Continuous Life Annuities

Policy	Notation for E(Y)	Formula(s) for E(Y)	Var(Y)
whole life	$\bar{a}_x$	$\bar{a}_x = \int_0^\infty v^t \, {}_tp_x \, dt$ or $\bar{a}_x = \frac{1-\bar{A}_x}{\delta}$	$\frac{{}^2\bar{A}_x - \bar{A}_x^2}{\delta^2}$
n-year temporary	$\bar{a}_{x:\overline{n}\vert}$	$\bar{a}_{x:\overline{n}\vert} = \int_0^n v^t \, {}_tp_x \, dt$ or $\bar{a}_{x:\overline{n}\vert} = \frac{1-\bar{A}_{x:\overline{n}\vert}}{\delta}$	$\frac{{}^2\bar{A}_{x:\overline{n}\vert} - (\bar{A}_{x:\overline{n}\vert})^2}{\delta^2}$
n-year deferred	${}_{n\vert}\bar{a}_x$	${}_{n\vert}\bar{a}_x = \int_n^\infty v^t \, {}_tp_x \, dt$	FP
n year certain and life	$\bar{a}_{\overline{x:\overline{n}\vert}}$	$\bar{a}_{\overline{x:\overline{n}\vert}} = \bar{a}_{\overline{n}\vert} + {}_{n\vert}\bar{a}_x$	FP
Continuously increasing whole life	$(\bar{I}\bar{a})_x$	$(\bar{I}\bar{a})_x = \int_0^\infty t \, v^t \, {}_tp_x \, dt$	FP
continuously increasing n-year temporary	$(\bar{I}\bar{a})_{x:\overline{n}\vert}$	$(\bar{I}\bar{a})_{x:\overline{n}\vert} = \int_0^n t \, v^t \, {}_tp_x \, dt$	FP.

$$Var(Y) = Var(Y_1) = \frac{2}{\delta}\left(v^n \, {}_{n\vert}\bar{a}_x - {}_{n\vert}{}^2\bar{a}_x\right) - \left({}_{n\vert}\bar{a}_x\right)^2$$

- where Y_1 is the present value random variable for an n-year deferred fully continuous whole life annuity.
- ${}^2\bar{a}_x$ be the APV of whole life annuity calculated based on a double force of interest.

Discrete life Annuities (Due)

Policy	Notation for $E(Y)$	Formula(s) for $E(Y)$	var (Y)
whole life	$\ddot{a}_x$	$\ddot{a}_x = \sum_{k=0}^{\infty} v^k \, {}_kp_x$ or $\ddot{a}_x = \frac{1-A_x}{d}$	$\frac{{}^2A_x - A_x^2}{d^2}$
n-year tempory	$\ddot{a}_{x:\overline{n}\rceil}$	$\ddot{a}_{x:\overline{n}\rceil} = \sum_{k=0}^{n-1} v^k \, {}_kp_x$ or $\ddot{a}_{x:\overline{n}\rceil} = \frac{1-A_{x:\overline{n}\rceil}}{d}$	$\frac{{}^2A_{x:\overline{n}\rceil} - A_{x:\overline{n}\rceil}^2}{d^2}$
n-year deferred	${}_{n\rvert}\ddot{a}_x$	${}_{n\rvert}\ddot{a}_x = \sum_{k=n}^{\infty} v^k \, {}_kp_x$	FP
n-year certain and life	$\ddot{a}_{\overline{x:\overline{n}\rceil}}$	$\ddot{a}_{\overline{x:\overline{n}\rceil}} = \ddot{a}_{\overline{n}\rceil} + {}_{n\rvert}\ddot{a}_x$	FP
Annually increasing whole life	$(I\ddot{a})_x$	$(I\ddot{a})_x = \sum_{k=0}^{\infty} (k+1) \, v^k \, {}_kp_x$	FP
Annually increasing n-year tempory	$(I\ddot{a})_{x:\overline{n}\rceil}$	$(I\ddot{a})_{x:\overline{n}\rceil} = \sum_{k=0}^{n-1} (k+1) \, v^k \, {}_kp_x$	FP

$$\boxed{{}_{n\rvert}a_x^{(m)} = {}_{n\rvert}\ddot{a}_x^{(m)} - \frac{{}_nE_x}{m}}$$

$$a_x = \ddot{a}_x - 1; \quad a_{x:\overline{n}\rceil} = \ddot{a}_{x:\overline{n}\rceil} - 1 + {}_nE_x$$

$${}_{n\rvert}a_x = {}_{n\rvert}\ddot{a}_x - {}_nE_x; \quad a_x^{(m)} = \ddot{a}_x^{(m)} - \frac{1}{m}; \quad a_{x:\overline{n}\rceil}^{(m)} = \ddot{a}_{x:\overline{n}\rceil}^{(m)} - \frac{1}{m} + \frac{{}_nE_x}{m}$$