Series-Parallel Networks

7.1 SERIES-PARALLEL NETWORKS

Series-parallel networks *are networks that contain both series and parallel circuit configurations.*

Only single-source dc network will be considered

General Approach:

1- Study the problem in total and make a mental sketch of the approach you plan to use.

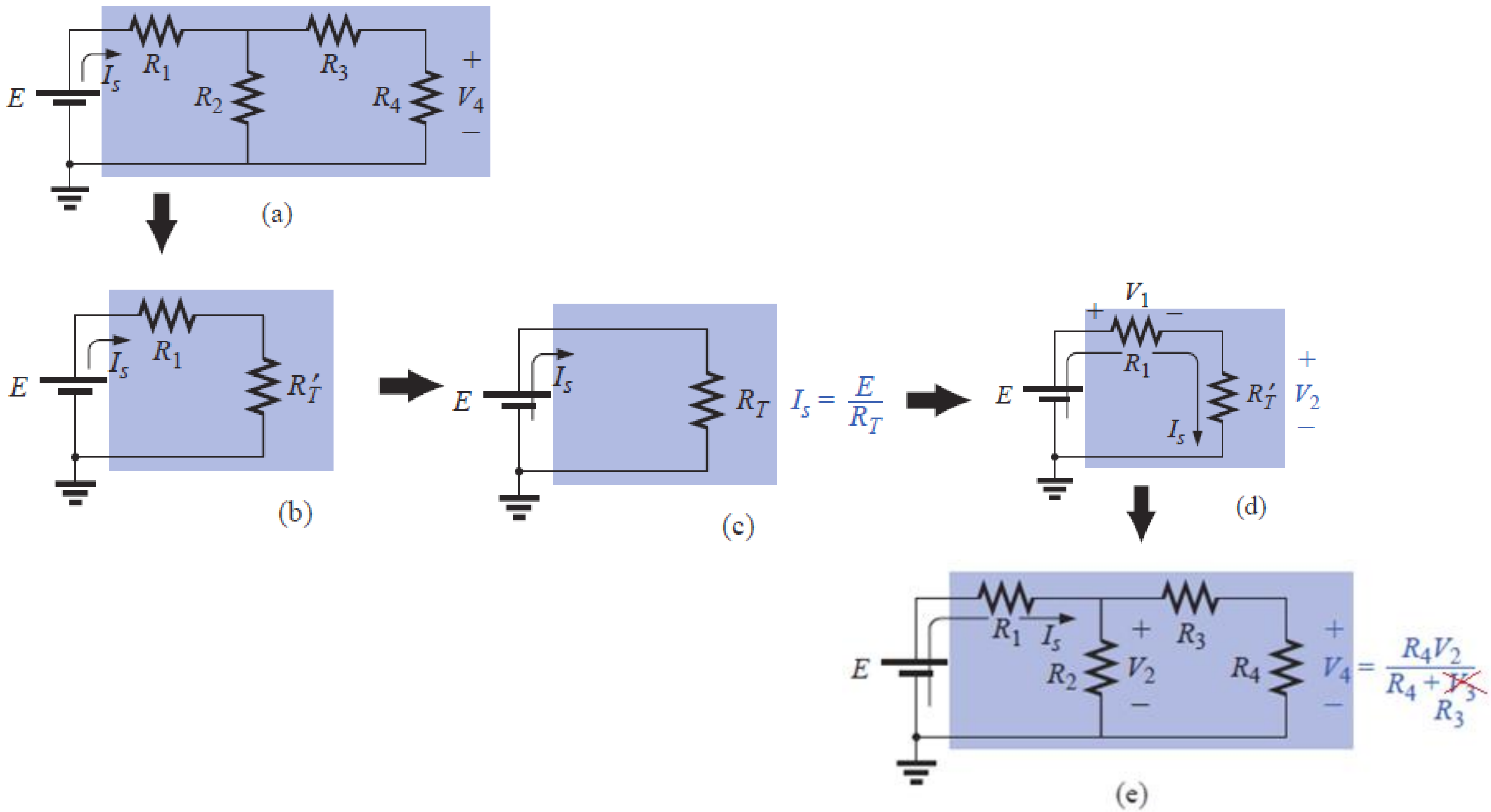
2- Examine each region independently $\Longrightarrow$ simplify the network

3- Redraw the circuit as often as possible with the reduced branches and undisturbed unknown.

4- When you have a solution, check that it is reasonable.

Reduce and Return Approach

Usually, the analysis is one that works back to the source, determines the source current, and then finds its way to the desired unknown.



The networks drawn during the reduction phase are often used for the return path.

Block Diagram Approach

block diagram approach $\Longrightarrow$

1. combinations of elements can be in series or parallel
2. show basic structures that involve similar analysis techniques.

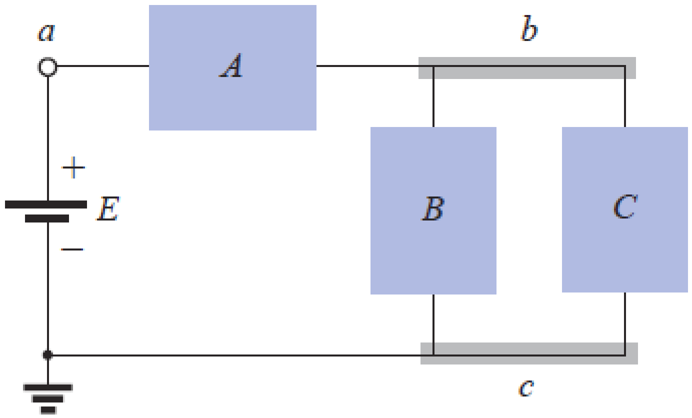


FIG. 7.2
Introducing the block diagram approach.

- blocks B and C are in parallel
- voltage source E is in series with block A
- parallel combination of B and C in series with A and the voltage source E

Notation:

- R_1 and R_2 in series: $R_{1,2} = R_1 + R_2$
- R_1 and R_2 in parallel: $R_{1\|2} \equiv R_1 \parallel R_2 = \frac{R_1 \cdot R_2}{R_1 + R_2}$

EXAMPLE 7.1 If each block of Fig. 7.2 were a single resistive element, the network of Fig. 7.3 might result.

The parallel combination of R_B and R_C results in

$$R_{B\|C} = R_B \parallel R_C = \frac{(12\ \text{k}\Omega)(6\ \text{k}\Omega)}{12\ \text{k}\Omega + 6\ \text{k}\Omega} = 4\ \text{k}\Omega$$

The equivalent resistance $R_{B\|C}$ is then in series with R_A, and the total resistance "seen" by the source is

$$\begin{aligned} R_T &= R_A + R_{B\|C} \\ &= 2\ \text{k}\Omega + 4\ \text{k}\Omega = \mathbf{6\ k\Omega} \end{aligned}$$

The result is an equivalent network, as shown in Fig. 7.4, permitting the determination of the source current I_s.

$$I_s = \frac{E}{R_T} = \frac{54\ \text{V}}{6\ \text{k}\Omega} = \mathbf{9\ mA}$$

and, since the source and R_A are in series,

$$I_A = I_s = 9\ \text{mA}$$

We can then use the equivalent network of Fig. 7.5 to determine I_B and I_C using the current divider rule:

$$I_B = \frac{6\ \text{k}\Omega(I_s)}{6\ \text{k}\Omega + 12\ \text{k}\Omega} = \frac{6}{18}I_s = \frac{1}{3}(9\ \text{mA}) = \mathbf{3\ mA}$$

$$I_C = \frac{12\ \text{k}\Omega(I_s)}{12\ \text{k}\Omega + 6\ \text{k}\Omega} = \frac{12}{18}I_s = \frac{2}{3}(9\ \text{mA}) = \mathbf{6\ mA}$$

or, applying Kirchhoff's current law,

$$I_C = I_s - I_B = 9\ \text{mA} - 3\ \text{mA} = \mathbf{6\ mA}$$

Note that in this solution, we worked back to the source to obtain the source current or total current supplied by the source. The remaining unknowns were then determined by working back through the network to find the other unknowns.

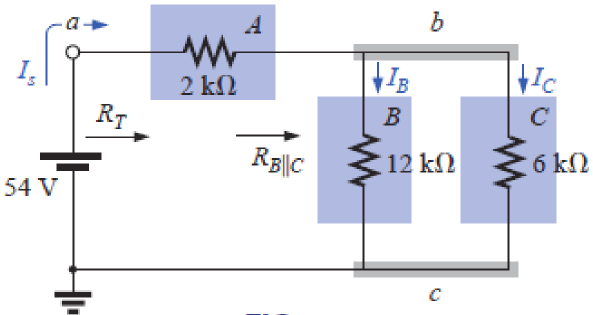


FIG. 7.3
Example 7.1.

