



KING SAUD UNIVERSITY
College of Science
Department of Mathematics

M-106

Second Semester (1430/1431)

Solution Final Exam

Name:	Number:
Name of Teacher:	Group No:

Max Marks: 50

Time: Three hours

Marks:

Multiple Choice (1-20)	
Question # 21	
Question # 22	
Question # 23	
Question # 24	
Question # 25	
Question # 26	
Total	

Multiple Choice

Q.No:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$\{a, b, c, d\}$	b	a	d	b	a	c	d	c'	a	a	b	a	a	a	b	c	a	d	b	a

Q. No: 1 If $\sum_{k=1}^6 (k^2 + 3k + 2\alpha) = 130$, then the value of α is equal to:

- (a) 2 (b) -2 (c) 1 (d) 3

Q. No: 2 The average value of the function $f(x) = \frac{1}{\sqrt{x^2+1}}$ on $[0, 1]$ is equal to:

- (a) $\sinh^{-1}(1)$ (b) $\cosh^{-1}(1)$ (c) $\sin^{-1}(1)$ (d) $\tan^{-1}(1)$

Q. No: 3 The number z that satisfies the conclusion of the Mean value Theorem for $f(x) = x^2$ on $[0, 2]$ is:

- (a) $\sqrt{\frac{8}{3}}$ (b) $\frac{8}{\sqrt{3}}$ (c) $\sqrt{\frac{2}{3}}$ (d) $\frac{2}{\sqrt{3}}$

Q. No: 4 If $F(x) = \int_{x-1}^{x+1} \tan(t^2) dt$, then $F'(x)$ is equal to:

- (a) $\tan(x^2 + 2x + 1) + \tan(x^2 - 2x + 1)$ (b) $\tan(x^2 + 2x + 1) - \tan(x^2 - 2x + 1)$
(c) $\tan(x^2 + 1) - \tan(x^2 - 1)$ (d) 0

Q. No: 5 The value of the integral $\int_0^1 3^x dx$ is equal to:

- (a) $\frac{2}{\ln 3}$ (b) $\frac{3}{\ln 3}$ (c) 3 (d) 2

Q. No: 6 If $f(x) = x^x$, then $f'(1)$ is equal to:

- (a) 0 (b) e (c) 1 (d) $\frac{1}{e}$

Q. No: 7 $\lim_{x \rightarrow \infty} \left(\frac{x}{\sqrt{x}-1} - \frac{x}{\sqrt{x}+1} \right)$ is equal to:

- (a) ∞ (b) 1 (c) 0 (d) 2

Q. No: 8 If $F(x) = \tan^{-1} x + \tan^{-1} \frac{1}{x}$ ($x \neq 0$), then $F'(x)$ is equal to:

- (a) $\frac{2}{1+x^2}$ (b) $\frac{-1}{1+x^2}$ (c) 0 (d) $\frac{1}{1+\left(\frac{1}{x^2}\right)}$

