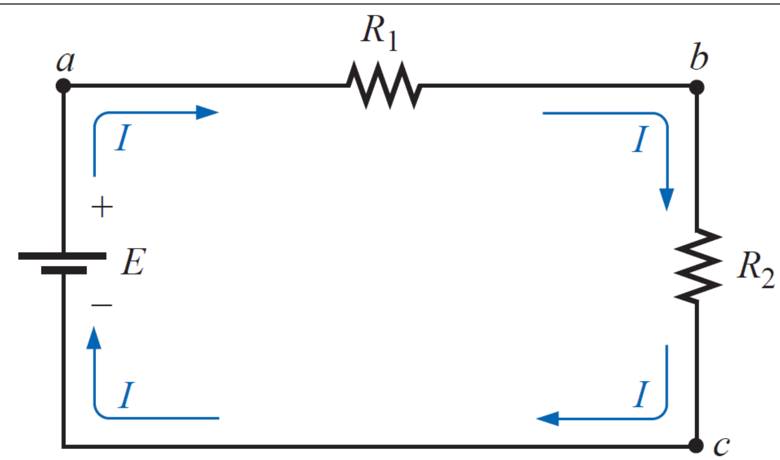
Series Circuit

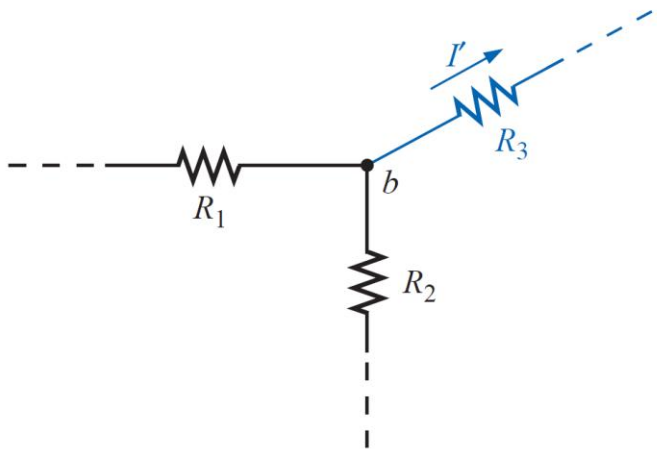
5.2 SERIES CIRCUITS

Two elements are in series if:

1- They have only one terminal in common
2- The common point between them is not connected to another current carrying element.

In the circuit E, R_1 and R_2 are in series. All elements in the circuit are in series: $\Longrightarrow$ Series Circuit The current is the same through series elements. A branch of a circuit is any portion of the circuit having one or more elements in series.	 (a) Series circuit

R_1 and R_2 are not in series because at point (b) the common between them is connected to R_3 which carries a current



(b) R_1 and R_2 are not in series

The total resistance of a series circuit is the sum of the resistance levels.

$$\boxed{R_T = R_1 + R_2 + R_3 + \cdots + R_N} \qquad \text{(ohms, } \Omega\text{)}$$

If $R_1 = R_2 = R_3 = \ldots\ldots = R_N = R \quad \Longrightarrow$

$$R_T = N \cdot R$$

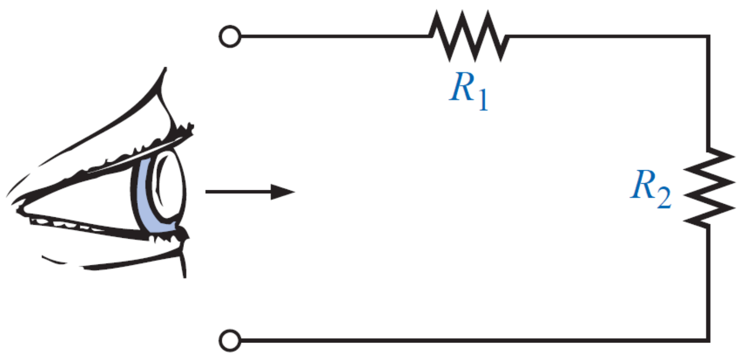


FIG. 5.5
Resistance "seen" by source.

Once R_T is known the circuit can be replaced by the one shown: and then

$$\boxed{I_s = \frac{E}{R_T}} \qquad \text{(amperes, A)}$$

E is fixed: $\Longrightarrow$ I_s depends on R_T.

$$V_1 = I_s \cdot R_1 \qquad V_2 = I_s \cdot R_2 \text{ , ...}$$

$$P_1 = V_1 \cdot I_1 = I_1^2 \cdot R_1 = \frac{V_1^2}{R_1} \text{ , ...}$$

$$P_{del} = E \cdot I_s$$

The total power delivered to a resistive circuit is equal to the total power dissipated by the resistive elements.

