First Midterm Exam

Sunday, Safar 02, 1439	PHYS 201	Academic year 1438-39 H
7:00 – 8:30 pm	Mathematical Physics I	First Semester

SOLUTION

P1 (3 Marks)

$$\begin{aligned} 3x \quad - 2z &= 4 \\ 7x + y + 4z &= -1 \\ 2y \quad\quad &= 5 \end{aligned}$$

P2 (4 Marks)

$$\begin{pmatrix} 1 & 2 & -3 & 6 \\ 2 & -1 & 4 & 1 \\ 1 & -1 & 1 & 3 \end{pmatrix} \xrightarrow{r_{12}^{-2}} \begin{pmatrix} 1 & 2 & -3 & 6 \\ 0 & -5 & 10 & -11 \\ 0 & -1 & 1 & 3 \end{pmatrix} \xrightarrow{r_2^{-\frac{1}{5}}} \begin{pmatrix} 1 & 2 & -3 & 6 \\ 0 & 1 & -2 & \frac{11}{5} \\ 1 & -1 & 1 & 3 \end{pmatrix}$$

$$\xrightarrow{r_{13}^{-1}} \begin{pmatrix} 1 & 2 & -3 & 6 \\ 0 & 1 & -2 & \frac{11}{5} \\ 0 & -3 & 4 & -3 \end{pmatrix} \xrightarrow{r_{23}^{3}} \begin{pmatrix} 1 & 2 & -3 & 6 \\ 0 & 1 & -2 & \frac{11}{5} \\ 0 & 0 & -2 & \frac{18}{8} \end{pmatrix}$$

$$\xrightarrow{r_{32}^{-1}} \begin{pmatrix} 1 & 2 & -3 & 6 \\ 0 & 1 & 0 & -\frac{7}{5} \\ 0 & 0 & -2 & \frac{18}{5} \end{pmatrix} \xrightarrow{r_3^{-\frac{1}{2}}} \begin{pmatrix} 1 & 2 & -3 & 6 \\ 0 & 1 & 0 & -\frac{7}{5} \\ 0 & 0 & 1 & -\frac{9}{5} \end{pmatrix}$$

$$\xrightarrow{r_{21}^{-2}} \begin{pmatrix} 1 & 0 & -3 & \frac{44}{5} \\ 0 & 1 & 0 & -\frac{7}{5} \\ 0 & 0 & 1 & -\frac{9}{5} \end{pmatrix} \xrightarrow{r_{31}^{3}} \begin{pmatrix} 1 & 0 & 0 & \frac{17}{5} \\ 0 & 1 & 0 & -\frac{7}{5} \\ 0 & 0 & 1 & -\frac{9}{5} \end{pmatrix}$$

$$\boxed{x_1 = \frac{17}{5} \quad x_2 = -\frac{7}{5} \quad x_3 = -\frac{9}{5}}$$

P 3 (4 Marks)

$$\left.\begin{array}{l} a - b = 9 \\ a + b = 3 \end{array}\right\} \Rightarrow 2a = 9 + 3 = 12 \Rightarrow a = 6$$

$$b = 3 - a = 3 - 6 = -3$$

$$\left.\begin{array}{l} 4d + c = 7 \\ 4d - 4c = 12 \end{array}\right\} \Rightarrow c + 4c = 7 - 12 = -5$$

$$5c = -5$$

$$c = -1$$

$$4d = 7 - c = 7 - (-1) = 7 + 1 = 8 \Rightarrow d = 2$$

So:

$a = 6$	$c = -1$
$b = -3$	$d = 2$

P 4 (4 Marks)

$$A = \begin{pmatrix} 3 & 0 & 0 \\ 0 & -1 & 3 \\ 0 & -3 & -1 \end{pmatrix}; \quad A^2 = \begin{pmatrix} 9 & 0 & 0 \\ 0 & -8 & -6 \\ 0 & 6 & -8 \end{pmatrix}$$