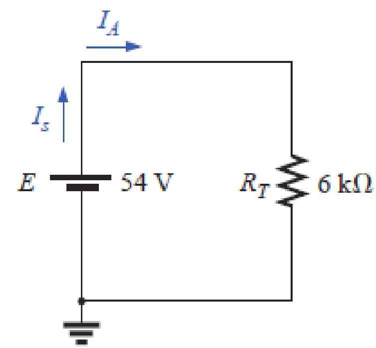


FIG. 7.4
Reduced equivalent of Fig. 7.3.

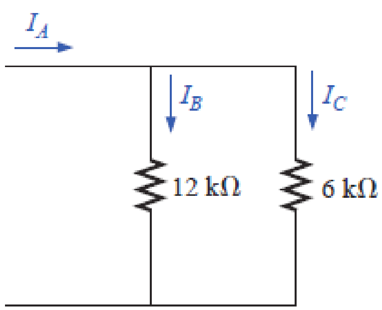


FIG. 7.5
Determining I_B and I_C for the network of Fig. 7.3.

EXAMPLE 7.2 It is also possible that the blocks A, B, and C of Fig. 7.2 contain the elements and configurations of Fig. 7.6. Working with each region:

$$A: \quad R_A = 4\ \Omega$$

$$B: \quad R_B = R_2 \parallel R_3 = R_{2\|3} = \frac{R}{N} = \frac{4\ \Omega}{2} = 2\ \Omega$$

$$C: \quad R_C = R_4 + R_5 = R_{4,5} = 0.5\ \Omega + 1.5\ \Omega = 2\ \Omega$$

Blocks B and C are still in parallel, and

$$R_{B\|C} = \frac{R}{N} = \frac{2\ \Omega}{2} = 1\ \Omega$$

with

$$R_T = R_A + R_{B\|C} = 4\ \Omega + 1\ \Omega = \mathbf{5\ \Omega}$$

(Note the similarity between this equation and that obtained for Example 7.1.)

and $$I_s = \frac{E}{R_T} = \frac{10\ \text{V}}{5\ \Omega} = \mathbf{2\ A}$$

We can find the currents I_A, I_B, and I_C using the reduction of the network of Fig. 7.6 (recall Step 3) as found in Fig. 7.7. Note that I_A, I_B, and I_C are the same in Figs. 7.6 and 7.7 and therefore also appear in Fig. 7.7. In other words, the currents I_A, I_B, and I_C of Fig. 7.7 will have the same magnitude as the same currents of Fig. 7.6.

$$I_A = I_s = \mathbf{2\ A}$$

and $$I_B = I_C = \frac{I_A}{2} = \frac{I_s}{2} = \frac{2\ \text{A}}{2} = \mathbf{1\ A}$$

Returning to the network of Fig. 7.6, we have

$$I_{R_2} = I_{R_3} = \frac{I_B}{2} = \mathbf{0.5\ A}$$

The voltages V_A, V_B, and V_C from either figure are

$$V_A = I_A R_A = (2\ \text{A})(4\ \Omega) = \mathbf{8\ V}$$
$$V_B = I_B R_B = (1\ \text{A})(2\ \Omega) = \mathbf{2\ V}$$
$$V_C = V_B = \mathbf{2\ V}$$

Applying Kirchhoff's voltage law for the loop indicated in Fig. 7.7, we obtain

$$\Sigma_{\circlearrowright} V = E - V_A - V_B = 0$$

$$E = V_A + V_B = 8\ \text{V} + 2\ \text{V}$$

or $$\underline{10\ \text{V} = 10\ \text{V} \quad \text{(checks)}}$$

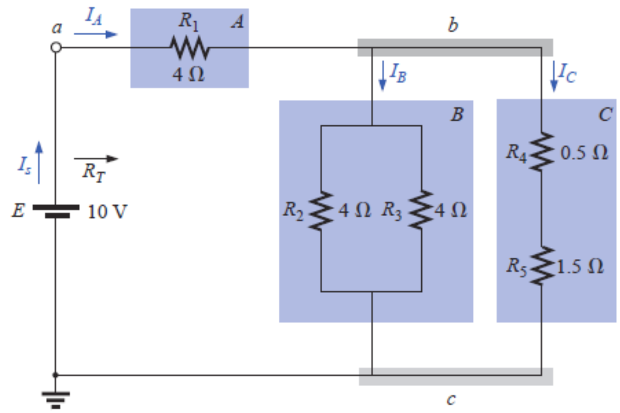


FIG. 7.6
Example 7.2.

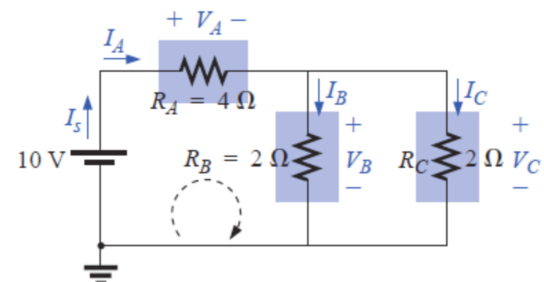


FIG. 7.7
Reduced equivalent of Fig. 7.6.

EXAMPLE 7.3 Another possible variation of Fig. 7.2 appears in Fig. 7.8.

$$R_A = R_{1\|2} = \frac{(9\ \Omega)(6\ \Omega)}{9\ \Omega + 6\ \Omega} = \frac{54\ \Omega}{15} = 3.6\ \Omega$$

$$R_B = R_3 + R_{4\|5} = 4\ \Omega + \frac{(6\ \Omega)(3\ \Omega)}{6\ \Omega + 3\ \Omega} = 4\ \Omega + 2\ \Omega = 6\ \Omega$$

$$R_C = 3\ \Omega$$

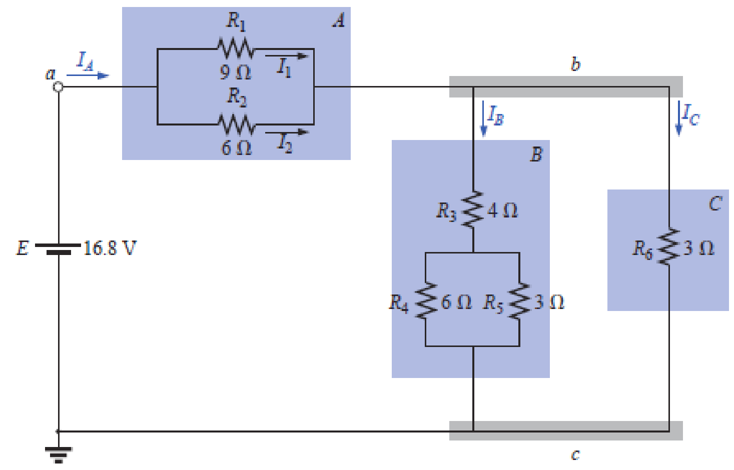


FIG. 7.8
Example 7.3.

The network of Fig. 7.8 can then be redrawn in reduced form, as shown in Fig. 7.9. Note the similarities between this circuit and the circuits of Figs. 7.3 and 7.7.

$$R_T = R_A + R_{B\|C} = 3.6\ \Omega + \frac{(6\ \Omega)(3\ \Omega)}{6\ \Omega + 3\ \Omega}$$

$$= 3.6\ \Omega + 2\ \Omega = \mathbf{5.6\ \Omega}$$

$$I_s = \frac{E}{R_T} = \frac{16.8\ \text{V}}{5.6\ \Omega} = \mathbf{3\ A}$$

$$I_A = I_s = \mathbf{3\ A}$$

Applying the current divider rule yields

$$I_B = \frac{R_C I_A}{R_C + R_B} = \frac{(3\ \Omega)(3\ \text{A})}{3\ \Omega + 6\ \Omega} = \frac{9\ \text{A}}{9} = \mathbf{1\ A}$$

By Kirchhoff's current law,

$$I_C = I_A - I_B = 3\ \text{A} - 1\ \text{A} = \mathbf{2\ A}$$

By Ohm's law,

$$V_A = I_A R_A = (3\ \text{A})(3.6\ \Omega) = \mathbf{10.8\ V}$$

$$V_B = I_B R_B = V_C = I_C R_C = (2\ \text{A})(3\ \Omega) = \mathbf{6\ V}$$

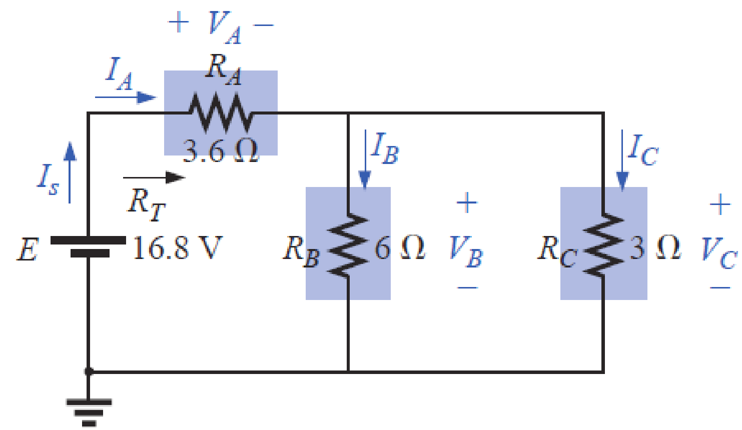


FIG. 7.9
Reduced equivalent of Fig. 7.8.