$$\boxed{P_{\text{del}} = P_1 + P_2 + P_3 + \cdots + P_N}$$

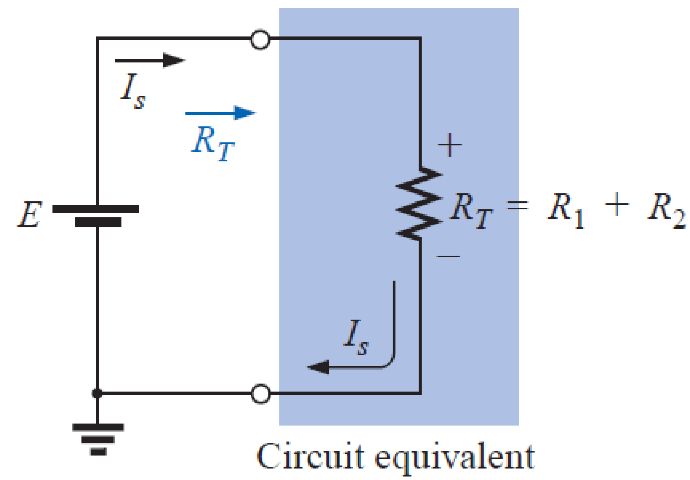


FIG. 5.6

Replacing the series resistors R_1 and R_2 of Fig. 5.5 with the total resistance.

EXAMPLE 5.1

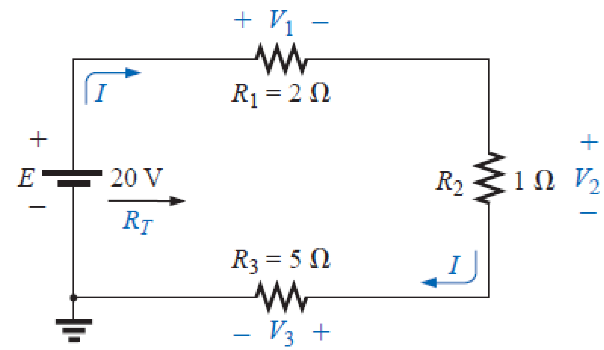
a. Find the total resistance for the series circuit of Fig. 5.7.

b. Calculate the source current I_s.

c. Determine the voltages V_1, V_2, and V_3.

d. Calculate the power dissipated by R_1, R_2, and R_3.

e. Determine the power delivered by the source, and compare it to the sum of the power levels of part (d).



Solutions:

a. $R_T = R_1 + R_2 + R_3 = 2\ \Omega + 1\ \Omega + 5\ \Omega = \mathbf{8\ \Omega}$

b. $I_s = \dfrac{E}{R_T} = \dfrac{20\text{ V}}{8\ \Omega} = \mathbf{2.5\ A}$

c. $V_1 = IR_1 = (2.5\text{ A})(2\ \Omega) = \mathbf{5\ V}$
$V_2 = IR_2 = (2.5\text{ A})(1\ \Omega) = \mathbf{2.5\ V}$
$V_3 = IR_3 = (2.5\text{ A})(5\ \Omega) = \mathbf{12.5\ V}$

d. $P_1 = V_1 I_1 = (5\text{ V})(2.5\text{ A}) = \mathbf{12.5\ W}$
$P_2 = I_2^2 R_2 = (2.5\text{ A})^2(1\ \Omega) = \mathbf{6.25\ W}$
$P_3 = V_3^2/R_3 = (12.5\text{ V})^2/5\ \Omega = \mathbf{31.25\ W}$

e. $P_{\text{del}} = EI = (20\text{ V})(2.5\text{ A}) = \mathbf{50\ W}$
$P_{\text{del}} = P_1 + P_2 + P_3$
$50\text{ W} = 12.5\text{ W} + 6.25\text{ W} + 31.25\text{ W}$
$\underline{50\text{ W} = 50\text{ W} \quad \text{(checks)}}$

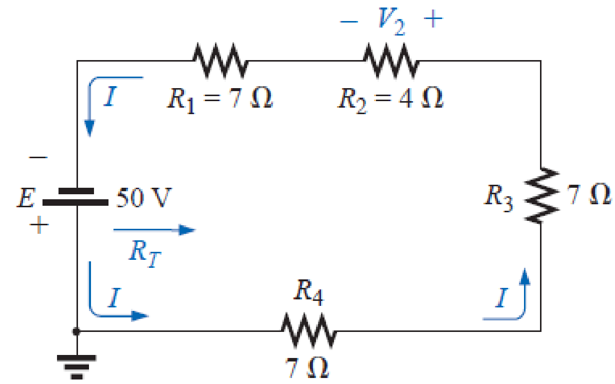
EXAMPLE 5.2 Determine R_T, I, and V_2 for the circuit of Fig. 5.8.

Solution: Note the current direction as established by the battery and the polarity of the voltage drops across R_2 as determined by the current direction. Since $R_1 = R_3 = R_4$,

$$R_T = NR_1 + R_2 = (3)(7\ \Omega) + 4\ \Omega = 21\ \Omega + 4\ \Omega = \mathbf{25\ \Omega}$$

$$I = \frac{E}{R_T} = \frac{50\ \text{V}}{25\ \Omega} = \mathbf{2\ A}$$

$$V_2 = IR_2 = (2\ \text{A})(4\ \Omega) = \mathbf{8\ V}$$



EXAMPLE 5.3 Given R_T and I, calculate R_1 and E for the circuit of Fig. 5.9.

