



FIRST MID TERM EXAM
(SOLUTION)

Name (in Arabic):

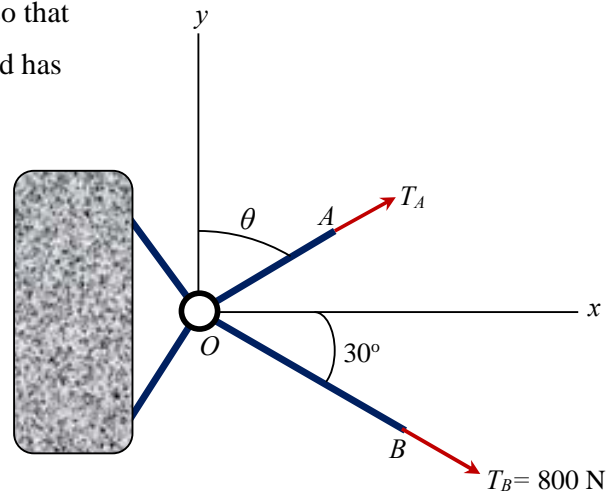
Student No.:

Section / Instructor:

Q. No.	Max. Marks	Marks Obtained
1	10	
2	10	
3	10	
Total	30	

Question # 1(a) (5 Marks)

Determine the magnitude of force T_A and its direction θ so that the resultant force is directed along the positive x axis and has a magnitude of 1250 N.



Solution

The force T_A can be obtained by applying the cosine law on ΔOAC :

$$c = \sqrt{a^2 + b^2 - 2ab \cos C}$$

$$\Rightarrow T_A = \sqrt{800^2 + 1250^2 - 2 \times 800 \times 1250 \times \cos 30^\circ}$$

$$\Rightarrow T_A = 685.9 \text{ N Ans.}$$

Applying the sine law on ΔOAC , we have

$$\frac{\sin(90 - \theta)}{800} = \frac{\sin 30^\circ}{685.9} \Rightarrow \sin(90 - \theta) = 0.583 \Rightarrow \theta = 54.33^\circ \text{ Ans.}$$

Alternatively,

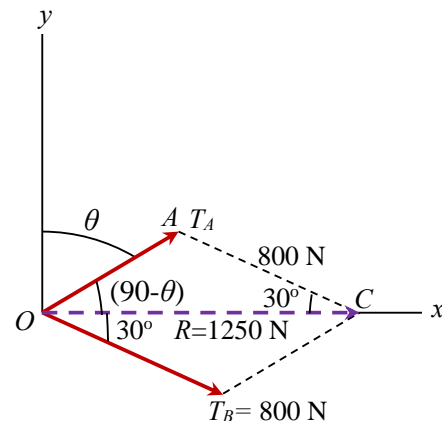
Note $R_y = 0$ and $R_x = R$

$$\uparrow R_y = \sum F_y = -800 \sin 30^\circ + T_A \cos \theta = 0 \Rightarrow T_A = \frac{400}{\cos \theta}$$

$$\rightarrow R_x = \sum F_x = T_A \sin \theta + 800 \cos 30^\circ = 1250$$

$$\Rightarrow \frac{400}{\cos \theta} \sin \theta + 692.82 = 1250 \Rightarrow 400 \tan \theta = 557.18$$

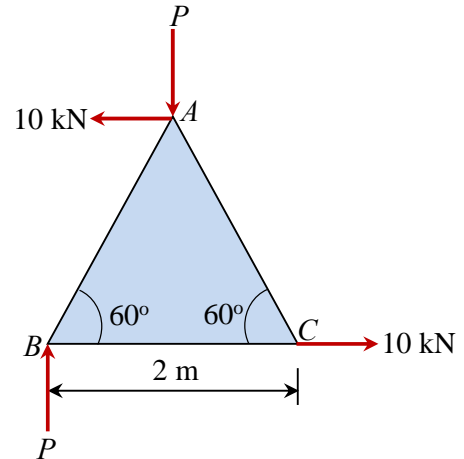
$$\Rightarrow \theta = 54.33^\circ \text{ and } T_A = \frac{400}{\cos \theta} = 685.9 \text{ N Ans.}$$



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Question # 1(b) (2.5 Marks)

Calculate the value of the force P that makes the resulting moment equals to zero.

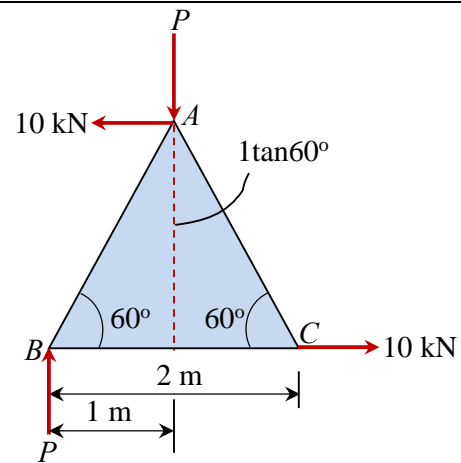


Solution

The given four forces form the two couples.

