

Show the output After one AES Round if (Show your work) :

Input = **ea835cF00445332d655d98ad8596b0c5**

Cipher Key = **ac7766f319fadc2128d12941575c006a**

Constant of multiplication by X = (0001 1011).

- Find the four state as follows:
- Sub-byte for all bytes of the state
- Shift Row all bytes of the state
- Mix-Column for the first byte of the resultant state
- Add-Round for the first byte of the resultant state

$$\begin{bmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{bmatrix}$$

(a) S-box

| | | y | | | | | | | | | | | | | | | |
|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | A | B | C | D | E | F |
| x | 0 | 63 | 7C | 77 | 7B | F2 | 6B | 6F | C5 | 30 | 01 | 67 | 2B | FE | D7 | AB | 76 |
| | 1 | CA | 82 | C9 | 7D | FA | 59 | 47 | F0 | AD | D4 | A2 | AF | 9C | A4 | 72 | C0 |
| | 2 | B7 | FD | 93 | 26 | 36 | 3F | F7 | CC | 34 | A5 | E5 | F1 | 71 | D8 | 31 | 15 |
| | 3 | 04 | C7 | 23 | C3 | 18 | 96 | 05 | 9A | 07 | 12 | 80 | E2 | EB | 27 | B2 | 75 |
| | 4 | 09 | 83 | 2C | 1A | 1B | 6E | 5A | A0 | 52 | 3B | D6 | B3 | 29 | E3 | 2F | 84 |
| | 5 | 53 | D1 | 00 | ED | 20 | FC | B1 | 5B | 6A | CB | BE | 39 | 4A | 4C | 58 | CF |
| | 6 | D0 | EF | AA | FB | 43 | 4D | 33 | 85 | 45 | F9 | 02 | 7F | 50 | 3C | 9F | A8 |
| | 7 | 51 | A3 | 40 | 8F | 92 | 9D | 38 | F5 | BC | B6 | DA | 21 | 10 | FF | F3 | D2 |
| | 8 | CD | 0C | 13 | EC | 5F | 97 | 44 | 17 | C4 | A7 | 7E | 3D | 64 | 5D | 19 | 73 |
| | 9 | 60 | 81 | 4F | DC | 22 | 2A | 90 | 88 | 46 | EE | B8 | 14 | DE | 5E | 0B | DB |
| | A | E0 | 32 | 3A | 0A | 49 | 06 | 24 | 5C | C2 | D3 | AC | 62 | 91 | 95 | E4 | 79 |
| | B | E7 | C8 | 37 | 6D | 8D | D5 | 4E | A9 | 6C | 56 | F4 | EA | 65 | 7A | AE | 08 |
| | C | BA | 78 | 25 | 2E | 1C | A6 | B4 | C6 | E8 | DD | 74 | 1F | 4B | BD | 8B | 8A |
| | D | 70 | 3E | B5 | 66 | 48 | 03 | F6 | 0E | 61 | 35 | 57 | B9 | 86 | C1 | 1D | 9E |
| | E | E1 | F8 | 98 | 11 | 69 | D9 | 8E | 94 | 9B | 1E | 87 | E9 | CE | 55 | 28 | DF |
| | F | 8C | A1 | 89 | 0D | BF | E6 | 42 | 68 | 41 | 99 | 2D | 0F | B0 | 54 | BB | 16 |

AES Single Round Example

The Input block of data to a single round of AES algorithm with 128 bits length is

| | | | |
|----|----|----|----|
| EA | 04 | 65 | 85 |
| 83 | 45 | 5D | 96 |
| 5C | 33 | 98 | B0 |
| F0 | 2D | AD | C5 |

and a round key input to this round is

| | | | |
|----|----|----|----|
| AC | 19 | 28 | 57 |
| 77 | FA | D1 | 5C |
| 66 | DC | 29 | 00 |
| F3 | 21 | 41 | 6A |

Find the data of output block from this round

Answer:

After Substitute Bytes Transformation

| | | | |
|----|----|----|----|
| 87 | F2 | 4D | 97 |
| EC | 6E | 4C | 90 |
| 4A | C3 | 46 | E7 |
| 8C | D8 | 95 | A6 |

After Shift Row Transformation

| | | | |
|----|----|----|----|
| 87 | F2 | 4D | 97 |
| 6E | 4C | 90 | EC |
| 46 | E7 | 4A | C3 |
| A6 | 8C | D8 | 95 |

After Mix Column Transformation

$$\begin{bmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{bmatrix} \begin{bmatrix} 87 & F2 & 4D & 97 \\ 6E & 4C & 90 & EC \\ 46 & E7 & 4A & C3 \\ A6 & 8C & D8 & 95 \end{bmatrix} = \begin{bmatrix} 47 & 40 & A3 & 4C \\ 37 & D4 & 70 & 9F \\ 94 & E4 & 3A & 42 \\ ED & A5 & A6 & BC \end{bmatrix}$$

- To find the first byte after Mix Column, we do matrix multiplication over $GF(2^8)$ as follows:

$$(02 * 87) \oplus (03 * 6E) \oplus 46 \oplus A6 = 47$$

$$\begin{aligned} \text{We have } 02 * 87 &= (0000\ 0010) * (1000\ 0111) = (0000\ 1110) \oplus (0001\ 1011) \\ &= (0001\ 0101) \end{aligned}$$

After Mix Column Transformation

- To find the first byte after Mix Column, we do matrix multiplication over $GF(2^8)$ as follows:

In particular, multiplication of a value by (i.e., by {02}) can be implemented as a 1-bit left shift followed by a conditional bitwise XOR with (0001 1011)

$$(02 * 87) \oplus (03 * 6E) \oplus 46 \oplus A6 = 47$$

$$\begin{aligned} \text{We have } 02 * 87 &= (0000\ 0010) * (1000\ 0111) = (0000\ 1110) \oplus (0001\ 1011) \\ &= (0001\ 0101) \end{aligned}$$

$$\begin{aligned} \text{and } (03 * 6E) &= (0000\ 0011) * (0110\ 1110) = (0000\ 0001) * (0110\ 1110) \oplus (0000\ 0010) * \\ &(0110\ 1110) \end{aligned}$$

$$= (0110\ 1110) \oplus (1101\ 1100) = (1011\ 0010)$$

$$\text{and } (46) = (0100\ 0110)$$

$$\text{and } (A6) = (1010\ 0110)$$

$$\begin{aligned} \text{Then the first byte} &= (0001\ 0101) \oplus (1011\ 0010) \oplus (0100\ 0110) \oplus (1010\ 0110) = (0100\ 0111) \\ &= (47) \end{aligned}$$

- After Add Round Key Transformation

| | | | | | | | | | | | | | |
|----|----|----|----|----------|----|----|----|----|---|----|----|----|----|
| 47 | 40 | A3 | 4C | \oplus | AC | 19 | 28 | 57 | = | EB | 59 | 8B | 1B |
| 37 | D4 | 70 | 9F | | 77 | FA | D1 | 5C | | 40 | 2E | A1 | C3 |
| 94 | E4 | 3A | 42 | | 66 | DC | 29 | 00 | | F2 | 38 | 13 | 42 |
| ED | A5 | A6 | BC | | F3 | 21 | 41 | 6A | | 1E | 84 | E7 | D2 |

The value of the first byte of after Add Round Key = $(47) \oplus (AC)$

$(47) \oplus (AC) = (0100\ 0111) \oplus (1010\ 1100) = (1110\ 1011) = (EB)$

The value of the first byte of after Add Round Key = $(47) \oplus (AC) = (EB)$