KING SAUD UNIVERSITY
College of Science
Department of Mathematics

First Semester (1433/1434)

Second Mid-Exam

Name:	Number:
Name of Teacher:	Group No:

Max Marks: 25 Time: 90 minutes

Marks:

Multiple Choice (1-10)	
Question # 11	
Question # 12	
Question # 13	
Question # 14	
Total	

Multiple Choice

Q.No:	1	2	3	4	5	6	7	8	9	10
$\{a,b,c,d\}$	b	d	d	b	d	a	c	c	c	d

Q. No: 1 The indefinite integral $\int \sin^2(\frac{x}{2})dx$ is equal to:

$(a)\ \frac{1}{2}x + \frac{1}{2}\sin(x) + c \quad (b)\ \frac{1}{2}(\ x - \sin(x)) + c \quad (c)\ \frac{1}{2}\cos(\frac{x}{2}) + c \quad (d) - \frac{1}{2}\cos(\frac{x}{2}) + c$

Q. No: 2 The substitution $u = \tan\left(\frac{x}{2}\right)$ transforms the integral $\int \frac{1}{1+\sin x}dx$ into:

$(a)\ \int du \quad (b)\ \int 2du \quad (c)\ \int \frac{1}{(u+1)^2}du \quad (d)\ \int \frac{2}{(u+1)^2}du$

Q. No: 3 To evaluate the integral $\int \sqrt{2x^2+4}dx$, we use the substitution:

$(a)\ x = \sqrt{2}\sec\theta \quad (b)\ x = 2\tan\theta \quad (c)\ x = 2\sec\theta \quad (d)\ x = \sqrt{2}\tan\theta$

Q. No: 4 $\lim_{x\to 0}(1+x)^{\frac{1}{x}}$ is equal to:

(a) 1 $\quad (b)$ e $\quad (c)$ 0 $\quad (d)$ ∞

Q. No: 5 If $\frac{x^3+1}{(3x^2+1)^2} = \frac{Ax+B}{3x^2+1} + \frac{Cx+D}{(3x^2+1)^2}$, then the value of A is equal to:

$(a)\ \frac{1}{2} \quad (b)\ \frac{3}{2} \quad (c)\ \frac{2}{3} \quad (d)\ \frac{1}{3}$

Q. No: 6 Evalute $\int \sin^3(x)dx$

$(a) - \cos(x) + \frac{\cos^3(x)}{3} + c \qquad (b)\ \cos(x) - \frac{\cos^3(x)}{3} + c$

$(c) - \cos(x) - \frac{\cos^3(x)}{3} + c \qquad (d) - \cos(x) + \frac{\cos^2(x)}{2} + c \ + c$

Q. No: 7 The improper integral $\int_{-\infty}^{0} \frac{e^x}{1+e^{2x}}dx$

(a) converges to 0 $\quad (b)$ diverges $\quad (c)$ converges to $\frac{\pi}{4}$ $\quad (d)$ converges to $\frac{\pi}{2}$

Q. No: 8 The indefinite integral $\int x\cos(x)dx$ is equal to:

(a) $\cos x - x\sin x + c$ (b) $-\cos x + x\sin x + c$
(c) $\cos x + x\sin x + c$ (d) $x\sin x + c$

Q. No: 9 The area of the region **bounded** by the graphs of equations: $y = x^2$ and $y = 2x$ is equal to:

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{4}{3}$ (d) $\frac{1}{2}$

Q. No: 10 To evalute $\int \frac{\sqrt[3]{x}}{1+\sqrt[4]{x}}dx$, we put:

(a) $u^3 = x$ (b) $x = u^4$ (c) $u = \sqrt{x}$ (d) $u = x^{\frac{1}{12}}$

Full Questions

Question No. 11: Evaluate $\displaystyle\int \frac{-x^2+2x+1}{(x-1)(x^2+1)}dx$ [**3**]
Solution: By using decomposition method we will have:

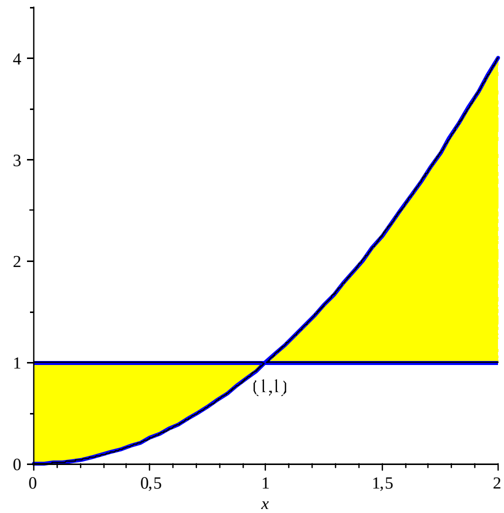
$$\frac{-x^2+2x+1}{(x-1)(x^2+1)} = \frac{1}{x-1} - \frac{2x}{x^2+1} \ [1]$$

Then

$$\begin{aligned}\int \frac{-x^2+2x+1}{(x-1)(x^2+1)}dx &= \int \frac{1}{x-1}dx - \int \frac{2x}{x^2+1}dx\ ,\\ &= \ln|x-1| - \ln(x^2+1) + c, \qquad [1]+[1]\end{aligned}$$

