KING SAUD UNIVERSITY **FULL MARKS: 30**

M - 107 **DEPARTMENT OF MATHEMATICS** **TIME: 90min**

(SEMESTER I, 1429-1430) SECOND MID-TERM

NOTE: Attempt all Questions.

Question: 1.(a) Let $a = 3i + 2j - 5k$ and $b = 2i + j + 3k$ find $comp_b a$.
[2+4+4]

(b) Find the work done by a constant force $F = 2i + j - 3k$, if its point of application moves from point P(4, -3, -2) to Q(6, 7, -3).

(c) Find direction cosines and direction angles of the vector $a = 3i - j + 5k$.

Question: 2.(a) Find the distance of the point P(2, 4, -5) from the line through points Q(3, 5, -1) and R(5, 2, 4).
[4+3+3]

(b) Find the volume of the parallelepiped having adjacent sides AB, AC and AD where A(1, -1,2), B(3, 4, -5), C(-3, 1, 1) and D(1, 2, 4).

(c)Determine whether the line
$x = 3 + 8t,\ y = 4 + 5t,\ z = -3 - t$ and the plane $x - 3y + 5z = 12$ are parallel . If the above line and the plane are not parallel find their point of intersection.

Question:3. (a) Find the parametric equation of the straight line passing through the point P(1, -1, 2) and orthogonal to plane 3x - 2y + z = 2.
[3+3+4]

(b) Find the equation of the plane through the point P(1, -1, 2) and parallel to plane 3x - 2y + z = 2.

(c) Find the shortest distance between skew lines , line l_1 through points A(1, 2,4 and B(3,2,5) and line l_2 through points C(6,3,3,) and D(4, 5,1).

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Question 1 (a) $\vec{a} = 3\vec{i} + 2\vec{j} - 5\vec{k}$, $\vec{b} = 2\vec{i} + \vec{j} + 3\vec{k}$

2

$$\text{comp}_{\vec{b}}\,\vec{a} = \vec{a} \cdot \frac{\vec{b}}{\|\vec{b}\|} = \frac{\vec{a} \cdot \vec{b}}{\|\vec{b}\|}$$

$\vec{a} \cdot \vec{b} = 6 + 2 - 15 = -7$ 1

$\|\vec{b}\| = \sqrt{4+1+9} = \sqrt{14}$, $\text{comp}_{\vec{b}}\,\vec{a} = \frac{-7}{\sqrt{14}}$ 1

4 (b) $\vec{d} = \vec{PQ} = \langle 2, 10, -1 \rangle$ 2

$W = \vec{F} \cdot \vec{d} = \langle 2, 1, -3 \rangle \langle 2, 10, -1 \rangle$

$= 4 + 10 + 3 = 17.$ 2

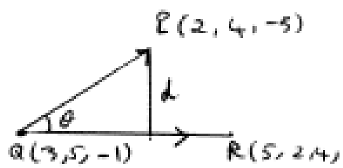
4 (c) $\vec{a} = 3\vec{i} - \vec{j} + 5\vec{k}$, $\|\vec{a}\| = \sqrt{9+1+25} = \sqrt{35}.$ 1

$$\begin{cases} \cos\alpha = \frac{a_1}{\|\vec{a}\|} = \frac{3}{\sqrt{35}} \\ \cos\beta = \frac{a_2}{\|\vec{a}\|} = \frac{-1}{\sqrt{35}} \\ \cos\gamma = \frac{a_3}{\|\vec{a}\|} = \frac{5}{\sqrt{35}} \end{cases} \qquad \begin{cases} \alpha = \cos^{-1}\left(\frac{3}{\sqrt{35}}\right) \\ \beta = \cos^{-1}\left(\frac{-1}{\sqrt{35}}\right) \\ \gamma = \cos^{-1}\left(\frac{5}{\sqrt{35}}\right) \end{cases}$$

2 1

Question 2 (a) $d = \frac{\|\vec{QP} \times \vec{QR}\|}{\|\vec{QR}\|}$

4



$$\vec{QP} \times \vec{QR} = \begin{vmatrix} i & j & k \\ -1 & -1 & -4 \\ 2 & -3 & 5 \end{vmatrix} = (-5-12)\vec{i} - (-5+8)\vec{j} + (3+2)\vec{k}$$