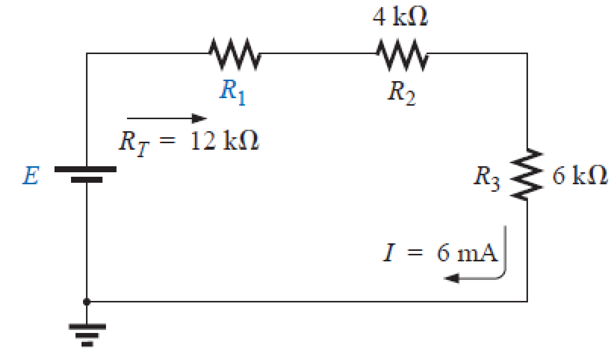
Solution:

$$R_T = R_1 + R_2 + R_3$$

$$12\ \text{k}\Omega = R_1 + 4\ \text{k}\Omega + 6\ \text{k}\Omega$$

$$R_1 = 12\ \text{k}\Omega - 10\ \text{k}\Omega = \mathbf{2\ k\Omega}$$

$$E = IR_T = (6 \times 10^{-3}\ \text{A})(12 \times 10^{3}\ \Omega) = \mathbf{72\ V}$$

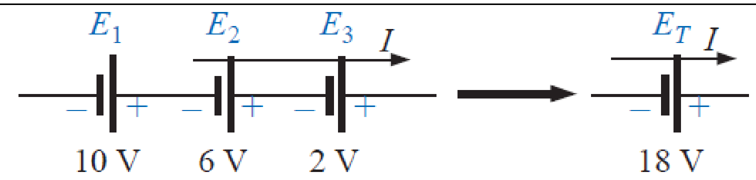


5.3 VOLTAGE SOURCES IN SERIES

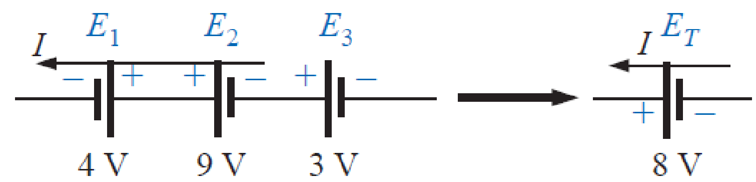
Voltage sources can be connected in series to increase or decrease the total voltage applied:

The net voltage is determined simply by summing the sources with the same polarity and subtracting the total of the sources with the opposite polarity.

Net polarity $\equiv$ polarity of the larger sum.



$$E_T = E_1 + E_2 + E_3 = 10\text{ V} + 6\text{ V} + 2\text{ V} = 18\text{ V}$$



$$E_T = E_2 + E_3 - E_1 = 9\text{ V} + 3\text{ V} - 4\text{ V} = 8\text{ V}$$

5.4 KIRCHHOFF'S VOLTAGE LAW

Kirchhoff's voltage law *(KVL) states that the algebraic sum of the potential rises and drops around a closed loop (or path) is zero.*

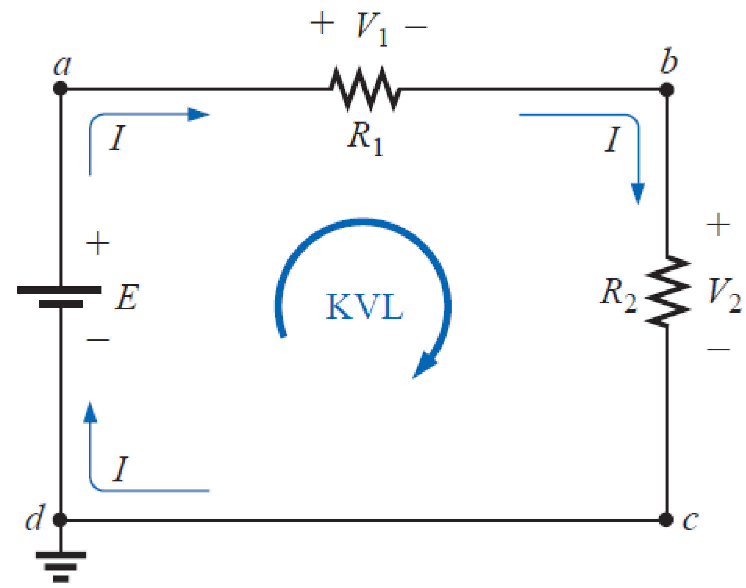
A **closed loop** is any continuous path that leaves a point in one direction and returns to that same point from another direction without leaving the circuit.

$$\boxed{\Sigma_{\circlearrowright} V = 0}$$

(Kirchhoff's voltage law in symbolic form)