$$M_t = 10 \times 1 \tan 60^\circ - P \times 1 = 0$$

$$\Rightarrow P = 10 \tan 60^\circ = 17.3 \text{ kN} \quad \text{Ans.}$$



Question # 1(c) (2.5 Marks)

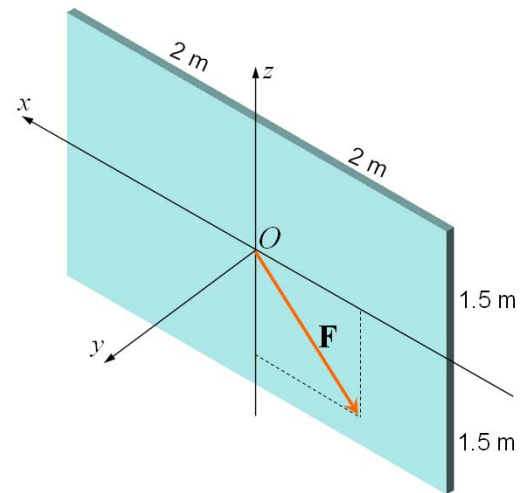
Determine the scalar projection of the force $\mathbf{F} = -10\mathbf{i} - 8\mathbf{k}$ kN on the x -, y - and z -axes.

Solution

Projection of the force on x -axis = $F_x = \mathbf{F} \cdot \mathbf{i} = -10$ kN *Ans.*

Projection of the force on y -axis = $F_y = \mathbf{F} \cdot \mathbf{j} = 0$ kN *Ans.*

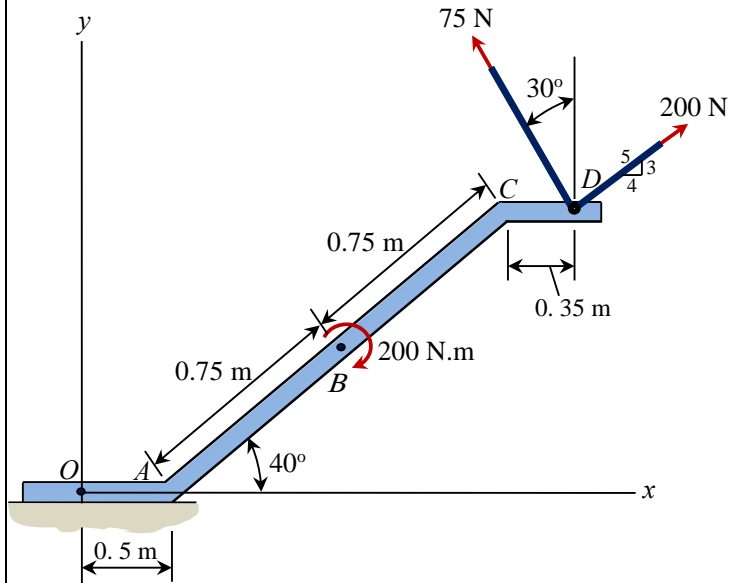
Projection of the force on z -axis = $F_z = \mathbf{F} \cdot \mathbf{k} = -8$ kN *Ans.*



Question # 2 (10 Marks)

For the force-system shown in the figure:

- i. Replace the two forces and one couple by an equivalent force-couple system (R and M) at point O .
- ii. Determine the direction of R .
- iii. Sketch the single resultant force R that represents the force-couple system alone and find its intersection with the x - and y -axes.



Solution

$$\cos \alpha = 4/5 = 0.8$$

$$\sin \alpha = 3/5 = 0.6$$

(i)

$$\rightarrow R_x = \sum F_x = 200 \cos \alpha - 75 \sin 30^\circ = 200 \times 0.8 - 75 \sin 30^\circ = 122.5 \text{ N} \rightarrow$$

$$\uparrow R_y = \sum F_y = 200 \sin \alpha + 75 \cos 30^\circ = 200 \times 0.6 + 75 \cos 30^\circ = 184.9 \text{ N} \uparrow$$

Therefore, $R = \sqrt{R_x^2 + R_y^2} = \sqrt{(122.5)^2 + (184.9)^2} = 221.8 \text{ N}$ *Ans.*

$$CCW(+)M_o = 75 \times \cos 30^\circ \times (0.5 + 1.5 \cos 40^\circ + 0.35) + 75 \sin 30^\circ \times 1.5 \sin 40^\circ$$

$$+ 200 \sin \alpha \times (0.5 + 1.5 \cos 40^\circ + 0.35) - 200 \cos \alpha \times 1.5 \sin 40^\circ - 200$$

