Problem Set (1): LP models

Problem 1:

A small manufacturer employs 5 skilled men and 10 semi-skilled men and makes an article in two qualities, a deluxe model and an ordinary model. The making of a deluxe model requires 2 hours work by a skilled man and 2 hours work by a semi-skilled man. The ordinary model requires 1 hour work by a skilled man and 3 hours work by a semi-skilled man. By work rules no man can work more than 8 hours a day. The manufacturer's clear profit of the deluxe model is L.E. 10 and of the ordinary model L.E. 8. Formulate the model of the problem.

Answer:

Maximize $Z = 10x_1 + 8x_2$, subject to: $2x_1 + x_2 \leq 40$, $2x_1 + 3x_2 \leq 80$, $x_1 \geq 0$, $x_2 \geq 0$.

Problem 2:

The manager of an oil refinery has to decide upon the optimal mix of two possible blending processes, of which the inputs and outputs per production run are as follows:

	Input		*Output*	
Process	*Crude A*	*Crude B*	*Gasoline X*	*Gasoline Y*
1	5	3	5	8
2	4	5	4	4

The maximum amount available of crude *A* and *B* are 200 units and 150 units respectively. Market requirements show that at least 100 units of gasoline *X* and 80 units of gasoline *Y* must be produced. The profits per production run from process 1 and process 2 are $3 and $4 respectively. Formulate the problem as linear programming problem.

Answer:

Maximize $Z = 3x_1 + 4x_2$, subject to: $5x_1 + 4x_2 \leq 200$, $3x_1 + 5x_2 \leq 150$, $5x_1 + 4x_2 \geq 100$, $8x_1 + 4x_2 \geq 80$ and $x_1, x_2 \geq 0$.

Problem 3:

A farmer has a 100 acre farm. He can sell tomatoes, lettuce or radishes. The price he can obtain is $1.00 per kg of tomatoes, $0.75 per head of lettuce and $2.00 per kg of radishes

The average yield per acre is 2000 kg of tomatoes, 3000 heads of lettuce and 1000 kg of radishes

Fertilizer is available at $0.5 per kg and the amount required per acre is 100kg each for tomatoes and lettuce and 50kg for radishes.

Labor required for sowing, cultivating and harvesting per acre is 5 man-days for tomatoes and radishes and 6 man-days for lettuce. A total of 400 man-days of labor are available at $20 per man-day.

Formulate the LP model for this problem in order to maximize the farmer's total profit

Solution:

Assume the farmer will cultivate x, y and z acres of tomatoes, lettuce and radishes respectively.

- The selling price for all products = $2000x+2250y+2000z$
- Cost of fertilizers = $50x+50y+25z$
- Cost of labor = $100x+120y+100z$
- Net profit = $(2000-50-100)x+(2250-50-120)y+(2000-25-100)z$
- Net profit =$1850x+2080y+1875z$

The constraints are:

- Total area of land: $x+y+z \leq 100$
- The available man-days: $5x+6y+5z \leq 400$
- Non negativity: $x, y, z \geq 0$

The LP model is:

Maximize

$$P = 1850x + 2080y + 1875z$$

Subject to:

$$x + y + z \leq 100$$

$$5x + 6y + 5z \leq 400$$

$$x, y, z \geq 0$$