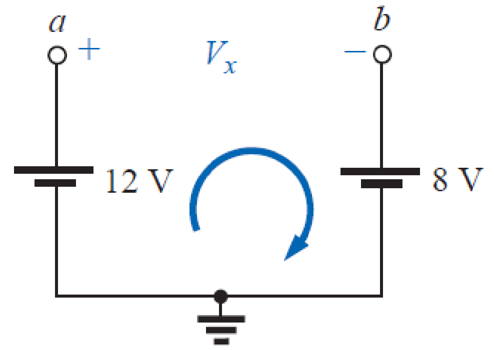
The applied voltage of a series circuit equals the sum of the voltage drops across the series elements:

$$\boxed{\Sigma_{\circlearrowright} V_{\text{rises}} = \Sigma_{\circlearrowright} V_{\text{drops}}}$$



abcda ≡ closed loop

$$+E - V_1 - V_2 = 0 \Longrightarrow E = V_1 + V_2$$

The application of Kirchhoff's voltage law need not follow a path that includes current-carrying elements. $+12V - V_x - 8V = 0 \Longrightarrow \; V_x = 4V$	 **FIG. 5.13** *Demonstration that a voltage can exist between two points not connected by a current-carrying conductor.*

!!!!! Polarity is very important when applying KVL !!!!!

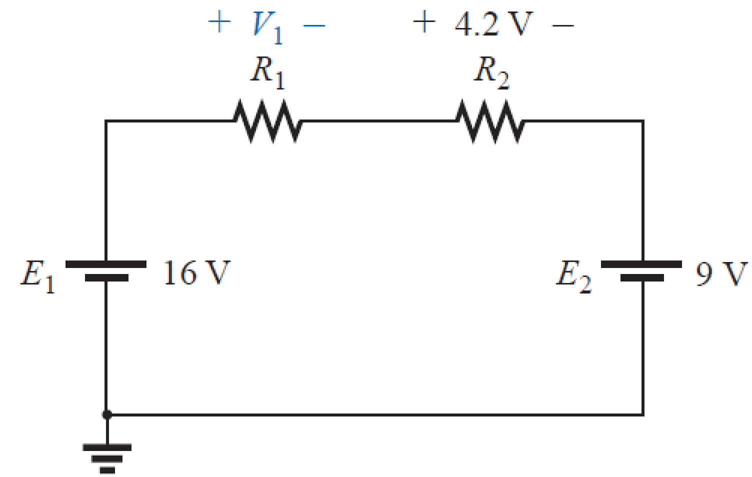
EXAMPLE 5.4 Determine the unknown voltages for the networks of the Figures.

Application of Kirchhoff's voltage law in clockwise direction results in:

$$+E_1 - V_1 - V_2 - E_2 = 0$$

$$V_1 = E_1 - V_2 - E_2 = 0$$

$$V_1 = 16V - 4.2V - 9V = 2.8V$$



(a)

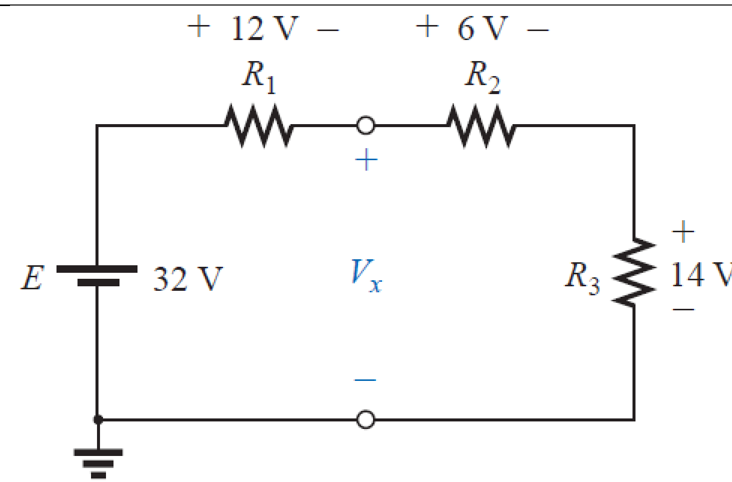
1- $E - V_1 - V_x = 0 \Longrightarrow$

$V_x = E - V_1 = 32V - 12V = \mathbf{20V}$

or

2- $V_x - V_2 - V_3 = 0 \Longrightarrow$

$V_x = V_2 + V_3 = 6V + 14V = \mathbf{20V}$



(b)

EXAMPLE 5.5 Find V_1 and V_2 for the network of Fig. 5.15.

Solution: For path 1, starting at point a in a clockwise direction:

$$+25\text{ V} - V_1 + 15\text{ V} = 0$$

and

$$V_1 = \mathbf{40\ V}$$

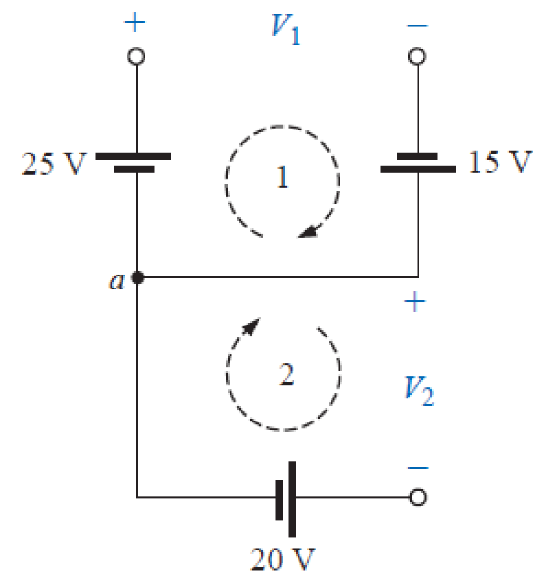
For path 2, starting at point a in a clockwise direction:

$$-V_2 - 20\text{ V} = 0$$

and

$$V_2 = \mathbf{-20\ V}$$

The minus sign simply indicates that the actual polarities of the potential difference are opposite the assumed polarity indicated in Fig. 5.15.



