King Saud University
College of Sciences
Department of Mathematics
Second MidTerm, First Term 1435/1436
M-106

Exercice 1 [5 Marks]:

a. Find the integral $\int \tan^2(x)dx$.

We have

$$\begin{aligned} \int \tan^2(x)dx &= \int (\sec^2(x) - 1)dx \text{ [0.5]} \\ &= \tan x - x + c \text{ [0.5]} \end{aligned}$$

b. Find the integral $\int \frac{\sinh(x)}{1+\cosh^2(x)}dx$.

We have

$$\begin{aligned} \int \frac{\sinh(x)}{1+\cosh^2(x)}dx &= \int \frac{1}{1+u^2}du \ (u = \cosh x,\ du = \sinh x dx) \qquad \text{[0.5]} \\ &= \tan^{-1}(u) + c = \tan^{-1}(\cosh x) + c \qquad \text{[0.5]} \end{aligned}$$

c. Find $\lim_{x \longrightarrow 49} \frac{x-49}{\sqrt{x}-7}$.

we have

$$\begin{aligned} \lim_{x \longrightarrow 49} \frac{x-49}{\sqrt{x}-7} &= \lim_{x \longrightarrow 49} \frac{1}{\frac{1}{2\sqrt{x}}} \text{ [0.5]} \\ &= 14 \text{ [0.5]} \end{aligned}$$

d. If $\frac{6x-11}{(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2}$, find then A.

We have

$$6x - 11 = A(x-1) + B \qquad \text{[0.5]}$$

Then

$$A = 6 \quad [0.5]$$

e. Evaluate the intergral $\int \ln(\frac{x}{4})dx$

We have

$$\begin{aligned}\int \ln(\frac{x}{4})dx &= 4\int \ln(u)du \ [0.5]\\ &= x\ln\frac{x}{4} - x + c \ \ [0.5]\end{aligned}$$

Exercice 2 [3 Marks]: Evaluate the integral $\int \tan^5(x)\sec(x)dx$.

We have

$$\begin{aligned}\int \tan^5(x)\sec(x)dx &= \int \tan^4(x)\tan(x)\sec(x)dx = \int (\sec^2(x)-1)^2\tan(x)\sec(x)dx \ [1]\\ &= \int (u^2-1)^2 du \quad [u = \sec x] \ [1]\\ &= \frac{u^5}{5} - 2\frac{u^3}{3} + u + c = \frac{(\sec x)^5}{5} - 2\frac{(\sec x)^3}{3} + \sec x + c \ \ [1]\end{aligned}$$

Exercice 3 [4 Marks]: Evaluate the integral $\int \sec^2(x)\ln(\sin(x))dx$.

We have

$$\begin{aligned}\int \sec^2(x)\ln(\sin(x))dx &= \tan(x)\ln(\sin x) - \int \tan x\frac{1}{\sin x}\cos x dx \ [2]\\ &= \tan(x)\ln(\sin x) - \int dx \ = \tan(x)\ln(\sin x) - x + c \ [2]\end{aligned}$$

Exercice 4 [4 Marks]: Evaluate the integral $\int \frac{\sqrt{x^2-25}}{x^4}dx$.

Let

$$x = 5\sec\theta \text{ then } dx = 5\sec\theta\tan\theta d\theta \ [1]$$

$$\begin{aligned}\int \frac{\sqrt{x^2-25}}{x^4}dx &= \int \frac{5\tan\theta}{(5\sec\theta)^4}5\sec\theta\tan\theta d\theta = \frac{1}{25}\int \sin^2\theta\cos\theta d\theta \ [1]\\ &= \frac{1}{25}\frac{\sin^3\theta}{3} + c \ \ [1]\\ &= \frac{1}{75}(\frac{\sqrt{x^2-25}}{x})^3 + c, \ \text{where } \sin\theta = \frac{\sqrt{x^2-25}}{x} \quad [1]\end{aligned}$$

Exercice 5 [4 Marks]: Evaluate the integral $\int \frac{1}{2+\cos(x)}dx$.

Let

$$u = \tan(\frac{x}{2}) \text{ then } \cos x = \frac{1-u^2}{1+u^2},\ dx = \frac{2}{1+u^2}du \quad [1+0.5+0.5]$$

We have

$$\begin{aligned} \int \frac{1}{2+\cos(x)}dx &= \int \frac{1}{2+\frac{1-u^2}{1+u^2}} \frac{2}{1+u^2}du = \int \frac{2}{u^2+3}du \quad [1] \\ &= \frac{2}{\sqrt{3}}\tan^{-1}(\frac{u}{\sqrt{3}}) + c = \frac{2}{\sqrt{3}}\tan^{-1}(\frac{\tan(\frac{x}{2})}{\sqrt{3}}) + c \quad [1] \end{aligned}$$

Exercice 6 [5 Marks]: Evaluate the integral $\int \frac{x+1}{x^3+x}dx$

We have

$$\frac{x+1}{x^3+x}dx = \frac{1}{x} + \frac{-x+1}{x^2+1} \quad [1.5]$$

and then

$$\begin{aligned} \int \frac{x+1}{x^3+x}dx &= \int \frac{1}{x}dx + \int \frac{-x+1}{x^2+1}dx \\ &= \int \frac{1}{x}dx - \int \frac{x}{x^2+1}dx + \int \frac{1}{x^2+1}dx \ [1] \\ &= \ln|x| - \frac{1}{2}\ln(x^2+1) + \tan^{-1}(x) + c \ [0.5+1+1] \end{aligned}$$