Returning to the original network (Fig. 7.8) and applying the current divider rule,

$$I_1 = \frac{R_2 I_A}{R_2 + R_1} = \frac{(6\ \Omega)(3\ \text{A})}{6\ \Omega + 9\ \Omega} = \frac{18\ \text{A}}{15} = \mathbf{1.2\ A}$$

By Kirchhoff's current law,

$$I_2 = I_A - I_1 = 3\ \text{A} - 1.2\ \text{A} = \mathbf{1.8\ A}$$

7.2 DESCRIPTIVE EXAMPLES

EXAMPLE 7.4 Find the current I_4 and the voltage V_2 for the network of Fig. 7.10.

Solution: In this case, particular unknowns are requested instead of a complete solution. It would, therefore, be a waste of time to find all the currents and voltages of the network. The method employed should concentrate on obtaining only the unknowns requested. With the block diagram approach, the network has the basic structure of Fig. 7.11, clearly indicating that the three branches are in parallel and the voltage across A and B is the supply voltage. The current I_4 is now immediately obvious as simply the supply voltage divided by the resultant resistance for B. If desired, block A could be broken down further, as shown in Fig. 7.12, to identify C and D as series elements, with the voltage V_2 capable of being determined using the voltage divider rule once the resistance of C and D is reduced to a single value. This is an example of how a mental sketch of the approach might be made before applying laws, rules, and so on, to avoid dead ends and growing frustration.

Applying Ohm's law,

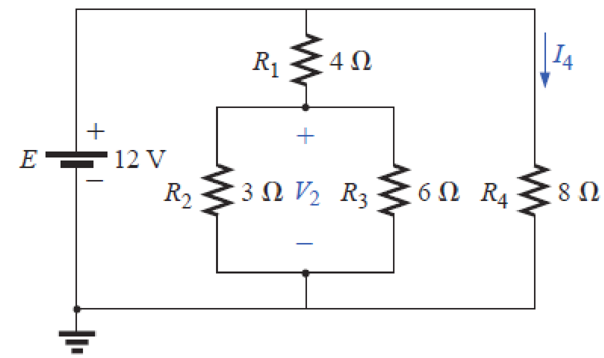
$$I_4 = \frac{E}{R_B} = \frac{E}{R_4} = \frac{12\text{ V}}{8\ \Omega} = \mathbf{1.5\ A}$$

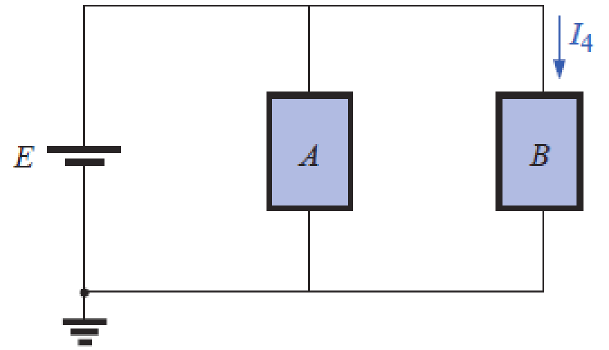
Combining the resistors R_2 and R_3 of Fig. 7.10 will result in

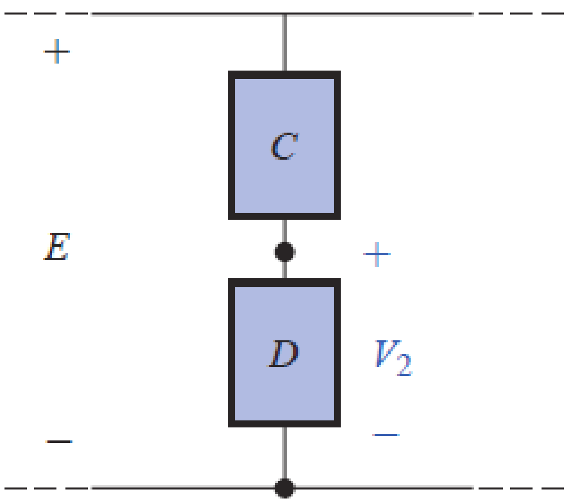
$$R_D = R_2 \parallel R_3 = 3\ \Omega \parallel 6\ \Omega = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 6\ \Omega} = \frac{18\ \Omega}{9} = 2\ \Omega$$

and, applying the voltage divider rule,

$$V_2 = \frac{R_D E}{R_D + R_C} = \frac{(2\ \Omega)(12\text{ V})}{2\ \Omega + 4\ \Omega} = \frac{24\text{ V}}{6} = \mathbf{4\ V}$$

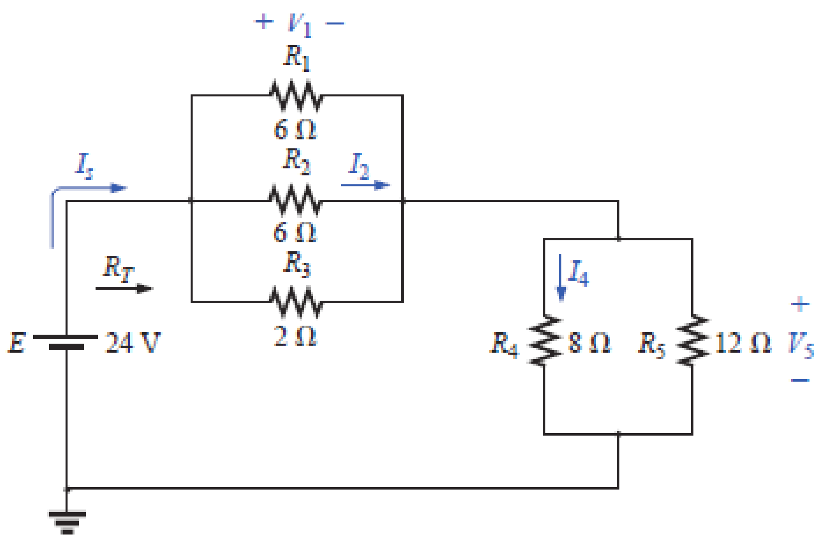


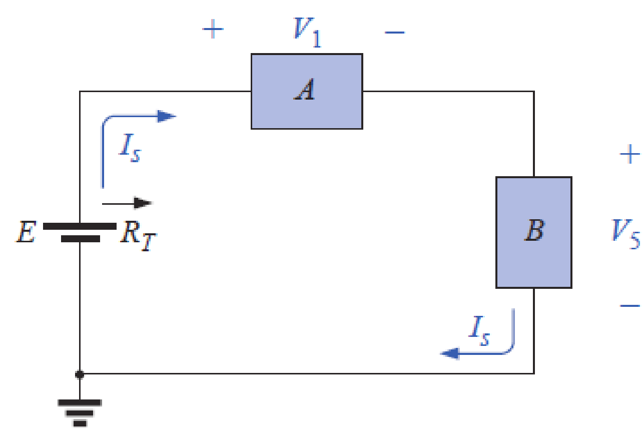




EXAMPLE 7.5 Find the indicated currents and voltages for the network of Fig. 7.13.

Solution: Again, only specific unknowns are requested. When the network is redrawn, it will be particularly important to note which unknowns are preserved and which will have to be determined using the original configuration. The block diagram of the network may appear as shown in Fig. 7.14, clearly revealing that *A* and *B* are in series. Note in this form the number of unknowns that have been preserved. The voltage V_1 will be the same across the three parallel branches of Fig. 7.13, and V_5 will be the same across R_4 and R_5. The unknown currents I_2 and I_4 are lost since they represent the currents through only one of the parallel branches. However, once V_1 and V_5 are known, the required currents can be found using Ohm's law.

