

## First Midterm Exam

Monday, Jumada II 17, 1439	PHYS 201	Academic year 1438-39 H
7:00 – 8:30 pm	Mathematical Physics I	Second Semester

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25

Instructions:

- No calculator is allowed.

### Problem 1 ( 5 Marks)

Solve the linear system of equations by Gauss-Jordan elimination:

$$\begin{cases} 2x_2 - x_3 = 0 \\ x_1 + x_2 + x_3 = 2 \\ 4x_1 + 2x_2 + 3x_3 = 4 \end{cases}$$

$$\begin{pmatrix} 0 & 2 & -1 & 0 \\ 1 & 1 & 1 & 2 \\ 4 & 2 & 3 & 4 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & 2 & -1 & 0 \\ 4 & 2 & 3 & 4 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & 2 & -1 & 0 \\ 0 & 2 & 1 & 4 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 1 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 1 & 2 \end{pmatrix} \\ \rightarrow \begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix} \rightarrow \begin{cases} x_1 = -1 \\ x_2 = 1 \\ x_3 = 2 \end{cases}$$

### Problem 2 ( 4 Marks)

Using the Gauss-Jordan elimination method, find the inverse of the matrix:

$$A = \begin{pmatrix} 2 & -1 \\ 1 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -1 & 1 & 0 \\ 1 & -1 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & 0 & 1 \\ 2 & -1 & 1 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & 0 & 1 \\ 0 & 1 & 1 & -2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 1 & -2 \end{pmatrix} \rightarrow A^{-1} = \begin{pmatrix} 1 & -1 \\ 1 & -2 \end{pmatrix}$$

**Problem 3 ( 4 Marks)**

1/ Write the matrix  $A$  as a product of elementary matrices.

$$A = \begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$R_{21}^{-1}R_{12}^{-2}R_{21}^{-1}A = I$$

2/ Deduce the inverse of  $A$ .

$$A^{-1} = R_{21}^{-1}R_{12}^{-2}R_{21}^{-1} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & -4 \\ -2 & 3 \end{pmatrix}$$

**Problem 4 ( 4 Marks)**

The solution of the following equation is:

$$\begin{pmatrix} 10 & 3 \\ 5 & 6 \end{pmatrix} = \begin{pmatrix} 9+x & 3 \\ 7+2y & 6 \end{pmatrix}$$

A)  $x = -1, y = -1$

B)  $x = 1, y = 1$

C)  $x = 1, y = -1$

D)  $x = -1, y = 1$

**Problem 5 ( 4 Marks)**

Let  $A$  be the matrix:

$$\begin{pmatrix} 2 & 0 & 1 \\ 0 & -1 & 3 \\ 0 & -3 & 1 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 2 & 0 & 1 \\ 0 & -1 & 3 \\ 0 & -3 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 & 1 \\ 0 & -1 & 3 \\ 0 & -3 & 1 \end{pmatrix} = \begin{pmatrix} 4 & -3 & 3 \\ 0 & -8 & 0 \\ 0 & 0 & -8 \end{pmatrix} \text{ So}$$

The trace of  $A^2$  is:

A) 10

B) 12

C) -10

D) -12

**Problem 6 ( 4 Marks)**

Let  $A, B$  and  $C$  the matrices:

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, B = \begin{pmatrix} -3 & -1 \\ 3 & 3/2 \end{pmatrix} \text{ and } C = AB - A^T$$

Then, the matrix  $C$  is:

A)  $C = \begin{pmatrix} 2 & -1 \\ 1 & -1 \end{pmatrix}$

B)  $C = \begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix}$

C)  $C = \begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix}$

D)  $C = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$