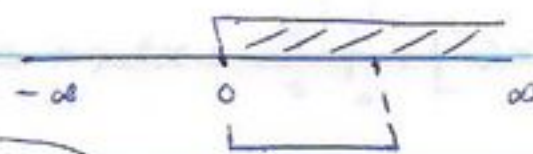
(c) $P(X < 1) = C \int_0^1 x e^{-x/2} dx$

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$$= -C [(-2x + 4) e^{-x/2} \Big|_0^1]$$

$$= -C [6 e^{-1/2} - 4]$$

$$= .360816 C$$



يمكن حلها كذلك بالتعويض
كما في الفقرات السابقة

Q10

(a) $P(Y \leq 5) = F(5) = 1 - \frac{4}{25} = \frac{16}{25} = .64$

(b) $P(Y > 8) = 1 - P(Y \leq 8) = 1 - F(8) = 1 - (1 - \frac{9}{64}) = \frac{9}{64} = .1406$

(c) $f(y) \xrightarrow{\text{تفاضل}} F(y)$

$f(y) = \frac{d}{dy} F(y) = (F(y))' = \frac{18y}{y^4} = \frac{18}{y^3}$

$$\therefore F(y) = \begin{cases} \frac{18}{y^3}, & y > 3 \\ 0, & \text{o.w} \end{cases}$$

(11)

7.11

$$(a) \int_{-\infty}^{\infty} f(x) dx = 1 \Leftrightarrow \int_0^1 x dx + \int_1^c (2-x) dx = 1 \Leftrightarrow \frac{x^2}{2} \Big|_0^1 + \left[2x - \frac{x^2}{2} \right] \Big|_1^c = 1$$

$$\Leftrightarrow \frac{1}{2} + \left[(2c - \frac{c^2}{2}) - (2 - \frac{1}{2}) \right] = 1 \Leftrightarrow \frac{1}{2} + \left[2c - \frac{c^2}{2} - \frac{3}{2} \right] = 1$$

$$\Leftrightarrow -\frac{1}{2}c^2 + 2c - 1 = 1 \Leftrightarrow -\frac{1}{2}c^2 + 2c - 2 = 0$$

بشكل مربع

$$a = -\frac{1}{2}, b = 2, c = -2$$

$$c = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{4 - 4}}{2(-\frac{1}{2})} = \frac{-2}{-1} = 2$$

$$-\frac{1}{2}[c^2 - 4c + 4] = 0$$

$$\Leftrightarrow c^2 - 4c + 4 = 0 \Leftrightarrow (c-2)^2 = 0$$

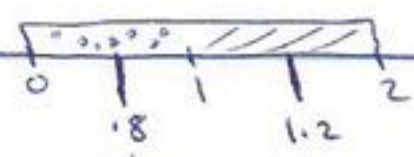
$$\Leftrightarrow c - 2 = 0 \Leftrightarrow c = 2$$

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 2-x, & 1 < x < 2 \\ 0, & \text{o.w.} \end{cases}$$

$$(b) F(x) = P(X \leq x) = \begin{cases} 0, & x < 0 \\ \int_0^x x dx = \frac{1}{2}x^2 \Big|_0^x = \frac{1}{2}x^2, & 0 \leq x < 1 \\ \int_0^1 x dx + \int_1^x (2-x) dx = \frac{x^2}{2} \Big|_0^1 + \left[2x - \frac{x^2}{2} \right] \Big|_1^x = 2x - \frac{x^2}{2} - 1, & 1 \leq x < 2 \\ 1, & 2 \leq x \end{cases}$$

$$(c) P(.8 < X < .6c) = P(.8 < X < 1.2), \text{ where } .6c = .6(2) = 1.2$$

من طريق



$$= \int_{.8}^1 x dx + \int_1^{1.2} (2-x) dx$$

F(x) من طريق

$$= P(X < 1.2) - P(X < .8)$$

$$= F(1.2) - F(.8)$$

$$= \left[2(1.2) - \frac{(1.2)^2}{2} - 1 \right] - \left[\frac{1}{2}(.8)^2 \right]$$

$$= .68 - .32 = .36$$