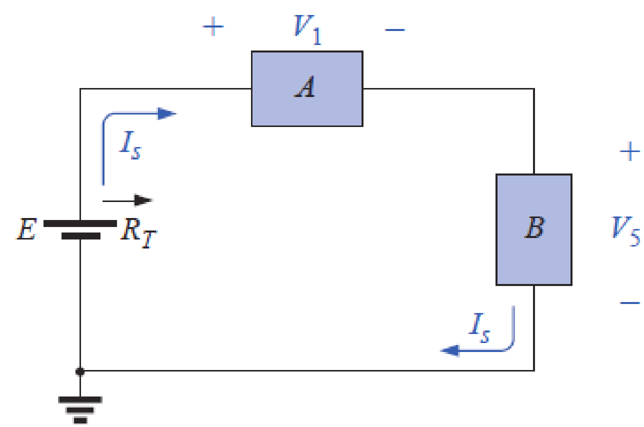




$$R_{1\|2} = \frac{R}{N} = \frac{6\ \Omega}{2} = 3\ \Omega$$

$$R_A = R_{1\|2\|3} = \frac{(3\ \Omega)(2\ \Omega)}{3\ \Omega + 2\ \Omega} = \frac{6\ \Omega}{5} = 1.2\ \Omega$$

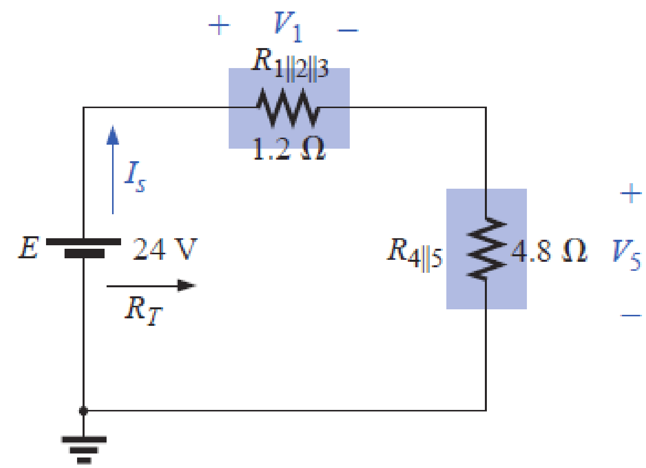
$$R_B = R_{4\|5} = \frac{(8\ \Omega)(12\ \Omega)}{8\ \Omega + 12\ \Omega} = \frac{96\ \Omega}{20} = 4.8\ \Omega$$



The reduced form of Fig. 7.13 will then appear as shown in Fig. 7.15, and

$$R_T = R_{1\|2\|3} + R_{4\|5} = 1.2\ \Omega + 4.8\ \Omega = \mathbf{6\ \Omega}$$

$$I_s = \frac{E}{R_T} = \frac{24\ \text{V}}{6\ \Omega} = \mathbf{4\ A}$$



with
$$V_1 = I_s R_{1\|2\|3} = (4\ \text{A})(1.2\ \Omega) = \mathbf{4.8\ V}$$
$$V_5 = I_s R_{4\|5} = (4\ \text{A})(4.8\ \Omega) = \mathbf{19.2\ V}$$

Applying Ohm's law,

$$I_4 = \frac{V_5}{R_4} = \frac{19.2\ \text{V}}{8\ \Omega} = \mathbf{2.4\ A}$$

$$I_2 = \frac{V_2}{R_2} = \frac{V_1}{R_2} = \frac{4.8\ \text{V}}{6\ \Omega} = \mathbf{0.8\ A}$$

EXAMPLE 7.6

a. Find the voltages V_1, V_3, and V_{ab} for the network of Fig. 7.16.
b. Calculate the source current I_s.

Solutions: This is one of those situations where it might be best to redraw the network before beginning the analysis. Since combining both sources will not affect the unknowns, the network is redrawn as shown in Fig. 7.17, establishing a parallel network with the total source voltage across each parallel branch. The net source voltage is the difference between the two with the polarity of the larger.

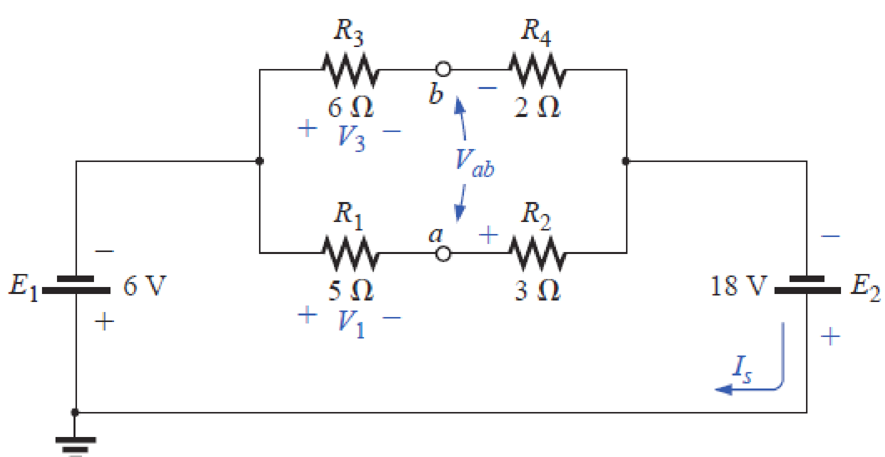


FIG. 7.16
Example 7.6.

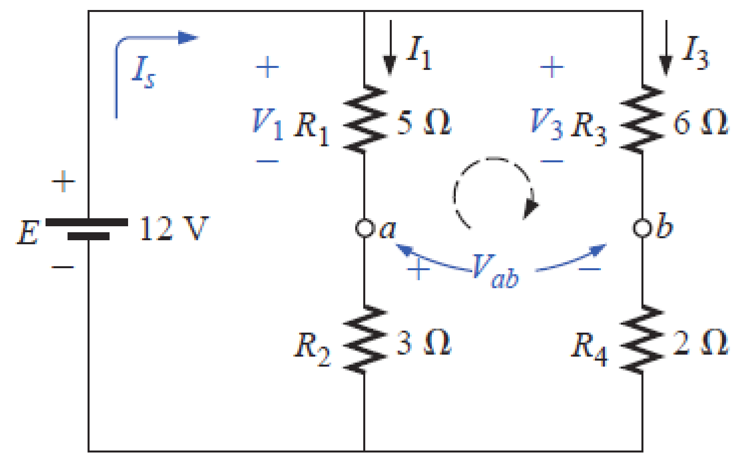


FIG. 7.17
Network of Fig. 7.16 redrawn.

a. Note the similarities with Fig. 7.12, permitting the use of the voltage divider rule to determine V_1 and V_3:

$$V_1 = \frac{R_1E}{R_1 + R_2} = \frac{(5\ \Omega)(12\ \text{V})}{5\ \Omega + 3\ \Omega} = \frac{60\ \text{V}}{8} = \mathbf{7.5\ V}$$

$$V_3 = \frac{R_3E}{R_3 + R_4} = \frac{(6\ \Omega)(12\ \text{V})}{6\ \Omega + 2\ \Omega} = \frac{72\ \text{V}}{8} = \mathbf{9\ V}$$

The open-circuit voltage V_{ab} is determined by applying Kirchhoff's voltage law around the indicated loop of Fig. 7.17 in the clockwise direction starting at terminal a.

$$+V_1 - V_3 + V_{ab} = 0$$

and $$V_{ab} = V_3 - V_1 = 9\ \text{V} - 7.5\ \text{V} = \mathbf{1.5\ V}$$

b. By Ohm's law,

$$I_1 = \frac{V_1}{R_1} = \frac{7.5\ \text{V}}{5\ \Omega} = 1.5\ \text{A}$$

$$I_3 = \frac{V_3}{R_3} = \frac{9\ \text{V}}{6\ \Omega} = 1.5\ \text{A}$$

Applying Kirchhoff's current law,

$$I_s = I_1 + I_3 = 1.5\ \text{A} + 1.5\ \text{A} = \mathbf{3\ A}$$

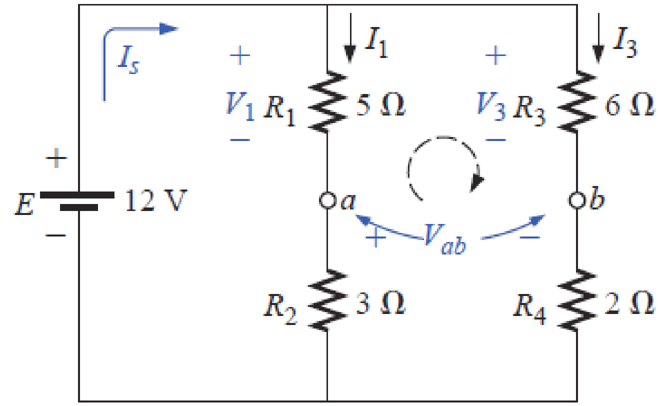


FIG. 7.17
Network of Fig. 7.16 redrawn.