5.5 INTERCHANGING SERIES ELEMENTS

The elements of a series circuit can be interchanged without affecting the total resistance, current, or power to each element.

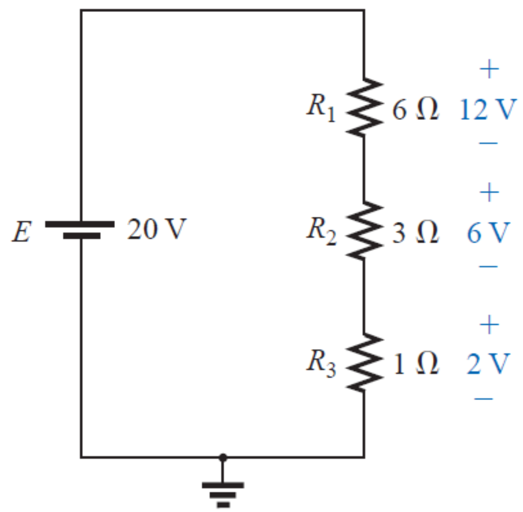
5.6 VOLTAGE DIVIDER RULE

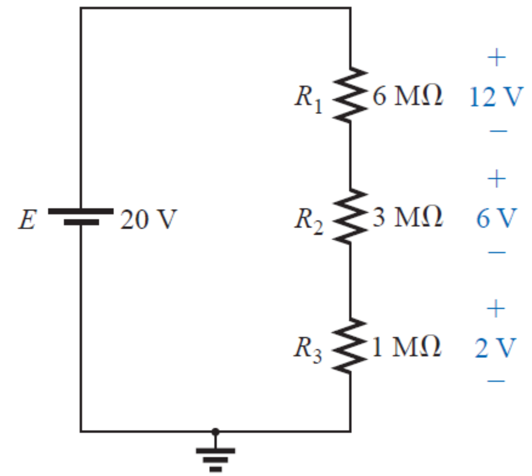
In a series circuit: ***the voltage across the resistive elements will divide as the magnitude of the resistance levels.***

$R_1 = 2R_2 \implies V_1 = 2V_2$

$R_2 = 3R_3 \implies V_2 = 3V_3$

The current I change by the values of R's, but the voltage remain the same.



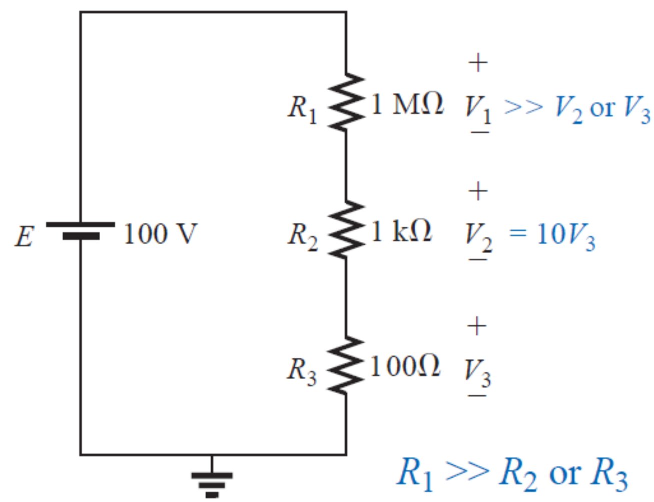


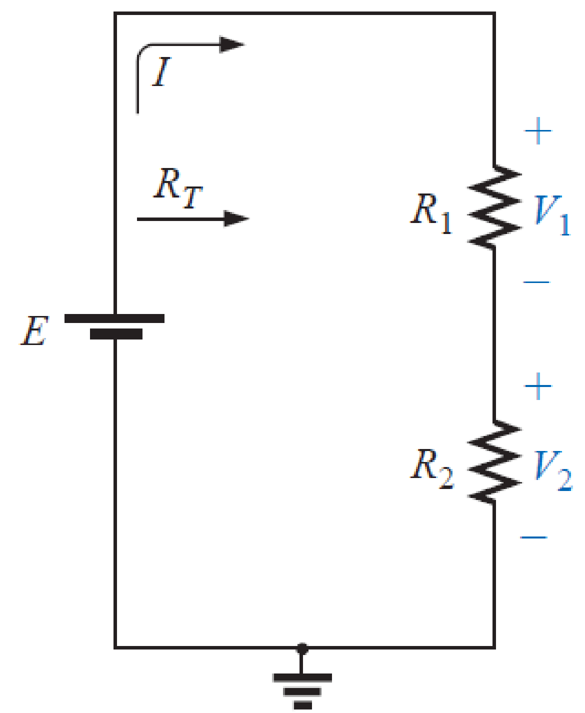
$R_1 = 1000\ R_2 \implies V_1 = 1000\ V_2$

