1

a/

$$A^{1}_{x:\overline{n|}} = \sum_{k=0}^{n-1} v^{k+1} {}_{k}p_x \, q_{x+k}$$

$$= v q_x + \sum_{k=1}^{n-1} v^{k+1} p_x \, {}_{k-1}p_{x+1} \, q_{x+k}$$

$$= v q_x + \sum_{k=0}^{n-2} v^{j+1} p_x \, {}_{j}p_{x+1} \, q_{x+1+j}$$

$$= v q_x + v p_x \sum_{k=0}^{n-2} v^{j+1} \, {}_{j}p_{x+1} \, q_{x+1+j}$$

$$= v q_x + v p_x \, A^{1}_{x+1:\overline{n-1|}}$$

b/

$$A_{x:\overline{n|}} = A^{1}_{x:\overline{n|}} + v^n \, {}_{n}p_x$$

$$= v q_x + v p_x \, A^{1}_{x+1:\overline{n-1|}} + v^n \, {}_{n}p_x$$

$$= v q_x + v p_x \left(A^{1}_{x+1:\overline{n-1|}} + v^{n-1} \, {}_{n-1}p_x \right)$$

$$= v q_x + v p_x \, A_{x+1:\overline{n-1|}}$$

c/

$$(DA)^{1}_{x:\overline{n|}} = \sum_{k=0}^{n-1} (n-k) v^{k+1} \, {}_{k}p_x \, \underbrace{q_{x+k}}_{=q_{x+1}}$$

$$= n v q_x + \sum_{k=1}^{n-1} (n-k) v^{k+1} p_x \, {}_{k-1}p_{x+1} \, q_{x+k}$$

$$= n v q_x + \sum_{j=0}^{n-2} (n-1-j) v^{j+2} p_x \, {}_{j}p_{x+1} \, q_{x+1+j}$$

$= n v q_x + v p_x \sum_{j=0}$

$= n v q_x + v p_x (DA)^{1}_{x+1:\overline{n-1}|}$

we have $Z = b_{T_x} v^{T_x} = e^{0{,}03 T_x} e^{-0{,}08 T_x}$

$= e^{-0.05 T_x} \; ; \quad T_x \leq 10.$

and zero otherwise

$E(Z) = \bar{A}^{1}_{x:\overline{n}|} = \frac{\mu}{\mu+\delta}\left(1 - e^{-(\mu+\delta)n}\right)$

$= \frac{0{,}02}{0{,}07}\left(1 - e^{-0{,}07(10)}\right)$

$= 0.14383$

$E(Z^2) = {}^2\bar{A}^{1}_{x:\overline{n}|} = \frac{\mu}{\mu+2\delta}\left(1 - e^{-(2\delta+\mu)n}\right)$

$= \frac{0{,}02}{0{,}07}\left(1 - e^{-0{,}12(10)}\right)$

$= 0{,}116468.$

$\Rightarrow$

$Var(Z) = 0.116468 - 0{,}14383^2$

$= 0{,}09578.$

3/

a/ $${}_kp_x = \frac{l_{x+k}}{l_x} = \sqrt{\frac{144-x-k}{144-x}}$$

b/ $$A^{\,1}_{50:\overline{3}|} = v q_{50} + v^2 p_{50} q_{51} + v^3 {}_2p_{50} q_{52}$$

$$= \frac{1}{1.06}\left(1-\sqrt{\frac{93}{94}}\right) + \frac{1}{1.06^2}\sqrt{\frac{93}{94}}\left(1-\sqrt{\frac{92}{93}}\right) + \frac{1}{1.06^3}\sqrt{\frac{92}{94}}\left(1-\sqrt{\frac{91}{92}}\right)$$

$$= 0,01433$$

c/ $$A_{50:\overline{3}|}^{\;\;1} = v^3 {}_3p_{50}$$

$$= \frac{1}{1.06^3}\sqrt{\frac{91}{94}} = 0,826112$$