EXAMPLE 7.7 For the network of Fig. 7.18, determine the voltages V_1 and V_2 and the current I.

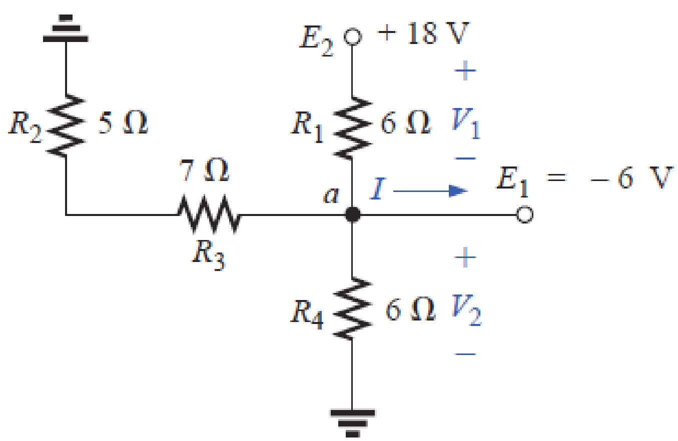
Solution: It would indeed be difficult to analyze the network in the form of Fig. 7.18 with the symbolic notation for the sources and the reference or ground connection in the upper left-hand corner of the diagram. However, when the network is redrawn as shown in Fig. 7.19, the unknowns and the relationship between branches become significantly clearer. Note the common connection of the grounds and the replacing of the terminal notation by actual supplies.

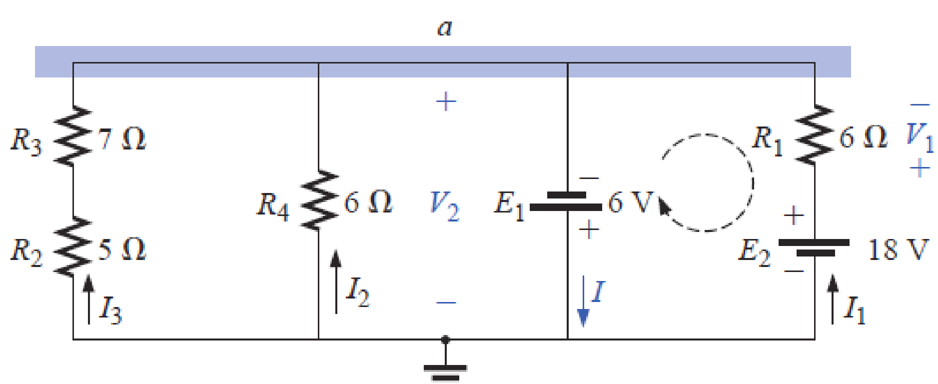
It is now obvious that

$$V_2 = -E_1 = \mathbf{-6\ V}$$

The minus sign simply indicates that the chosen polarity for V_2 in Fig. 7.18 is opposite to that of the actual voltage. Applying Kirchhoff's voltage law to the loop indicated, we obtain

$$-E_1 + V_1 - E_2 = 0$$

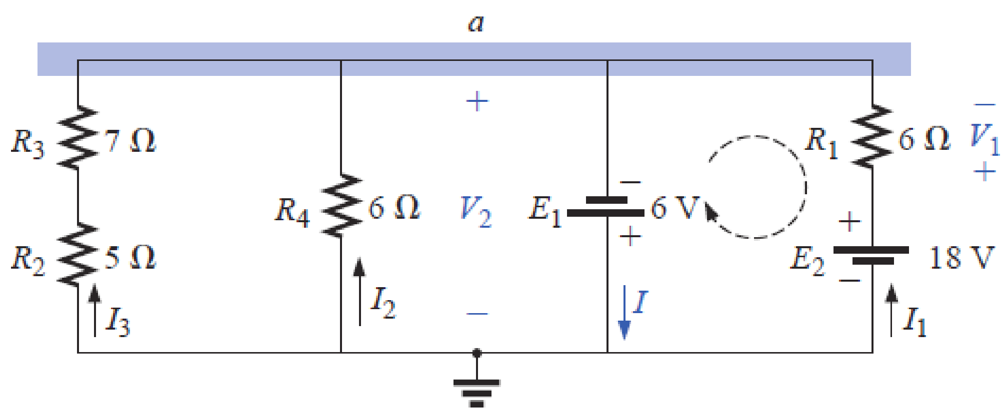




and $$V_1 = E_2 + E_1 = 18\text{ V} + 6\text{ V} = \mathbf{24\ V}$$

Applying Kirchhoff's current law to node a yields

$$\begin{aligned}
I &= I_1 + I_2 + I_3 \\
&= \frac{V_1}{R_1} + \frac{E_1}{R_4} + \frac{E_1}{R_2 + R_3} \\
&= \frac{24\text{ V}}{6\ \Omega} + \frac{6\text{ V}}{6\ \Omega} + \frac{6\text{ V}}{12\ \Omega} \\
&= 4\text{ A} + 1\text{ A} + 0.5\text{ A} \\
I &= \mathbf{5.5\ A}
\end{aligned}$$



EXAMPLE 7.8 For the transistor configuration of Fig. 7.20, in which V_B and V_{BE} have been provided:

a. Determine the voltage V_E and the current I_E.
b. Calculate V_1.
c. Determine V_{BC} using the fact that the approximation $I_C = I_E$ is often applied to transistor networks.
d. Calculate V_{CE} using the information obtained in parts (a) through (c).

Solutions:

a. From Fig. 7.20, we find

$$V_2 = V_B = 2\text{ V}$$

Writing Kirchhoff's voltage law around the lower loop yields

$$V_2 - V_{BE} - V_E = 0$$

or $$V_E = V_2 - V_{BE} = 2\text{ V} - 0.7\text{ V} = \mathbf{1.3\text{ V}}$$

and $$I_E = \frac{V_E}{R_E} = \frac{1.3\text{ V}}{1000\ \Omega} = \mathbf{1.3\text{ mA}}$$

b. Applying Kirchhoff's voltage law to the input side (left-hand region of the network) will result in

$$V_2 + V_1 - V_{CC} = 0$$

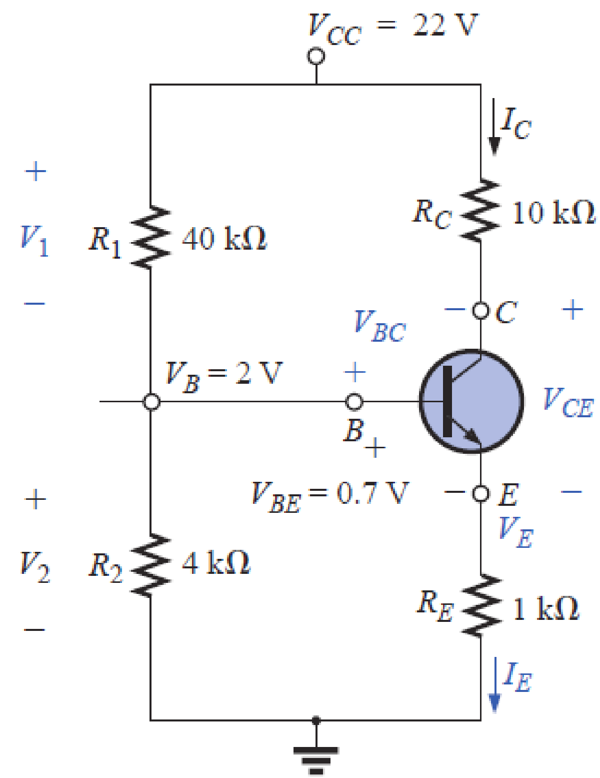


FIG. 7.20
Example 7.8.

and $\quad V_1 = V_{CC} - V_2$

but $\quad V_2 = V_B$

and $\quad V_1 = V_{CC} - V_2 = 22\text{ V} - 2\text{ V} = \mathbf{20\ V}$

c. Redrawing the section of the network of immediate interest will result in Fig. 7.21, where Kirchhoff's voltage law yields

$$V_C + V_{R_C} - V_{CC} = 0$$

and $\quad V_C = V_{CC} - V_{R_C} = V_{CC} - I_C R_C$

but $\quad I_C = I_E$

and
$$V_C = V_{CC} - I_E R_C = 22\text{ V} - (1.3\text{ mA})(10\text{ k}\Omega)$$
$$= 9\text{ V}$$

Then
$$V_{BC} = V_B - V_C$$
$$= 2\text{ V} - 9\text{ V}$$
$$= \mathbf{-7\ V}$$

d.
$$V_{CE} = V_C - V_E$$
$$= 9\text{ V} - 1.3\text{ V}$$
$$= \mathbf{7.7\ V}$$

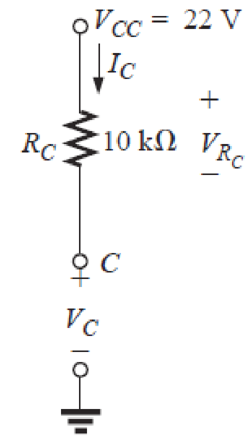


FIG. 7.21
Determining V_C for the network of Fig. 7.20.