$R_1 = 10000R_3 \implies V_1 = 10000V_3$

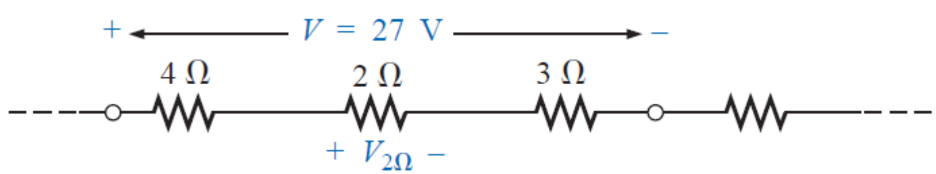
$$I = \frac{E}{R_T} = \frac{100}{1001100} \cong 99.89\ \mu A$$

$$V_1 = IR_1 = 99.89\ V$$
$$V_2 = IR_2 = 99.89\ mV$$
$$V_3 = IR_3 = 9.989\ mV$$



$R_T = R_1 + R_2 \quad \Longrightarrow \quad I = \frac{E}{R_T}$ $V_1 = IR_1 = \frac{E}{R_T}R_1 = \frac{R_1}{R_T} \cdot E$ $V_2 = IR_2 = \frac{E}{R_T}R_2 = \frac{R_2}{R_T} \cdot E$ In General: $V_x = IR_x = \frac{R_x}{R_T} \cdot E$ (Voltage Divider Rule)	

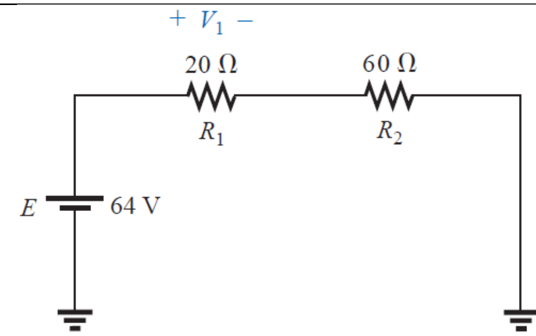
The voltage across a resistor in a series circuit is equal to the value of that resistor times the total impressed voltage across the series elements divided by the total resistance of the series elements.

$V_{2\Omega} = \frac{(2\ \Omega)(27\ \text{V})}{4\ \Omega + 2\ \Omega + 3\ \Omega} = \frac{54\ \text{V}}{9} = \mathbf{6\ V}$	

EXAMPLE 5.10 Determine the voltage V_1 for the network of Fig. 5.27.

Solution: Eq. (5.10):

$$V_1 = \frac{R_1E}{R_T} = \frac{R_1E}{R_1 + R_2} = \frac{(20\ \Omega)(64\ \text{V})}{20\ \Omega + 60\ \Omega} = \frac{1280\ \text{V}}{80} = \mathbf{16\ V}$$



EXAMPLE 5.11 Using the voltage divider rule, determine the voltages V_1 and V_3 for the series circuit of Fig. 5.28.

Solution:

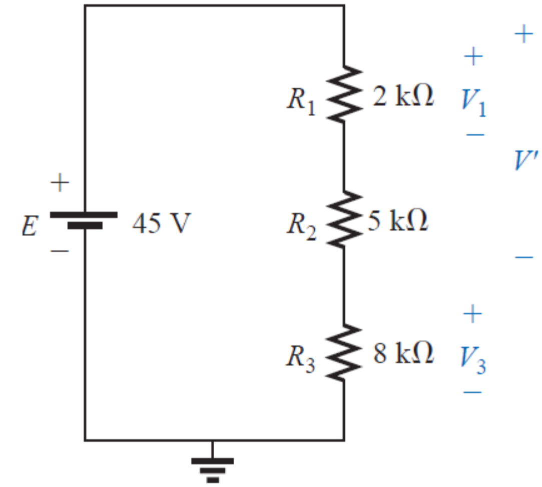
$$V_1 = \frac{R_1E}{R_T} = \frac{(2\ \text{k}\Omega)(45\ \text{V})}{2\ \text{k}\Omega + 5\ \text{k}\Omega + 8\ \text{k}\Omega} = \frac{(2\ \text{k}\Omega)(45\ \text{V})}{15\ \text{k}\Omega}$$

$$= \frac{(2 \times 10^3\ \Omega)(45\ \text{V})}{15 \times 10^3\ \Omega} = \frac{90\ \text{V}}{15} = \mathbf{6\ V}$$

$$V_3 = \frac{R_3E}{R_T} = \frac{(8\ \text{k}\Omega)(45\ \text{V})}{15\ \text{k}\Omega} = \frac{(8 \times 10^3\ \Omega)(45\ \text{V})}{15 \times 10^3\ \Omega}$$

$$= \frac{360\ \text{V}}{15} = \mathbf{24\ V}$$

$$V' = \frac{R'E}{R_T} = \frac{(2\ \text{k}\Omega + 5\ \text{k}\Omega)(45\ \text{V})}{15\ \text{k}\Omega} = \frac{(7\ \text{k}\Omega)(45\ \text{V})}{15\ \text{k}\Omega} = \mathbf{21\ V}$$