P 5 (4 Marks)

$$A^{-1} = 5\begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 20 & 10 \\ 5 & 15 \end{pmatrix}$$

$$\left(\begin{array}{cc|cc} 20 & 10 & 1 & 0 \\ 5 & 15 & 0 & 1 \end{array}\right) \xrightarrow{r_2^4} \left(\begin{array}{cc|cc} 20 & 10 & 1 & 0 \\ 20 & 60 & 0 & 4 \end{array}\right) \xrightarrow{r_{12}^{-1}} \left(\begin{array}{cc|cc} 20 & 10 & 1 & 0 \\ 0 & 50 & -1 & 4 \end{array}\right)$$

$$\xrightarrow{r_1^5} \left(\begin{array}{cc|cc} 100 & 50 & 5 & 0 \\ 0 & 50 & -1 & 4 \end{array}\right) \xrightarrow{r_{21}^{-1}} \left(\begin{array}{cc|cc} 100 & 0 & 6 & -4 \\ 0 & 50 & -1 & 4 \end{array}\right) \xrightarrow{r_1^{\frac{1}{2}}} \left(\begin{array}{cc|cc} 50 & 0 & 3 & -2 \\ 0 & 50 & -1 & 4 \end{array}\right)$$

So $\boxed{A = \frac{1}{50}\begin{pmatrix} 3 & -2 \\ -1 & 4 \end{pmatrix}}$

P6 (3 + 3 Marks)

1-

$$\begin{pmatrix} 6 & 4 & 1 & 0 \\ 2 & 3 & 0 & 1 \end{pmatrix} \xrightarrow{r_1^2,\, r_2^3} \begin{pmatrix} 6 & 8 & 2 & 0 \\ 6 & 9 & 0 & 3 \end{pmatrix} \xrightarrow{r_{12}^{-1}} \begin{pmatrix} 6 & 8 & 2 & 0 \\ 0 & 1 & -2 & 3 \end{pmatrix} \xrightarrow{r_2^8} \begin{pmatrix} 6 & 8 & 2 & 0 \\ 0 & 8 & -16 & 24 \end{pmatrix}$$

$$\xrightarrow{r_{21}^{-1}} \begin{pmatrix} 6 & 0 & 18 & -24 \\ 0 & 8 & -16 & 24 \end{pmatrix} \xrightarrow{r_1^4,\, r_2^3} \begin{pmatrix} 24 & 0 & 4\times 18 & -4\times 24 \\ 0 & 24 & -3\times 16 & 3\times 24 \end{pmatrix}$$

$$\frac{1}{24}\begin{pmatrix} 24 & 0 & 4\times 18 & -4\times 24 \\ 0 & 24 & -3\times 16 & 3\times 24 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 3 & -4 \\ 0 & 1 & -2 & 3 \end{pmatrix}$$

So $\boxed{A^{-1} = \begin{pmatrix} 3 & -4 \\ -2 & 3 \end{pmatrix}}$. We can verif that $A.A^{-1} = I_2$

2-

$$A^2 = \begin{pmatrix} 3\times 3 + 4\times 2 & 3\times 4 + 4\times 3 \\ 2\times 3 + 3\times 2 & 2\times 4 + 3\times 3 \end{pmatrix} = \begin{pmatrix} 17 & 24 \\ 12 & 17 \end{pmatrix}$$

$$A^2 - 2A^T = \begin{pmatrix} 17 & 24 \\ 12 & 17 \end{pmatrix} - 2\begin{pmatrix} 3 & 2 \\ 4 & 3 \end{pmatrix} = \begin{pmatrix} 17 & 24 \\ 12 & 17 \end{pmatrix} - \begin{pmatrix} 6 & 4 \\ 8 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} 11 & 20 \\ 4 & 11 \end{pmatrix}$$

and $tr(A^2 - 2A^T) = tr\begin{pmatrix} 11 & 20 \\ 4 & 11 \end{pmatrix} = 11 + 11 = 22$

$$\boxed{tr(A^2 - 2A^T) = 22}$$