EXAMPLE 7.9 Calculate the indicated currents and voltage of Fig. 7.22.

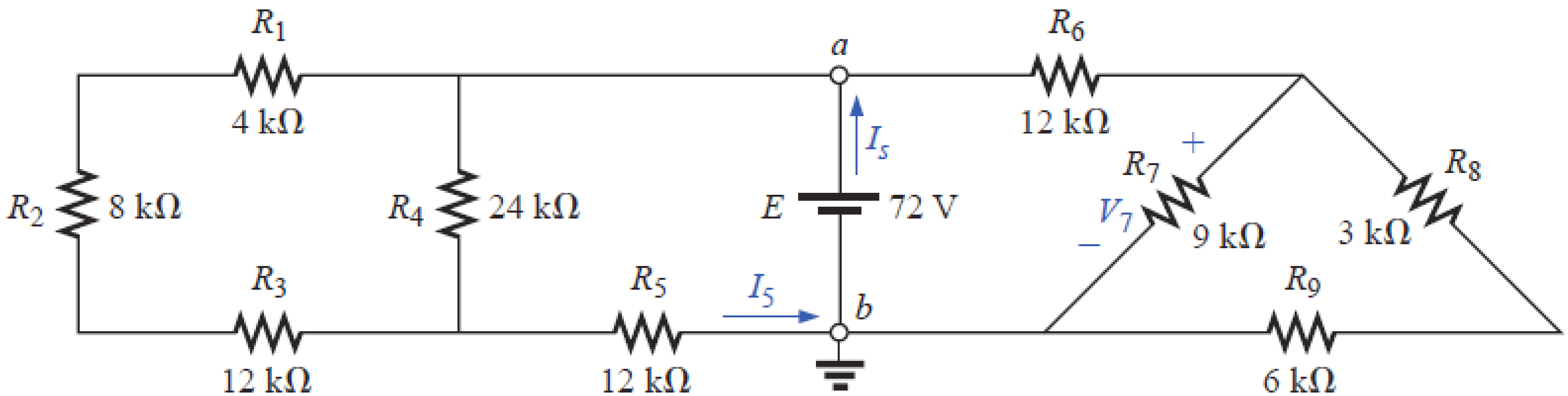
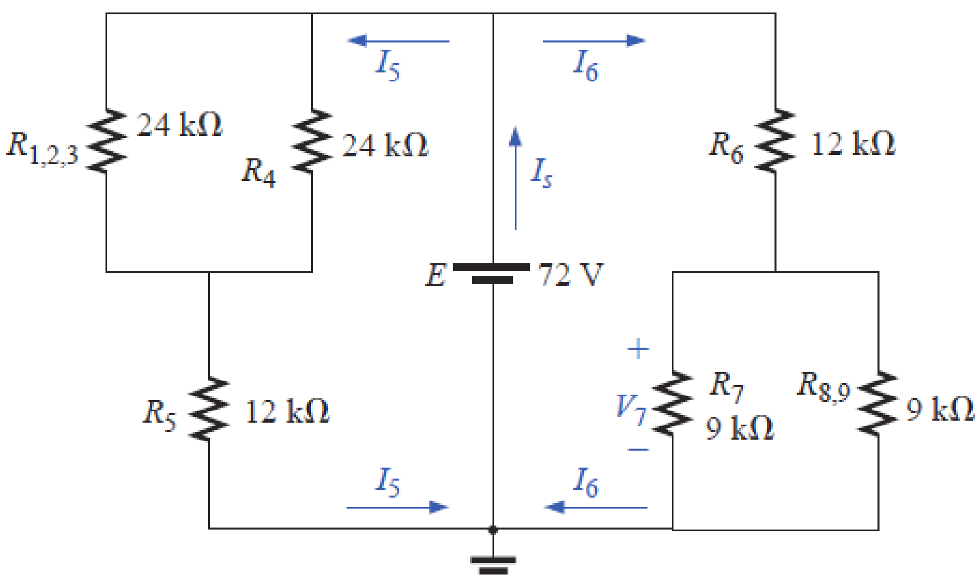


FIG. 7.22
Example 7.9.

Solution: Redrawing the network after combining series elements yields Fig. 7.23, and



$$I_5 = \frac{E}{R_{(1,2,3)\|4} + R_5} = \frac{72\text{ V}}{12\text{ k}\Omega + 12\text{ k}\Omega} = \frac{72\text{ V}}{24\text{ k}\Omega} = \mathbf{3\text{ mA}}$$

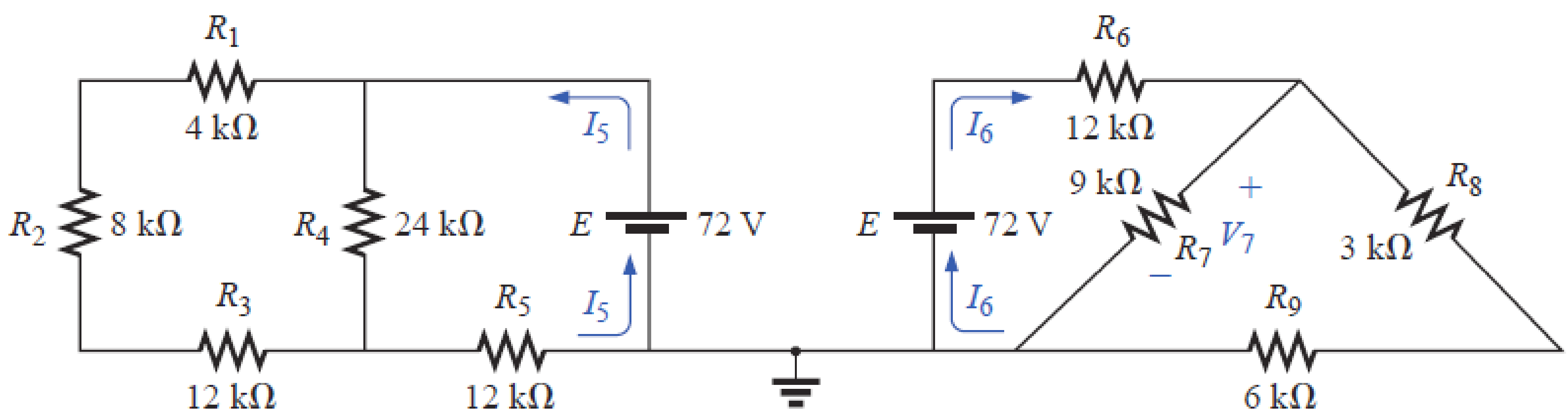
with

$$V_7 = \frac{R_{7\|(8,9)}E}{R_{7\|(8,9)} + R_6} = \frac{(4.5\text{ k}\Omega)(72\text{ V})}{4.5\text{ k}\Omega + 12\text{ k}\Omega} = \frac{324\text{ V}}{16.5} = \mathbf{19.6\text{ V}}$$

$$I_6 = \frac{V_7}{R_{7\|(8,9)}} = \frac{19.6\text{ V}}{4.5\text{ k}\Omega} = \mathbf{4.35\text{ mA}}$$

and $$I_s = I_5 + I_6 = 3\text{ mA} + 4.35\text{ mA} = \mathbf{7.35\text{ mA}}$$

Since the potential difference between points a and b of Fig. 7.22 is fixed at E volts, the circuit to the right or left is unaffected if the network is reconstructed as shown in Fig. 7.24.



We can find each quantity required, except I_s, by analyzing each circuit independently. To find I_s, we must find the source current for each circuit and add it as in the above solution; that is, $I_s = I_5 + I_6$.