5.7 NOTATION

Voltage Sources and Ground:

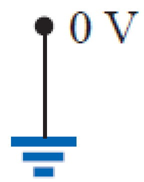
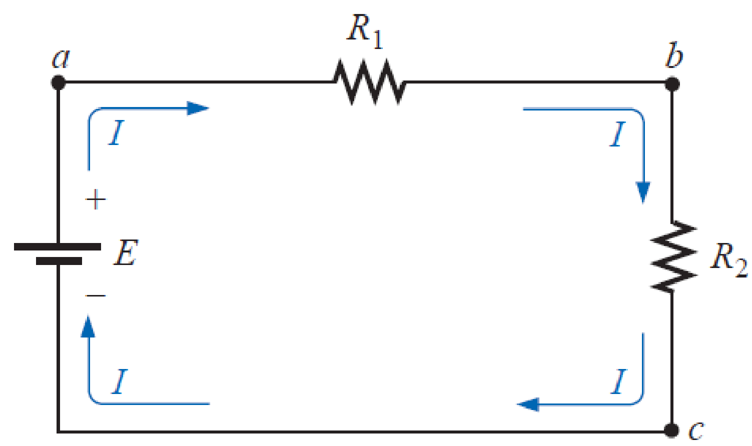
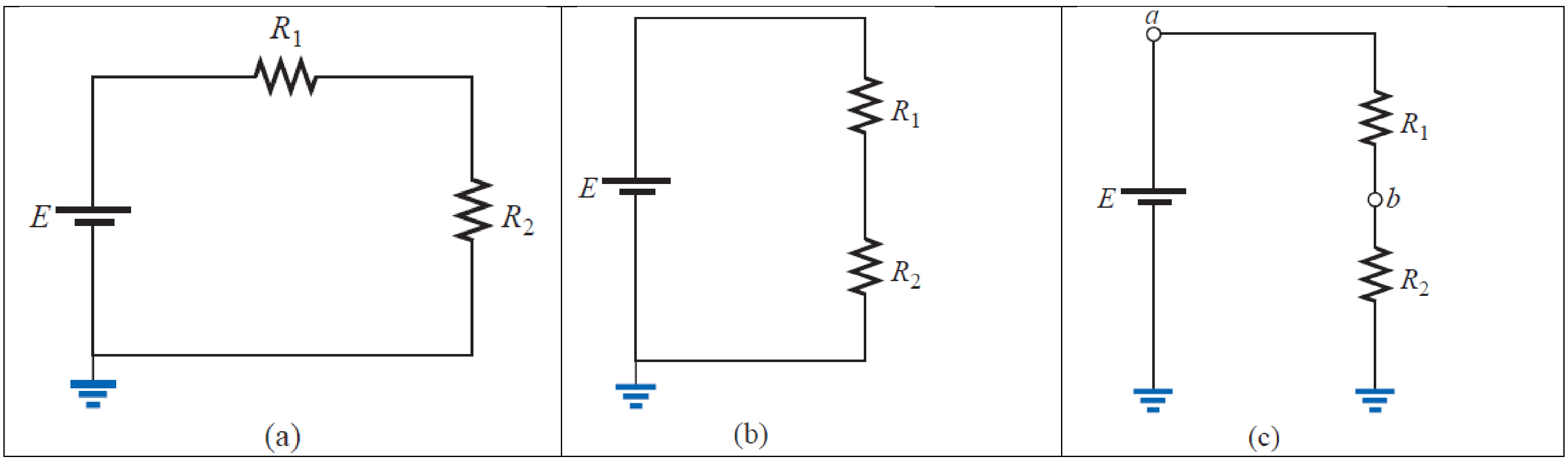
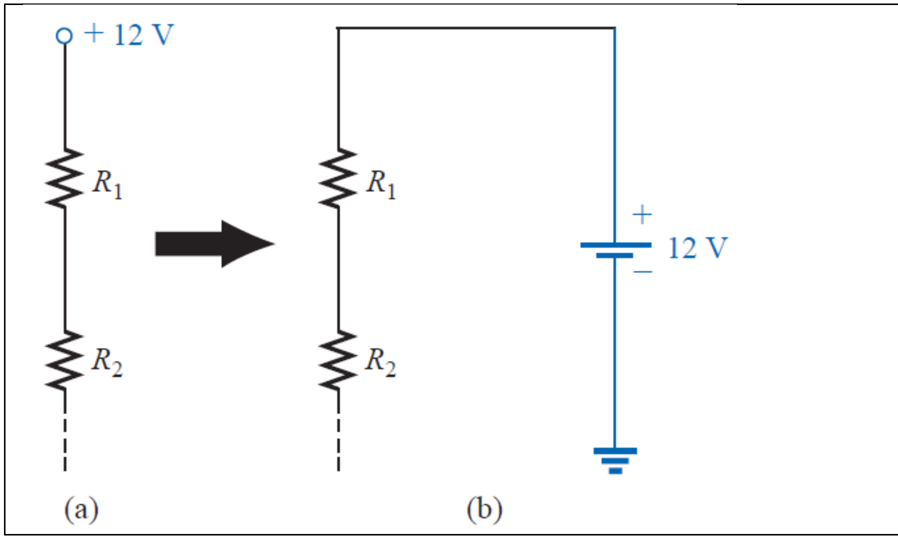
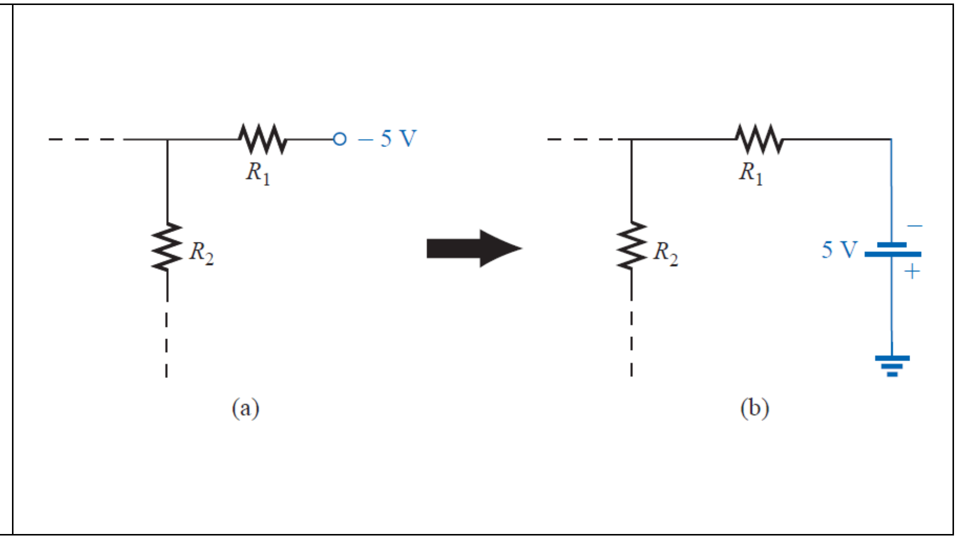