- Q. No: 9 The integral $\int \frac{x}{\cos^2(x^2)} dx$ is equal to:
- (a) $\frac{1}{2} \tan(x^2) + c$ (b) $\tan(x^2) + c$ (c) $\frac{1}{2} \tan(x) + c$ (d) $-\frac{1}{\cos(x^2)}$
- Q. No: 10 To evaluate the integral $\int \frac{3}{x^2\sqrt{9x^2-4}} dx$, we use the substitution:
- (a) $x = \frac{2}{3} \sec \theta$ (b) $x = \frac{2}{3} \tan \theta$ (c) $x = \frac{3}{2} \sec \theta$ (d) $x = \frac{4}{9} \tan \theta$
- Q. No: 11 The partial fraction decomposition of $\frac{x^2-2}{1-x^3}$ takes the form:
- (a) $\frac{A}{1-x} + \frac{Bx+C}{1-x+x^2}$ (b) $\frac{A}{1-x} + \frac{Bx+C}{1+x+x^2}$ (c) $\frac{A}{1-x} + \frac{Bx+C}{1+x^2}$ (d) $\frac{A}{1-x} + \frac{C}{1+x+x^2}$
- Q. No: 12 The substitution $u = \tan\left(\frac{x}{2}\right)$ transforms the integral $\int \frac{1}{1+\cos x} dx$ into:
- (a) $\int du$ (b) $\int \frac{1}{1+u} du$ (c) $\int \frac{2}{1-u} du$ (d) $\int \frac{2}{1+u} du$
- Q. No: 13 The area of the region bounded by the graphs of $y = \cos x$, $y = \sin x$, $x = 0$ and $x = \frac{\pi}{4}$ is equal to:
- (a) $\sqrt{2} - 1$ (b) 0 (c) $\sqrt{2} + 1$ (d) $1 - \sqrt{2}$
- Q. No: 14 The slope of the tangent line at the point corresponding to $t = \frac{\pi}{2}$ on the curve given parametrically by the equations $x = \sin^2 t$, $y = \cos t$; $\frac{\pi}{2} \leq t \leq 2\pi$ is:
- (a) ∞ (b) -1 (c) 0 (d) 1
- Q. No: 15 If a graph has polar equation $r = \frac{1}{2\sin\theta + \cos\theta}$, then its equation in xy -system is:
- (a) $x + 2y + 1 = 0$ (b) $x + 2y - 1 = 0$ (c) $2x + y + 1 = 0$ (d) $2x + y - 1 = 0$
- Q. No: 16 The length of the curve $C : x = 2 \cos t$, $y = 2 \sin t$; $0 \leq t \leq 1$ is equal to:
- (a) 1 (b) $\sqrt{2}$ (c) 2 (d) 4
- Q. No: 17 The surface area resulting by revolving the graph of the equation $x = y$, $0 \leq y \leq 4$ around the y -axis is equal to:
- (a) $16\sqrt{2}\pi$ (b) $\sqrt{2}\pi$ (c) $16\sqrt{2}$ (d) $8\sqrt{2}\pi$

Q. No: 18 If a point has xy -coordinates $(x, y) = (1, 1)$ then one of its (r, θ) -coordinates is:

(a) $\left(1, \frac{\pi}{2}\right)$ (b) $\left(1, \frac{\pi}{4}\right)$ (c) $\left(2, \frac{\pi}{4}\right)$ (d) $\left(\sqrt{2}, \frac{\pi}{4}\right)$

Q. No: 19 The slope of the tangent line to the graph of the equation $r = 2$ at $\theta = \frac{\pi}{4}$ is:

(a) 1 (b) -1 (c) 0 (d) ∞

Q. No: 20 The polar equation that has the same graph as the equation:

$x^4 + 2x^2y^2 + y^4 = 2xy$, is:

(a) $r^2 = \sin 2\theta$ (b) $r^2 = \frac{\sin 2\theta}{1 + \frac{1}{2} \sin 2\theta}$ (c) $r^2 = \cos 2\theta$ (d) $r^2 = \sin \theta \cos \theta$

Full Questions

Question No: 21 Approximate the integral $\int_0^2 \frac{1}{x^5 + 1} dx$ using the **Trapezoidal rule**

with $n = 4$. [4]

Solution:

Let $f(x) = \frac{1}{x^5 + 1}$. $\Delta x = \frac{1}{2}$

So $x_0 = 0$, $x_1 = \frac{1}{2}$, $x_2 = 1$, $x_3 = \frac{3}{2}$ and $x_4 = 2$.

$$\begin{aligned} \int_0^2 \frac{1}{x^5 + 1} dx &\approx \frac{2-0}{2 \times 4} \{f(0) + 2f\left(\frac{1}{2}\right) + 2f(1) + 2f\left(\frac{3}{2}\right) + f(2)\} \\ &\approx \frac{1}{4} \{1 + (2 \times 0.9697) + (2 \times \frac{1}{2}) + (2 \times 0.11636) + 0.0303\} \\ &\approx \frac{1}{4} \{1 + 1.9394 + 1 + 0.23272 + 0.0303\} \\ &\approx \frac{1}{4} \{4.2024\} \\ &\approx 1.0506. \end{aligned}$$