EXAMPLE 7.10 This example demonstrates the power of Kirchhoff's voltage law by determining the voltages V_1, V_2, and V_3 for the network of Fig. 7.25. For path 1 of Fig. 7.26,

$$E_1 - V_1 - E_3 = 0$$

and $$V_1 = E_1 - E_3 = 20\text{ V} - 8\text{ V} = \mathbf{12\text{ V}}$$

For path 2,

$$E_2 - V_1 - V_2 = 0$$

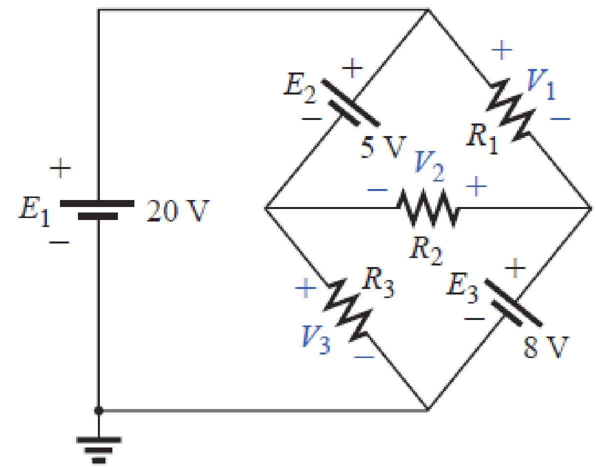
and $$V_2 = E_2 - V_1 = 5\text{ V} - 12\text{ V} = \mathbf{-7\text{ V}}$$

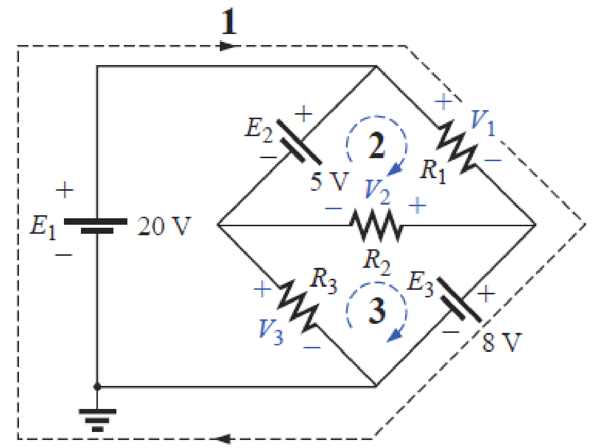
indicating that V_2 has a magnitude of 7 V but a polarity opposite to that appearing in Fig. 7.25. For path 3,

$$V_3 + V_2 - E_3 = 0$$

and $$V_3 = E_3 - V_2 = 8\text{ V} - (-7\text{ V}) = 8\text{ V} + 7\text{ V} = \mathbf{15\text{ V}}$$

Note that the polarity of V_2 was maintained as originally assumed, requiring that -7 V be substituted for V_2.





7.3 LADDER NETWORKS

The figure shows a three-section **ladder network**,

Ladder network ≡ repetitive structure.

Two approaches are used to solve networks of this type.

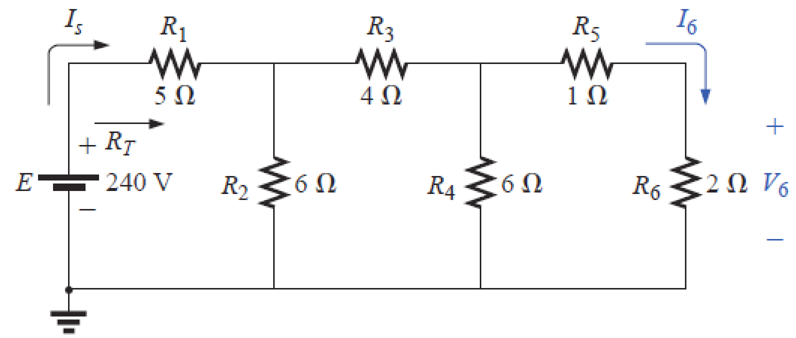


FIG. 7.27
Ladder network.

Method 1:

Calculate the total resistance and the resulting source current and work back until the desired unknown:

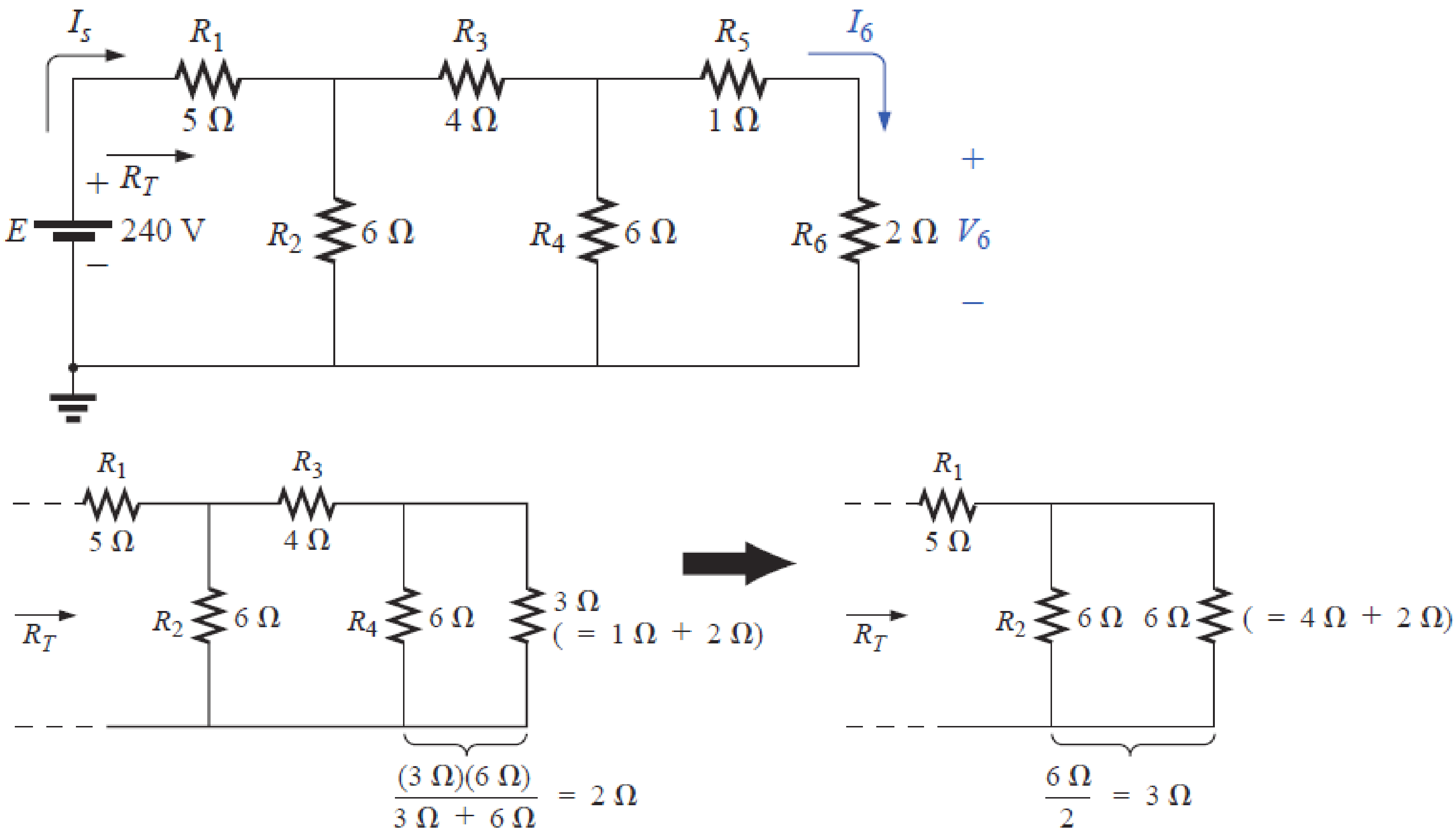


FIG. 7.28

Working back to the source to determine R_T for the network of Fig. 7.27.

Combining parallel and series elements:

$$R_T = 5\ \Omega + 3\ \Omega = 8\ \Omega$$

$$I_s = \frac{E}{R_T} = \frac{240\ \text{V}}{8\ \Omega} = 30\ \text{A}$$

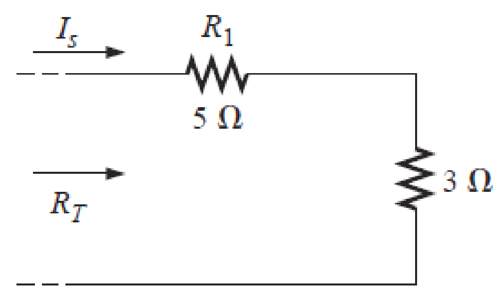


FIG. 7.29
Calculating R_T and I_s.

