College of Sciences
Department of
Physics and Astronomy

كلية العلوم
قسم الفيزياء والفلك

جامعة الملك سعود
King Saud University

Second Midterm Exam

Monday, Rajab 16, 1439	PHYS 201	Academic year 1438-39 H
7:00 – 8:30 pm	Mathematical Physics I	Second Semester

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SOLUTION

Instructions:

- **No calculator is allowed.**

Problem 1 (5 Marks)

We consider the matrix:

$$A = \begin{pmatrix} 1 & 2 & -2 \\ 1 & 1 & -1 \\ 2 & 1 & -2 \end{pmatrix}$$

Calculate:

1/ det(A) $= 1$

2/ Minor matrix $M_{\text{ij}} = \begin{pmatrix} -1 & 0 & -1 \\ -2 & 2 & -3 \\ 0 & 1 & -1 \end{pmatrix}$

3/ Cofactor matrix $C_{\text{ij}} = \begin{pmatrix} -1 & 0 & -1 \\ 2 & 2 & 3 \\ 0 & -1 & -1 \end{pmatrix}$

4/ Adj(A) $= \begin{pmatrix} -1 & 2 & 0 \\ 0 & 2 & -1 \\ -1 & 3 & -1 \end{pmatrix}$

5/ $A^{-1} = \begin{pmatrix} -1 & 2 & 0 \\ 0 & 2 & -1 \\ -1 & 3 & -1 \end{pmatrix}$

Problem 2 (5 Marks)

Solve the system of linear equations using the Cramer's rule:

$$\begin{cases} x_1 + 2x_2 - x_3 = 6 \\ x_1 + x_2 + 3x_3 = -3 \\ x_1 - x_2 + x_3 = -1 \end{cases}$$

$$x_1 = \frac{\begin{vmatrix} 6 & 2 & -1 \\ -3 & 1 & 3 \\ -1 & -1 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 2 & -1 \\ 1 & 1 & 3 \\ 1 & -1 & 1 \end{vmatrix}} = \frac{20}{10} = 2$$

$$x_2 = \frac{\begin{vmatrix} 1 & 6 & -1 \\ 1 & -3 & 3 \\ 1 & -1 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 2 & -1 \\ 1 & 1 & 3 \\ 1 & -1 & 1 \end{vmatrix}} = \frac{10}{10} = 1, \quad x_3 = \frac{\begin{vmatrix} 1 & 2 & 6 \\ 1 & 1 & -3 \\ 1 & -1 & -1 \end{vmatrix}}{\begin{vmatrix} 1 & 2 & -1 \\ 1 & 1 & 3 \\ 1 & -1 & 1 \end{vmatrix}} = \frac{-20}{10} = -2$$

Problem 3 (5 Marks)

Find the eigenvalues of the matrix:

$$A = \begin{pmatrix} 1 & 3 & 3 \\ 1 & -1 & -4 \\ -1 & -1 & 2 \end{pmatrix}$$

Hint: $\lambda^3 - 2\lambda^2 = \lambda(\lambda-1)^2 - \lambda$

$\det(A) = \lambda^3 - 2\lambda^2 - 5\lambda + 6$
$= \lambda(\lambda-1)^2 - \lambda - 5\lambda + 6$
$= \lambda(\lambda-1)^2 - 6\lambda + 6$
$= (\lambda-1)[\lambda(\lambda-1) - 6]$
$= (\lambda-1)(\lambda^2 - \lambda - 6)$
$= (\lambda-1)(\lambda+2)(\lambda-3)$
$\lambda_1 = 1;\ \lambda_2 = -2;\ \lambda_3 = 3$

MCQ 1 (2 Marks)
A determinant is equal to zero if a column is:

A/ multiple of a row
✓B/ multiple of a column
C/ divisor of a row
D/ none of the above

MCQ 2 (2 Marks)
A and B are general square matrices. We have:

A/ det(A+B) = det(A) + det(B)
B/ det(A+B) = det(A) det(B)
✓C/ det(A+B) ≠ det(A) + det(B)
D/ det(A+B) = det(A) + det(B) + 2 det(AB)

MCQ 3 (2 Marks)
The determinant of A = (a_{ij}), an upper triangular matrix 3x3 is:

A/ $a_{11}\, a_{12}\, a_{13}$
B/ $a_{11} + a_{22} + a_{33}$
✓C/ $a_{11}\, a_{22}\, a_{33}$
D/ $a_{11} + a_{12} + a_{13}$

MCQ 4 (2 Marks)
If A is an n x n matrix and c is a scalar, then:

A/ det(cA) = c det(A)
✓B/ det(cA) = c^n det(A)
C/ det(cA) = cn det(A)
D/ det(cA) = det(A)

MCQ 5 (2 Marks)

(1, 2, 2) is an eigenvector of $\begin{pmatrix} 4 & -2 & 1 \\ 2 & 0 & 1 \\ 2 & -2 & 3 \end{pmatrix}$. The corresponding eigenvalue is:

A/ 1 ✓B/ 2 C/ -1 D/ -2