Question No: 22 Evaluate $\int \frac{x^2}{x^2 - 1} dx$. [4]

Solution:

$$\frac{x^2}{x^2 - 1} = 1 + \frac{1}{2(x-1)} - \frac{1}{2(x+1)}$$

So

$$\begin{aligned} \int \frac{x^2}{x^2 - 1} dx &= \int 1 dx + \int \frac{1}{2(x-1)} dx - \int \frac{1}{2(x+1)} dx \\ &= x + \frac{1}{2} \ln|x - 1| - \frac{1}{2} \ln|x + 1| + c. \end{aligned}$$

Question No: 23 Determine whether the improper integral $\int_0^2 \frac{1}{(x-1)^{\frac{2}{3}}} dx$ converges or diverges and if it converges find its value. [5]

Solution:

$$\begin{aligned} \int_0^2 \frac{1}{(x-1)^{\frac{2}{3}}} dx &= \int_0^1 \frac{1}{(x-1)^{\frac{2}{3}}} dx + \int_1^2 \frac{1}{(x-1)^{\frac{2}{3}}} dx \\ &= \lim_{t \rightarrow 1^-} \int_0^t (x-1)^{-\frac{2}{3}} dx + \lim_{t \rightarrow 1^+} \int_t^2 (x-1)^{-\frac{2}{3}} dx \\ &= \lim_{t \rightarrow 1^-} \left[\frac{(x-1)^{\frac{1}{3}}}{\frac{1}{3}} \right]_0^t + \lim_{t \rightarrow 1^+} \left[\frac{(x-1)^{\frac{1}{3}}}{\frac{1}{3}} \right]_t^2 \\ &= \lim_{t \rightarrow 1^-} 3 \left[(t-1)^{\frac{1}{3}} - (-1)^{\frac{1}{3}} \right] + \lim_{t \rightarrow 1^+} 3 \left[(1)^{\frac{1}{3}} - (t-1)^{\frac{1}{3}} \right]_t^2 \\ &= 3 + 3 \\ &= 6 \end{aligned}$$

Hence $\int_0^2 \frac{1}{(x-1)^{\frac{2}{3}}} dx$ converges and $\int_0^2 \frac{1}{(x-1)^{\frac{2}{3}}} dx = 6$.

Question No: 24 Find the length of the arc of the graph of the function

$$f(x) = \int_0^x \sqrt{\cos t} dt; 0 \leq x \leq \frac{\pi}{2}. \quad [5]$$

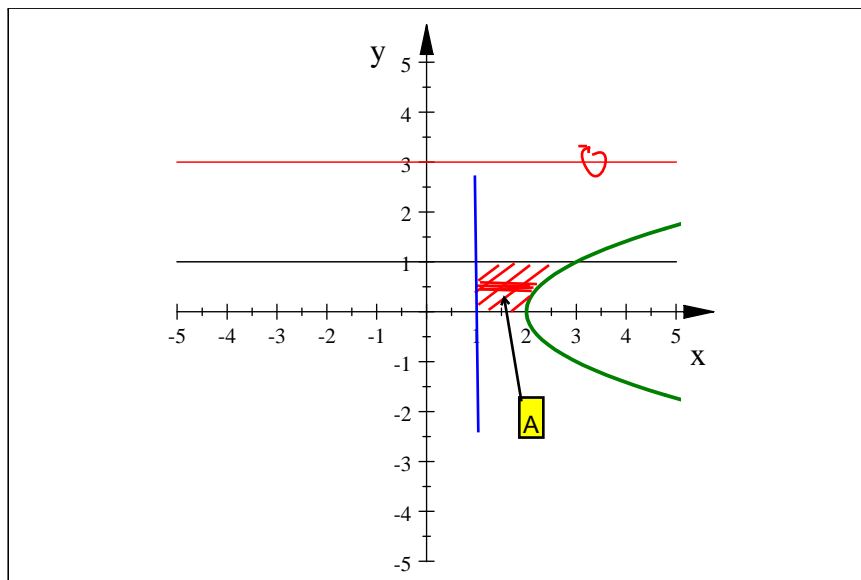
Solution:

