KING SAUD UNIVERSITY
College of Science
Department of Mathematics

Second Semester (1432-33/2011-12)

First Mid-Exam
M-106

Programmable Calculators are Not Authorized

The Exam paper contains 5 pages
(**10 Multiple choice questions and 4 Full questions**)

Name:	Number:
Name of Teacher:	Group No:

Max Marks: 20 Time: 90 minutes

Marks:

Multiple Choice (1-10)	
Question # 11	
Question # 12	
Question # 13	
Question # 14	
Total	

Multiple Choice

Q. No:	1	2	3	4	5	6	7	8	9	10
$\{a, b, c, d\}$	c	b	c	d	c	d	c	d	a	c

Q. No: 1 If $\sum_{k=1}^{25} (k^2 + \alpha k) = 0$, then the value of α is equal to:

(a) -16 (b) 16 (c) -17 (d) -1

Q. No: 2 The value of the integral $\int_{-1}^{1} 2\,|x|^3\, dx$ is equal to

(a) 2 (b) 1 (c) 0 (d) -1

Q. No: 3 The value of the integral $\int \frac{\sin(\tan(x))}{\cos^2(x)} dx$ is equal to:

(a) $\cos(\tan(x)) + c$ (b) $\sin(\tan(x)) + c$ (c) $-\cos(\tan(x)) + c$ (d) $-\sin(\tan(x)) + c$

Q. No: 4 The derivative of the integral $\int_{0}^{x} [1 + \frac{d\tan(t)}{dt}] dt$ is equal to:

(a) $1 + \tan x$ (b) $1 - \tan x$ (c) $1 - \sec^2 x$ (d) $1 + \sec^2 x$

Q. No: 5 If $G(x) = \int_{e}^{x^2} \frac{\ln(t)}{4} dt$ then $G'(e)$ is equal to:

(a) $2e$ (b) 1 (c) e (d) $4e$

Q. No: 6 If $\log_2\left(\frac{x-1}{x}\right) = 1$, then x is equal to:

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) -1

Q. No: 7 The value of the integral $\int_{0}^{1} 5^x dx$ is equal to:

(a) $\frac{4\ln 5}{5}$ (b) $\frac{\ln 5}{4}$ (c) $\frac{4}{\ln 5}$ (d) $\frac{5\ln 5}{4}$

Q. No: 8 The integral $\int x\sqrt{x^2+1}dx$ is equal to:

$(a)\ \frac{1}{2}x^2\sqrt{x^2+1}+c$ $(b)\ \frac{2}{3}(x^2+1)^{3/2}+c$ $(c)\ -\frac{2}{3}(x^2+1)^{3/2}+c$ $(d)\ \frac{1}{3}(x^2+1)^{3/2}+c$

Q. No: 9 The value of the integral $\int_0^1 \frac{e^x}{(e^x+1)^2}dx$ is equal to:

$(a)\ \frac{e-1}{2(1+e)}$ $(b)\ 0$ $(c)\ -1$ $(d)\ \frac{1}{(1+e)^2}$

Q. No: 10 The value of the integral $\int \frac{dx}{\sqrt{16-25x^2}}$ is equal to:

$(a)\ -\frac{\cos^{-1}(\frac{x}{16})}{25}+c$ $(b)\ \frac{\cos^{-1}(\frac{x}{16})}{25}+c$ $(c)\ \frac{\sin^{-1}(\frac{5x}{4})}{5}+c$ $(d)\ -\frac{\sin^{-1}(\frac{5x}{4})}{5}+c$

Full Questions

Question No: 11: Use Trapezoidal rule to approximate the integral $\int_1^3 \sqrt{x^2+3}dx$ with $n=4$. [3]:

Answer: $[a,b]=[1,3]$, $n=4$, and $f(x)=\sqrt{x^2+3}$,

$$\begin{aligned} \Delta x &= \frac{b-a}{n} = \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2} = 0.5 \ (0.5) \\ x_0 &= 1,\ x_1 = 1.5, x_2 = 2, x_3 = 2.5, x_4 = 3.\ (0.5) \end{aligned}$$

$$\begin{aligned} \int_1^3 \sqrt{x^2+3}dx &\approx \frac{3-1}{2(4)}[f(1)+2f(1.5)+2f(2)+2f(2.5)+f(3)] \qquad (1) \\ &\approx \frac{3-1}{2(4)}[2+2(2.29129)+2(2.64575)+2(3.04138)+3.4641] \quad (0.5) \\ &\approx \frac{21.4209}{4} \approx 5.3552 \qquad (0.5) \end{aligned}$$

Question No: 12: If $f(x)=x^{\cosh(x)}$, then find $f'(x)$. [**2**]

Answer: by using the natural logaritm we have:

$$\ln(f(x)) = \ln(x^{\cosh(x)}) \ = \cosh(x)\ln(x) \ (0.5)$$

then

$$\begin{aligned} \frac{d\ \ln(f(x))}{dx} &= \frac{d(\cosh(x)\ln(x))}{dx} \ (0.5) \\ \frac{f'(x)}{f(x)} &= \frac{d(\cosh(x))}{dx}\ln(x) + \cosh(x)\frac{d(\ln(x))}{dx} = \sinh(x)\ln(x) + \frac{\cosh(x)}{x} \ (0.5) \end{aligned}$$

and then

$$f'(x) = \left[\sinh(x)\ln(x) + \frac{\cosh(x)}{x}\right] f(x) = \left[\sinh(x)\ln(x) + \frac{\cosh(x)}{x}\right] x^{\cosh(x)} \ (0.5)$$

Question No: 13: Find the number z that satisfies the conclusion of the Mean value Theorem for the function $f(x)=\cos(2x)$ where $x \in [0, \frac{\pi}{2}]$. And also find the average value f_{av} of $f(x)$. [3]

Answer: By using the Mean-Value theorem we have

$$\int_0^{\pi/2} \cos(2x)dx = (\frac{\pi}{2} - 0)\cos(2z), \ (1)$$

where z is a real number in $[0, \frac{\pi}{2}]$. Then

$$\frac{\pi}{2}\cos(2z) = \int_0^{\pi/2} \cos(2x)dx = [\frac{1}{2}\sin(2x)]_0^{\frac{\pi}{2}} = \frac{1}{2}(\sin(\pi) - \cos(0) = 0, \ (0.5)$$

therefore $\cos(2z) = 0$, and then

$$z = \frac{\pi}{4} \in [0, \frac{\pi}{2}], \ (1)$$

and also we have

$$f_{av} = \frac{1}{\frac{\pi}{2} - 0}\int_0^{\pi/2} \cos(2x)dx = 0, \ (0.5)$$

Question No: 14: Evaluate the following integral

$$J = \int \frac{\cos(x)}{\sin(x)\sqrt{4 - \sin^2(x)}}dx$$

Answer: Put

$$u = \sin(x) \text{ then } du = \cos(x)dx, \ (0.5)$$

and we have

$$\begin{aligned} J &= \int \frac{du}{u\sqrt{4-u^2}} = \int \frac{du}{u\sqrt{2^2 - u^2}}, \ (0.5) \\ &= -\frac{1}{2}\,\text{sech}^{-1}(\frac{u}{2}) + c = -\frac{1}{2}\,\text{sech}^{-1}(\frac{|\sin(x)|}{2}) + c, \quad (1) \end{aligned}$$