FIG. 5.31
Ground potential.






+ 12 V
R_1
R_2
(a)
R_1
R_2
+
12 V
–
(b)


R_1
– 5 V
R_2
(a)
R_1
R_2
–
5 V
+
(b)

Double-Subscript Notation

Voltage is always **across** (between) two points resulted in a double –subscript notation that defines the first subscript as the higher potential

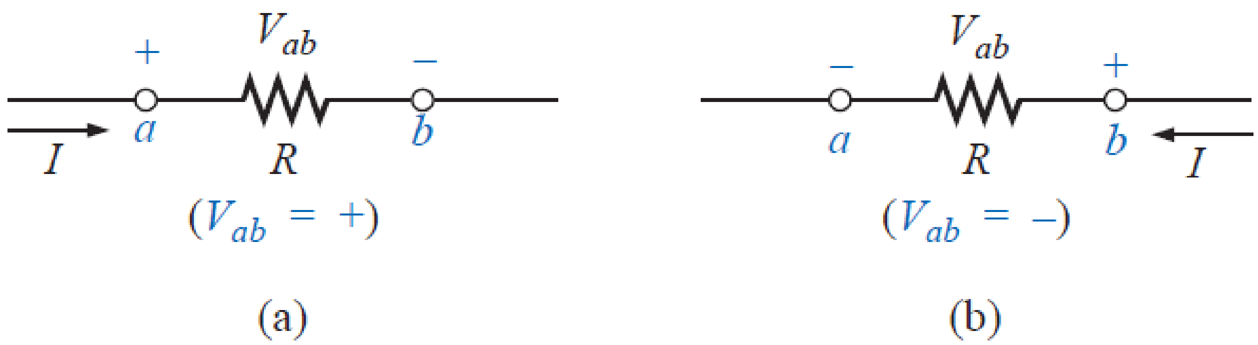


FIG. 5.36

Defining the sign for double-subscript notation.

The double-subscript notation V_{ab} specifies point "a" as the higher potential. If this is not the case, a negative sign must be associated with the magnitude of V_{ab}.

The voltage V_{ab} is the voltage at point "a" with respect to (w.r.t.) point "b".

Single-Subscript Notation:

If one of the point is specified as ground (reference) then a single subscript is employed, that provide the voltage with respect to ground.

If the voltage is less than zero volts, a negative sign must be associated with the magnitude of V_a .

In general:

$$\boxed{V_{ab} = V_a - V_b}$$

$$V_{ab} = V_a - V_b = 10\,\text{V} - 4\,\text{V}$$
$$= 6\,\text{V}$$