Working our way back to I_6.

$$I_1 = I_s \quad \text{and} \quad I_3 = \frac{I_s}{2} = \frac{30\ \text{A}}{2} = 15\ \text{A}$$

Finally:

$$I_6 = \frac{(6\ \Omega)I_3}{6\ \Omega + 3\ \Omega} = \frac{6}{9}(15\ \text{A}) = 10\ \text{A}$$

$$V_6 = I_6 R_6 = (10\ \text{A})(2\ \Omega) = \mathbf{20\ V}$$

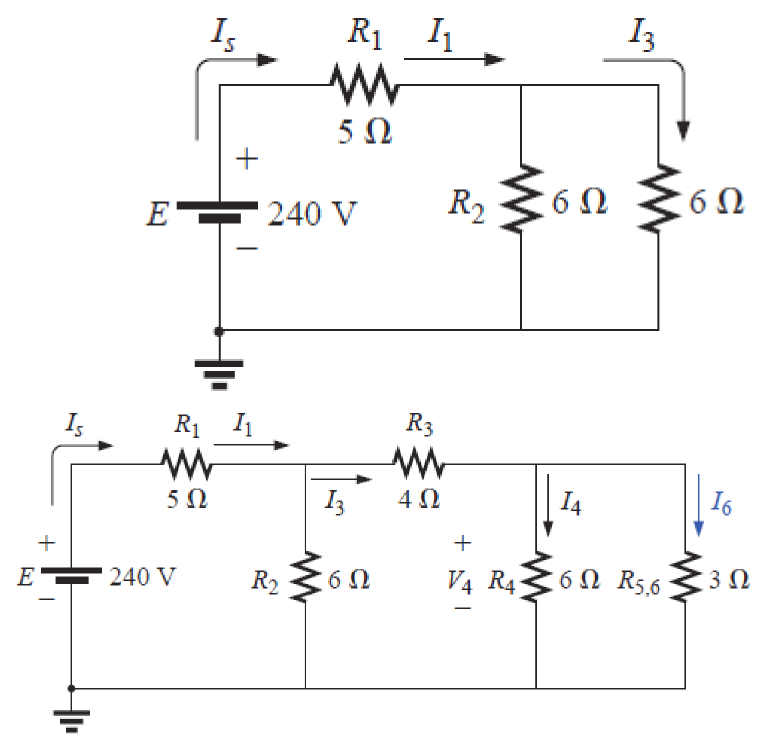


FIG. 7.31
Calculating I_6.

Method 2

Assign a letter symbol to the last branch current and work back through the network to the source, maintaining this assigned current or other current of interest. The desired current can then be found directly.

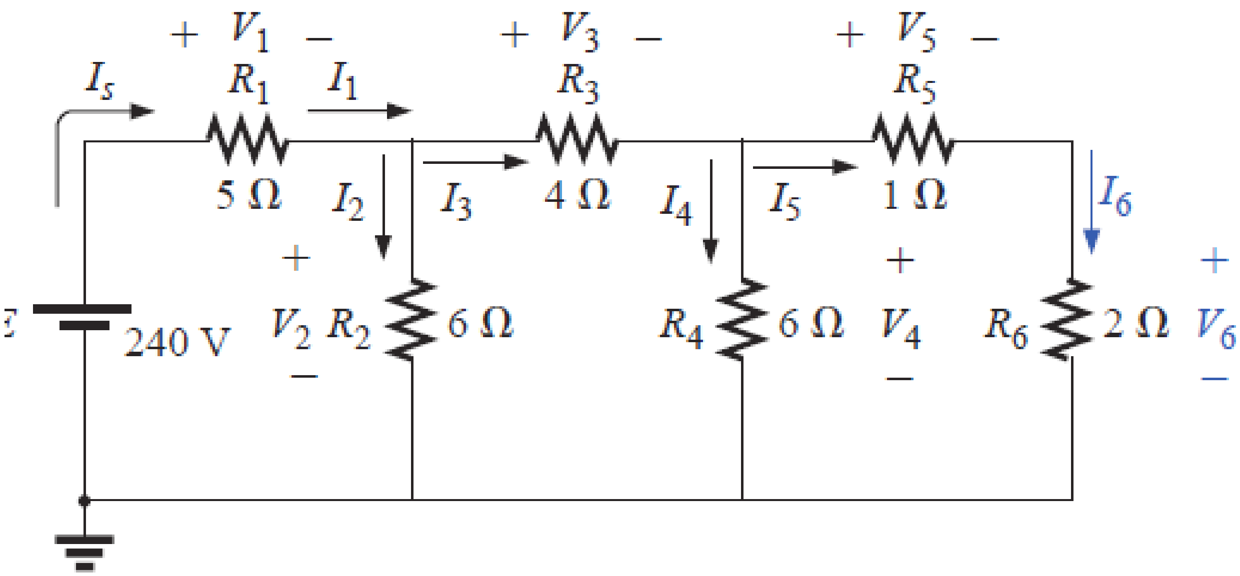
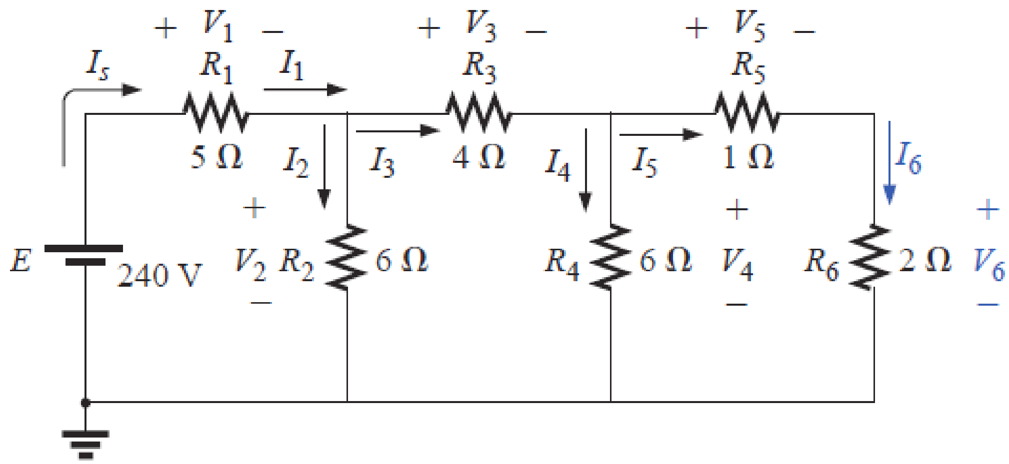


FIG. 7.32

An alternative approach for ladder networks.

The assigned notation for the current through the final branch is I_6:



$$I_6 = \frac{V_4}{R_5 + R_6} = \frac{V_4}{1\ \Omega + 2\ \Omega} = \frac{V_4}{3\ \Omega}$$

or $$V_4 = (3\ \Omega)I_6$$

so that $$I_4 = \frac{V_4}{R_4} = \frac{(3\ \Omega)I_6}{6\ \Omega} = 0.5I_6$$

and $$I_3 = I_4 + I_6 = 0.5I_6 + I_6 = 1.5I_6$$

$$V_3 = I_3R_3 = (1.5I_6)(4\ \Omega) = (6\ \Omega)I_6$$

Also, $$V_2 = V_3 + V_4 = (6\ \Omega)I_6 + (3\ \Omega)I_6 = (9\ \Omega)I_6$$

so that $$I_2 = \frac{V_2}{R_2} = \frac{(9\ \Omega)I_6}{6\ \Omega} = 1.5I_6$$

and $$I_s = I_2 + I_3 = 1.5I_6 + 1.5I_6 = 3I_6$$

with $$V_1 = I_1R_1 = I_sR_1 = (5\ \Omega)I_s$$

so that $$E = V_1 + V_2 = (5\ \Omega)I_s + (9\ \Omega)I_6 = (5\ \Omega)(3I_6) + (9\ \Omega)I_6 = (24\ \Omega)I_6$$

and $$I_6 = \frac{E}{24\ \Omega} = \frac{240\text{ V}}{24\ \Omega} = 10\text{ A}$$

with $$V_6 = I_6R_6 = (10\text{ A})(2\ \Omega) = \mathbf{20\ V}$$

as was obtained using method 1.