$= -17\vec{i} - 3\vec{j} + 5\vec{k}$ 2

$\|\vec{QP} \times \vec{QR}\| = \sqrt{289+9+25} = \sqrt{323} \approx 17.97$

$\vec{QR} = \langle 2, -3, 5 \rangle$, $\|\vec{QR}\| = \sqrt{4+9+25} = \sqrt{38} \approx 6.16$

$d = \frac{17.97}{6.16} \approx 2.92$

2

(b) $\vec{AB} = \langle 2, 5, -7 \rangle = \vec{a}$

(3) $\vec{AC} = \langle -4, 2, -1 \rangle = \vec{b}$ (1)

$\vec{AD} = \langle 0, 3, 2 \rangle = \vec{c}$

$$V = |(\vec{a} \times \vec{b}) \cdot \vec{c}| = \begin{vmatrix} 2 & 5 & -7 \\ -4 & 2 & -1 \\ 0 & 3 & 2 \end{vmatrix}$$

$= 2(4+3) - 5(-8+0) - 7(-12-0)$ (2)

$= 14 + 40 + 84 = 138$ unit^3.

(c) The direction of the line is $\vec{a}(8, 5, -1)$

(3) The normal of the plane is $\vec{n}(1, -3, 5)$

so $\vec{a} \not\parallel \vec{n}$ ($\vec{a}$ is not parallele to $\vec{n}$) (1)

Let (x, y, z) be a point of the intersection.

$3 + 8t - 3(4 + 5t) + 5(-3 - t) = 12$

$3 + 8t - 12 - 15t - 15 - 5t = 12$ $\quad x = 3 - 24 = -21$

$-24 - 12t = 12 \Rightarrow t = -3 \Rightarrow$ $\quad y = 4 - 1 = -11$

$z = -3 + 3 = 0$ (2)

$\boxed{M(-21, -11, 0)}$

Question (3) a) The normal of the plane $3x - 2y + z = 2$

is $\vec{n}(3, -2, 1)$ (1). So $\vec{n}$ is parallele to the direction of the line. So

$$\frac{x-1}{3} = \frac{y+1}{-2} = \frac{z-2}{1} = t \quad (2)$$

hence $x = 1 + 3t$, $y = -1 - 2t$, $z = 2 + t$ is the parametric equation of the line.

(2)

(b) The normal of the plane is $\vec{n}\,(3,-2,1)$ ①

③

The equation of the plane is

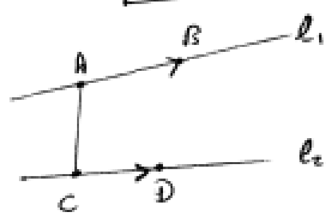
$$3(x-1)+(y+1)(-2)+(z-2)2=0$$

$$3x-2y+2z-3-2+4=0$$ ②

$$\boxed{3x-2y+2z-1=0}$$

④ (c)

A = (1, 2, 4)
B (3, 2, 5)
C (6, 3, 3)
D (4, 5, 1)



$$h=\frac{|(\vec{AB}\times\vec{CD})\cdot\vec{CA}|}{\|\vec{AB}\times\vec{CD}\|}$$

$$h=\frac{|12|}{\sqrt{24}}=\frac{12}{2\sqrt{6}}=\frac{6}{\sqrt{6}}=\sqrt{6}$$

$\vec{AB}=(2,0,1)$

$\vec{CD}=(-2,2,-2)$

$$\vec{AB}\times\vec{CD}=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 0 & 1 \\ -2 & 2 & -2 \end{vmatrix}$$ ②

$$=-2\vec{i}-(-4+2)\vec{j}+4\vec{k}$$

$$=-2\vec{i}+2\vec{j}+4\vec{k}$$

$(\vec{AB}\times\vec{CD})\cdot\vec{CA}$

$=\langle -2,+2,4\rangle\langle -5,-1,1\rangle$

$=10-2+4=12$

$\|\vec{AB}\times\vec{CD}\|=\sqrt{4+4+16}$

$=\sqrt{24}$ ②