KING SAUD UNIVERSITY
College of Science
Department of Mathematics

M-106

First Semester (1431/1432)

Solution Badeel Second Mid-Exam

Name:	Number:
Name of Teacher:	Group No:

Max Marks: 20 Time: 90 minutes

Marks:

Multiple Choice (1-10)	
Question # 11	
Question # 12	
Question # 13	
Question # 14	
Total	

Multiple Choice

Q.No:	1	2	3	4	5	6	7	8	9	10
$\{a, b, c, d\}$	b	a	c	d	b	b	a	b	c	a

Q. No: 1 $\lim\limits_{x\to\infty} (e^x + x)^{\frac{1}{x}}$ is equal to:

(a) e^2 (b) e (c) 1 (d) ∞

Q. No: 2 The partial fraction decomposition of $\dfrac{x^3}{(x-1)^2}$ takes the form:

(a) $x + \frac{A}{x-1} + \frac{B}{(x-1)^2}$ (b) $\frac{A}{x-1} + \frac{B}{(x-1)^2}$ (c) $x + \frac{A}{(x-1)^2}$ (d) $\frac{A}{x+1} + \frac{Bx+C}{(x-1)^2}$

Q. No: 3 To evaluate the integral $\displaystyle\int \frac{x^3}{\sqrt{9-x^2}}dx$, we use the substitution:

(a) $x = 3\tan\theta$ (b) $\theta = 3\sin x$ (c) $x = 3\sin\theta$ (d) $x = 3\sec\theta$

Q. No: 4 The value of the integral $\displaystyle\int_0^{\frac{\pi}{3}} \sin^4(3x)\cos^3(3x)\,dx$ is equal to:

(a) 1 (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) 0

Q. No: 5 The substitution $u = \tan\left(\frac{x}{2}\right)$ transforms the integral $\displaystyle\int \frac{1}{1+\sin x+\cos x}dx$ into:

(a) $\int \frac{1}{1+2u}du$ (b) $\int \frac{1}{1+u}du$ (c) $\int \frac{1}{1-u}du$ (d) $\int \frac{1}{u^2+u+1}du$

Q. No: 6 To evaluate the integral $\displaystyle\int \frac{\sec x}{\cot^5(x)}dx$, we use the substitution:

(a) $u = \tan x$ (b) $u = \sec x$ (c) $u = \cot x$ (d) $u = \sin x$

Q. No: 7 The improper integral $\displaystyle\int_0^1 \frac{1}{x^{2\alpha}}dx$ converges if

(a) $\alpha < \frac{1}{2}$ (b) $\alpha = \frac{1}{2}$ (c) $\frac{1}{2} < \alpha < \frac{3}{2}$ (d) $\alpha \geq \frac{3}{2}$

Q. No: 8 The value of the integral $\displaystyle\int \frac{1}{\sqrt{x^2+2x+2}}dx$ is equal to:

(a) $\sinh^{-1}(x-1)+c$ (b) $\sinh^{-1}(x+1)+c$ (c) $\sin^{-1}(x+1)+c$
(d) $\sin^{-1}(x-1)+c$

Q. No: 9 The improper integral $\int_0^\infty xe^{-x^2}dx$

(a) diverges (b) converges to 1 (c) converges to $\frac{1}{2}$ (d) converges to -1

Q. No: 10 The value of the integral $\int \sec^4 x \tan^2 x dx$ is equal to:

(a) $\frac{1}{3}\tan^3 x + \frac{1}{5}\tan^5 x + c$ (b) $\frac{1}{3}\tan^3 x - \frac{1}{5}\tan^5 x + c$

(c) $\frac{-1}{3}\tan^3 x + \frac{1}{5}\tan^5 x + c$ (d) $\frac{-1}{3}\tan^3 x - \frac{1}{5}\tan^5 x + c$

Full Questions

Question No: 11 **Evaluate** $\int \frac{\sin^{-1}(\ln x)}{x}dx$ [**3**]

Solution:

Let $u = \ln x \Rightarrow du = \frac{1}{x}dx$

$$\begin{aligned}\int \frac{\sin^{-1}(\ln x)}{x}dx &= \int \sin^{-1}(u)\,du \\ &= u\sin^{-1}u + \sqrt{1-u^2} + c \qquad \text{(By Integration by parts)} \\ &= (\ln x)\sin^{-1}(\ln x) + \sqrt{1-(\ln x)^2} + c\end{aligned}$$

Question No: 12 **Evaluate** $\int \frac{1}{x\sqrt{1+x^2}}dx$ [**2**]

Solution:

Let $x = \tan\theta \Rightarrow dx = \sec^2\theta d\theta$

$$\int \frac{1}{x\sqrt{1+x^2}}dx = \int \csc\theta d\theta = \ln|\csc\theta - \cot\theta| + c = \ln\left|\frac{\sqrt{1+x^2}}{x} - \frac{1}{x}\right| + c$$

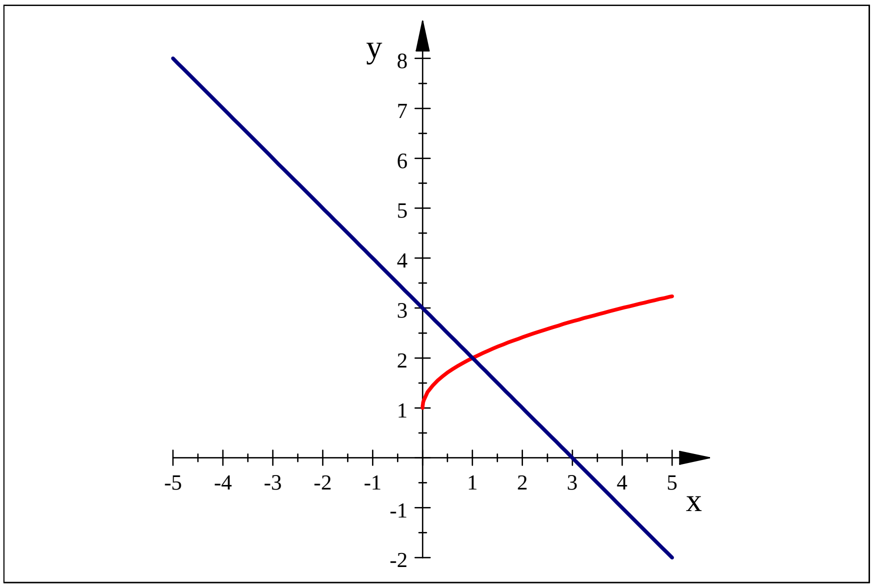
Question No: 13 **Evaluate** $\displaystyle\int \frac{x-3}{x^3-1}dx$ [**3**]

Solution:

$$\int \frac{x-3}{x^3-1}dx = \int \frac{\frac{-2}{3}}{x-1}dx + \int \frac{\frac{2}{3}x+\frac{7}{3}}{x^2+x+1}dx$$
$$= \tfrac{-2}{3}\ln|x-1| + \tfrac{1}{3}\ln|x^2+x+1| + \tfrac{4}{\sqrt{3}}\tan^{-1}\left(\tfrac{2}{\sqrt{3}}\left(x+\tfrac{1}{2}\right)\right)$$

Question No: 14 Let R be the region bounded by the graphs of $y = 1+\sqrt{x}$, $y = 3-x$, $x-$axis and $y-$axis. **Sketch** the region R and **set up** (Do not evaluate) an integral that can be used to find its **area**. [**2**]

Solution:



$y = 1+\sqrt{x}$ and $y = 3-x$

$$A = \int_0^1 (1+\sqrt{x})\,dx + \int_1^3 (3-x)\,dx$$