$$\begin{aligned} L_0^{\frac{\pi}{2}} &= \int_0^{\frac{\pi}{2}} \sqrt{1 + (f'(x))^2} dx \\ &= \int_0^{\frac{\pi}{2}} \sqrt{1 + (\sqrt{\cos x})^2} dx \\ &= \int_0^{\frac{\pi}{2}} \sqrt{1 + \cos x} dx \\ &= \int_0^{\frac{\pi}{2}} \sqrt{2 \cos^2 \left(\frac{x}{2}\right)} dx \\ &= \sqrt{2} \int_0^{\frac{\pi}{2}} \left| \cos \left(\frac{x}{2}\right) \right| dx = \sqrt{2} \int_0^{\frac{\pi}{2}} \cos \left(\frac{x}{2}\right) dx \\ &= \sqrt{2} \left[2 \sin \left(\frac{x}{2}\right) \right]_0^{\frac{\pi}{2}} = 2 \end{aligned}$$

Question No: 25 Let R be the region bounded by the graph of the equations $x = y^2 + 2$, $x = 1$ and the horizontal lines $y = 0$, $y = 1$.

Sketch the region R and **set up** (Do not evaluate) an integral that can be used to find the **volume** of the solid generated by revolving the region R around the **line** $y = 3$. (Use **Cylindrical Shell**) [6]

Solution:



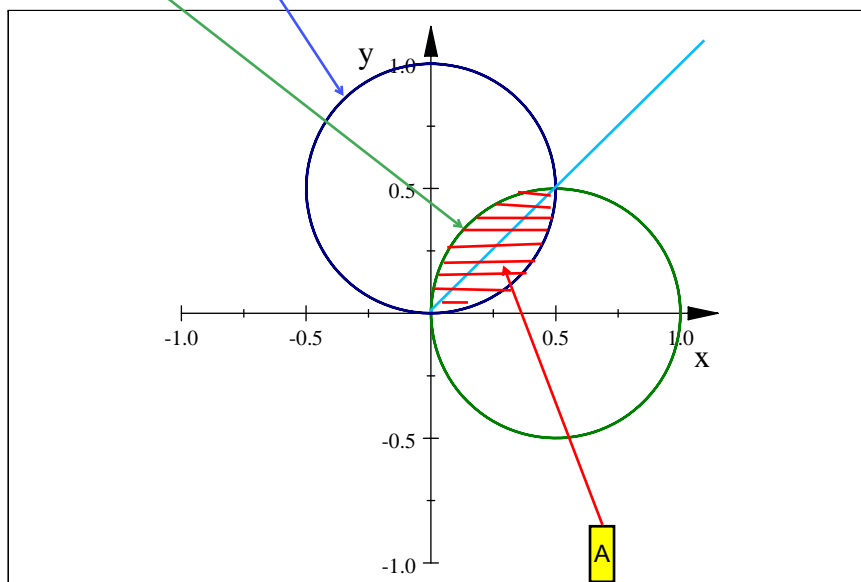
$$A = 2\pi \int_0^1 (3 - y)(y^2 + 1) dy$$

Question No: 26 Let R be the region that lies inside the graphs of **both** the equations:

$r = \cos \theta$ and $r = \sin \theta$.

Sketch the region R and **set up** (Do not evaluate) an integral that can be used to find its **area**. [6]

Solution:



$$A = \frac{1}{2} \int_0^{\frac{\pi}{4}} \sin^2 \theta d\theta + \frac{1}{2} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos^2 \theta d\theta$$