Question No. 12: Sketch and **Find** the area between the curves $f(x) = x^2$ and $g(x) = 1$ on the interval $[0, 2]$. [**4**]

Solution:



, [1]

We have

$$\begin{aligned} Area &= \int_0^1 (1 - x^2)dx + \int_1^2 (x^2 - 1)dx, \ [2] \\ &= [x - \frac{x^3}{3}]_0^1 + [\frac{x^3}{3} - x]_1^2, \ [0.5] \\ &= \frac{2}{3} + \frac{4}{3} = 2. \ [0.5] \end{aligned}$$

Question No. 13: Evaluate $\displaystyle\int \frac{1}{(1+x^2)^2} dx$ [**6**]

Solution: **Method N. 1:**

Let

$$x = \tan\theta, \text{ where } -\frac{\pi}{2} < \theta < \frac{\pi}{2}, \text{ then we have } dx = \sec^2(\theta)d\theta, \ [1]$$

and also we have

$$(1 + x^2)^2 = (1 + \tan^2\theta)^2 = \sec^4(\theta), \ [1]$$

Thus

$$\begin{aligned} \int \frac{1}{(1+x^2)^2} dx &= \int \frac{\sec^2(\theta)d\theta}{\sec^4(\theta)} = \int \frac{d\theta}{\sec^2(\theta)} = \int \cos^2(\theta)d\theta = \int \frac{1+\cos(2\theta)}{2} d\theta, \ [1] \\ &= \frac{1}{2}(\theta + \frac{1}{2}\sin(2\theta)) + c = \frac{1}{2}(\theta + \sin(\theta)\cos(\theta)) + c, \ [1] \end{aligned}$$

we have $x = \tan(\theta)$ then

$$\begin{aligned}\tan(\theta) &= x \Rightarrow \theta = \tan^{-1}(x), \text{ [0.5]}\\ \sin(\theta) &= \frac{x}{\sqrt{1+x^2}}, \ \cos(\theta) = \frac{1}{\sqrt{1+x^2}}, \text{ [1]}\end{aligned}$$

and we can get

$$\int \frac{1}{(1+x^2)^2}dx = \frac{1}{2}(\tan^{-1}(x) + \frac{x}{\sqrt{1+x^2}}\frac{1}{\sqrt{1+x^2}}) + c \ = \frac{\tan^{-1}(x)}{2} + \frac{x}{2(1+x^2)} + c, \text{ [0.5]}$$

Method N 2: We have

$$\begin{aligned}\int \frac{1}{(1+x^2)^2}dx &= \int \frac{1+x^2-x^2}{(1+x^2)^2}dx & \text{[1.5]}\\ &= \int (\frac{1}{(1+x^2)^2} - \frac{x}{2}\frac{2x}{(1+x^2)^2})dx & \text{[1.5]}\\ &= \tan^{-1}(x) - \left(-\frac{x}{2}\frac{1}{1+x^2} + \frac{1}{2}\int \frac{dx}{1+x^2}\right) & \text{[1.5]}\\ &= \frac{1}{2}\tan^{-1}(x) + \frac{x}{2}\frac{1}{1+x^2} + c & \text{[1.5]}\end{aligned}$$

Method N. 3: By using decomposition method we can find:

$$\begin{aligned}\frac{1}{(1+u)^2} &= \frac{1}{2}\left(\frac{1}{1+u} + \frac{1-u}{(1+u)^2}\right), \quad \text{where} \quad u = x^2 \quad \text{[1.5]}\\ \frac{1}{(1+x^2)^2} &= \frac{1}{2}\left(\frac{1}{1+x^2} + \frac{1-x^2}{(1+x^2)^2}\right)\\ &= \frac{1}{2}\left(\frac{1}{1+x^2} + \frac{1+x^2-2x^2}{(1+x^2)^2}\right) \quad \text{[1]}\end{aligned}$$

and also we have

$$\frac{1}{(1+x^2)^2} = \frac{1}{2}\left(\frac{1}{1+x^2} + \frac{(\frac{dx}{dx})(1+x^2) - x\frac{d(1+x^2)}{dx}}{(1+x^2)^2}\right) \quad \text{[1.5]}$$

Thus

$$\int \frac{1}{(1+x^2)^2}dx = \frac{\tan^{-1}(x)}{2} + \frac{x}{2(1+x^2)} + c, \text{ [1+1]}$$

Question No. 14: Evaluate $\lim_{x\to 0} \frac{x - \ln(x+1)}{x\ln(x+1)}$ [**2**]

Solution: We have

$$\lim_{x\to 0}\frac{x-\ln(x+1)}{x\ln(x+1)};\ \left(\frac{0}{0}\right)$$

We apply l'Hopital rule we have

$$\lim_{x\to 0}\frac{x-\ln(x+1)}{x\ln(x+1)}=\lim_{x\to 0}\frac{1-\dfrac{1}{x+1}}{\dfrac{x}{x+1}+\ln(x+1)}=\lim_{x\to 0}\frac{x}{x+(x+1)\ln(x+1)};\left(\frac{0}{0}\right)\ [1]$$

Applying again l'Hopital rule we get

$$\lim_{x\to 0}\frac{x}{x+(x+1)\ln(x+1)}=\lim_{x\to 0}\frac{1}{1+\ln(x+1)+1}=\frac{1}{2}\ [1]$$