106Midterm2 Solutions (Sem1-37/38)

Question1 a) $lny = tanxlntanx \quad (\mathbf{0,5}) + \mathbf{2} + (\mathbf{0,5})$

$$\lim_{x\to 0^+} lny = \lim_{x\to 0^+} \frac{lntanx}{\frac{1}{tanx}} = \lim_{x\to 0^+} \frac{sec^2x}{tanx} \frac{tan^2x}{(-sec^2x)} = -\lim_{x\to 0^+} tanx = 0$$

So $\lim_{x\to 0^+} tanx^{tanx} = e^0 = 1$

Note this is the same as $\lim_{u\to 0^+} u^u$

b) $\int sinh^{-1} xdx = xsinh^{-1}x - \int \frac{xdx}{\sqrt{1+x^2}} \quad (\mathbf{1,5}) + (\mathbf{1,5})$

$$= xsinh^{-1}x - \sqrt{1+x^2} + C$$

c) $\int tan^5 xsecx = \int tanx^4 \, sextanxdx = \int (sec^2x - 1)^2 secxtanxdx$

$$= \frac{(secx)^5}{5} - \frac{2}{3} sec^3x + secx + C \quad (\mathbf{1}) + (\mathbf{1}) + (\mathbf{1})$$

Question2

a) $\int \frac{dx}{\sqrt{-x^2+6x-5}} = \int \frac{dx}{\sqrt{4-(x-3)^2}} = sin^{-1}\frac{(x-3)}{2} + C \quad (\mathbf{1}) + (\mathbf{1})$

b) $\int \frac{\sqrt{9-x^2}}{x^2} dx = \int \frac{9cos^2\theta d\theta}{9sin^2\theta} = \int (csc^2\theta - 1)d\theta \quad x = 3sin\theta$

$$= -cot\theta - \theta + C$$

$$= -\frac{\sqrt{9-x^2}}{x} - sin^{-1}\left(\frac{x}{3}\right) + C \quad (\mathbf{1}) + (\mathbf{1}) + (\mathbf{1})$$

c) $\frac{2x^2-x+2}{x(x^2+2)} = \frac{1}{x} + \frac{x-1}{x^2+2}$ $\quad \mathbf{(1,5) + (1,5)}$

$$\int \frac{(2x^2 - x + 2)dx}{x(x^2+2)} = \ln\,|x| + \frac{1}{2}\ln(x^2+2) - \frac{1}{\sqrt{2}}tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + C$$

Question3

a) $\int \frac{dx}{1-sinx} = 2\int \frac{du}{u^2-2u+1}$ $\quad u = \tan\left(\frac{x}{2}\right), dx = \frac{2du}{1+u^2}$ $\quad \mathbf{(1,5)}$

$$= 2\int \frac{du}{(u-1)^2} = \frac{-2}{u-1} + C = \frac{-2}{\tan\left(\frac{x}{2}\right)-1} + C \quad \mathbf{(1,5)}$$

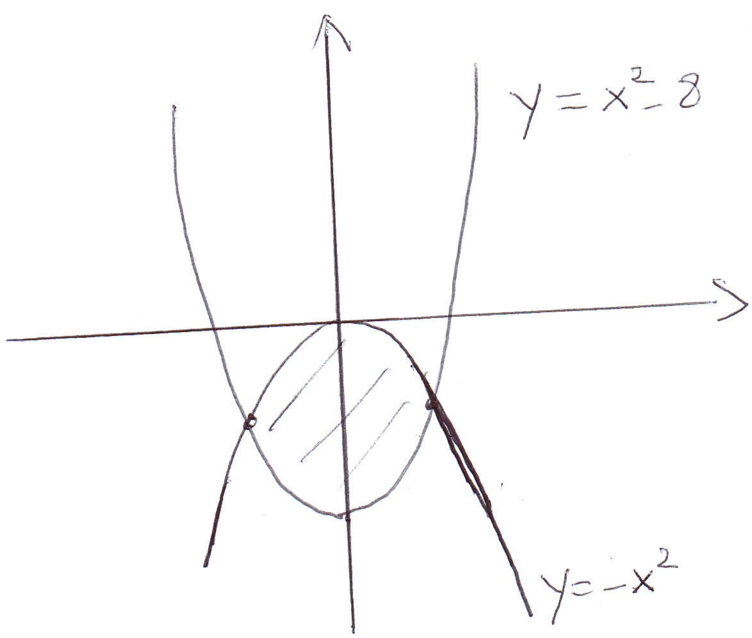
b) $\int_c^0 xe^x dx = [xe^x - e^x]_c^0 = -1 - ce^c + e^c$ $\quad \mathbf{(1)}$

$\lim_{c\to-\infty}(-1 - ce^c + e^c) = -1$,

so $\int_{-\infty}^0 xe^x dx\ cv\ and\ \int_{-\infty}^0 xe^x dx = -1$ $\quad \mathbf{(1)}$

c) Intersection points: $x^2 - 8 = -x^2\ so\ x = \pm 2$ $\quad \mathbf{(0,5)}$

$$A = 2\int_0^2 [-x^2 - (x^2-8)]dx = 2\left[-\frac{2}{3}x^3 + 8x\right]_0^2 = \frac{64}{3} \quad \mathbf{(1,5)}$$



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