$\Rightarrow M_o = 51.6 \text{ kN.m (CCW)}$ *Ans.*

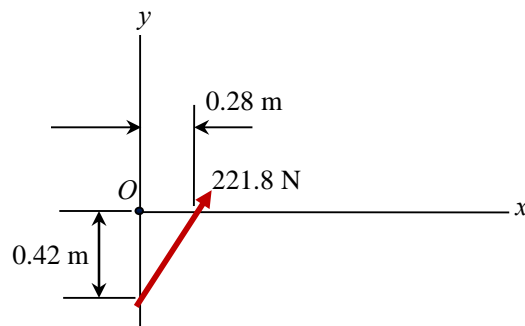
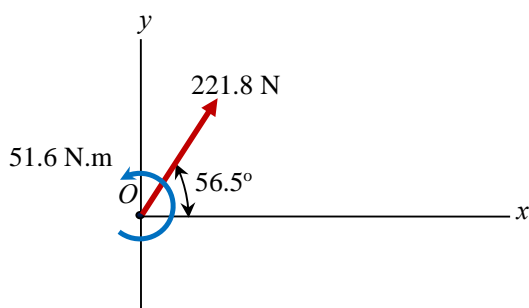
(ii)

$$\theta = \tan^{-1} \left(\frac{R_y}{R_x} \right) = \tan^{-1} \left(\frac{184.9}{122.5} \right) = 56.5^\circ \text{ } \textit{Ans.}$$

(iii)

x -intercept, $x = \frac{M_o}{R_y} = \frac{51.6}{184.9} = 0.28 \text{ m}$ *Ans.*

y -intercept, $y = -\frac{M_o}{R_x} = -\frac{51.6}{122.5} = -0.42 \text{ m}$ *Ans.*

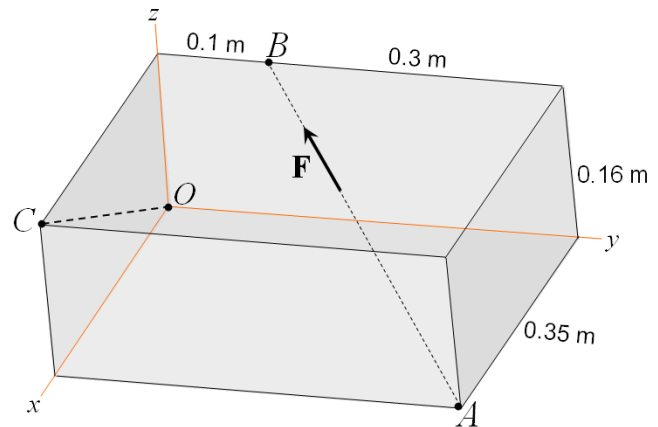


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Question # 3 (10 Marks)

The force $F = 976$ N is acting along the line AB as shown in the figure. Determine the following:

- The *magnitude* of the moment about line OC (M_{OC}), and express it in vector form (\vec{M}_{OC}).
- The moment about point B .



Solution

The coordinates of points O , A , B and C are: $O(0, 0, 0)$; $A(0.35, 0.4, 0)$; $B(0, 0.1, 0.16)$ and $C(0.35, 0, 0.16)$

(i) $\vec{M}_O = \vec{r}_{OB} \times \vec{F}$, where

$$\vec{r}_{OB} = (0-0)\vec{i} + (0.1-0)\vec{j} + (0.16-0)\vec{k} = 0.1\vec{j} + 0.16\vec{k}$$

$$\vec{F} = 976\vec{n}_{AB} = 976 \left(\frac{(0-0.35)\vec{i} + (0.1-0.4)\vec{j} + (0.16-0)\vec{k}}{\sqrt{(0-0.35)^2 + (0.1-0.4)^2 + (0.16-0)^2}} \right) = -700\vec{i} - 600\vec{j} + 320\vec{k} \text{ N}$$

Therefore,

$$\vec{M}_O = \vec{r}_{OB} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0.1 & 0.16 \\ -700 & -600 & 320 \end{vmatrix} \Rightarrow \vec{M}_O = 128\vec{i} - 112\vec{j} + 70\vec{k} \text{ N.m}$$

$M_{OC} = \vec{M}_O \cdot \vec{n}_{OC}$, where \vec{n}_{OC} is the unit vector along the line OC .

$$\vec{n}_{OC} = \frac{(0.35-0)\vec{i} + (0-0)\vec{j} + (0.16-0)\vec{k}}{\sqrt{(0.35-0)^2 + (0-0)^2 + (0.16-0)^2}} = \frac{0.35\vec{i} + 0.16\vec{k}}{\sqrt{0.35^2 + 0.16^2}} = 0.91\vec{i} + 0.42\vec{k}$$

$$\text{Therefore, } M_{OC} = \vec{M}_O \cdot \vec{n}_{OC} = (128\vec{i} - 112\vec{j} + 70\vec{k}) \cdot (0.91\vec{i} + 0.42\vec{k}) = 145.9 \text{ N.m} \quad \text{Ans.}$$

The above moment can be expressed in a vector form as

$$\vec{M}_{OC} = M_{OC}\vec{n}_{OC} = 145.9(0.91\vec{i} + 0.42\vec{k}) = 132.8\vec{i} + 61.3\vec{k} \text{ N.m} \quad \text{Ans.}$$

(ii) Since the line of action of the force is passing through the point B , the moment of the force about point B will be zero. That is, $M_B = 0$ Ans.