Thermodynamics of Rotating Kaluza-Klein Black Holes in Gravity's Rainbow

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عمادة البحث العلمي

Introduction

Most of quantum gravity programmes predict that the spacetime admits a minimal length scale. Therefore , there is a maximal energy E_P that can be put into a system. This basic, yet important and universal prediction of quantum gravity programmes leads to phenomenological investigation of quantum gravity. The deformation of desperation relations is made by gravity's rainbow [13], where different wavelengths of light (having different energies) experience gravity differently. More generally, gravity is energy-dependent phenomena. In this research we will study the rainbow deformation of rotating Kaluza-Klien black holes. In gravity's rainbow, the geometry depends on the energy of the probe, and thus probes of of different energy see the geometry differently. Thus, a single metric is replaced by a family of energy dependent metrics forming a rainbow of metrics. Now the UV modification of the energy-momentum dispersion relation can be expressed as

$$E^2 f^2(E/E_P) - p^2 g^2(E/E_P) = m^2 \tag{1}$$

where E_P is the Planck energy, E is the energy at which the geometry is probed, and $f(E/E_P)$ and $g(E/E_P)$ are the rainbow functions. The deformation of geometry by the rainbow functions has been studied extensively, [2, 3]. Gravity's rainbow has also been used to address the black hole information paradox [5]. In this research, we study deformed rotating Kaluza-Kleinblack hole by the rainbow functions, and investigate its thermodynamic properties.

Kaluza Klein Black Holes

Kaluza-Klein black holes are a 5d uplifted solution of rotating black holes with electric Q and magnetic P charges [8, 15, 12]. This is a general solution to the dyonic solution (where $Q = P$). This solution is considered from the 4d Einstein-Maxwell-dilaton theory [14], or as a rotating D0-D6 bound state in string theory [10]. The KK solution in 5d pure Einstein gravity has the following metric:

$$ds^2_{(5)} = \frac{H_2}{H_1}(Rd\hat{y} + A)^2 - \frac{H_3}{H_2}\left(d\hat{t} + B\right)^2 + H_1\left(\frac{d\hat{r}^2}{\Xi} + d\theta^2 + \frac{\Xi}{H_3}\sin^2\theta d\phi\right) \tag{2}$$

Where: H_1, H_2 A and B are functions of four parameters p, q, j and μ. With R being the radius of the compactified fifth K-K dimension $\hat{y}$ with the condition $\hat{y} = \hat{y} + 2\pi$. There are four physical parameters that characterises the rotating K-K black hole, the mass M, electric and magnetic charges Q, P and the angular momentum J. They are given in terms of the parameters μ, q, p and j :

$$M = \frac{p+q}{4} \qquad Q = \frac{1}{2}\left(\frac{q(q^2 - 4\mu^2)}{p+q}\right)^{1/2} \tag{3}$$

$$P = \frac{1}{2}\left(\frac{p(p^2 - 4\mu^2)}{p+q}\right)^{1/2} \qquad J = \frac{\sqrt{pq}(pq + 4\mu^2)}{4(p+q)} j \tag{4}$$

The Hawking temperature is then:

$$T_0 = \frac{\mu\hbar}{\pi\sqrt{pq}\left(\frac{2\mu}{\sqrt{1-j^2}} + \frac{4\mu^2+pq}{p+q}\right)} \tag{5}$$

Using the relation $dS = dM/T$ we can obtain the entropy:

$$S_0 = \frac{2\pi\frac{p+q)}{4}\left(\frac{3(p+q)}{4\sqrt{1-j^2}} + 12\mu + \frac{pq}{\mu}\right)}{3\hbar} \tag{6}$$

K-K black holes in gravity's rainbow

The rotating K-K black hole is deformed by the rainbow functions discussed earlier where E is the energy of a ' quantum' particle near the outer horizon $\hat{r} \sim r_+$. In order to estimate E, we may use the uncertainty relation for position and momentum , and write $\Delta p \geq 1/\Delta x$. Thus, we can obtain a bound on energy of a black hole, $E \geq 1/\Delta x$ [4]. It should be noted that this uncertainty relation holds for the rotating K-K black hole like any other 4-D black hole, in gravity's rainbow [1] we write, $E \geq 1/\Delta x \approx 1/r_+$. One may define the rainbow functions $f(E)$ and $g(E)$ in many ways, However, in this study these functions are chosen such that they are compatible with loop quantum gravity and non-commutative geometry [7, 11].

$$f(E) := 1 \qquad g(E) := \sqrt{1 - \eta(E/E_p)^\nu}, \tag{7}$$

Here, η and ν are free parameters. Now, we use (5), and (7) to obtain the formula for the modified temperature :

$$T = \frac{\mu\hbar\sqrt{1 - \eta(1/r_+ E_p)^\nu}}{\pi\sqrt{pq}\left(\frac{2\mu}{\sqrt{1-j^2}} + \frac{4\mu^2+pq}{p+q}\right)} \tag{8}$$

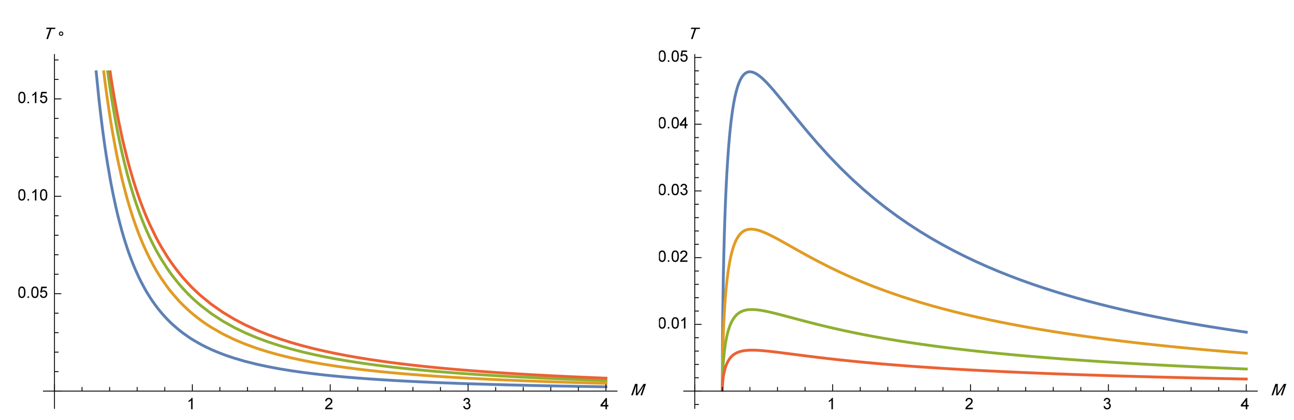

Figure 1: Ordinary (left) and deformed (right) Hawking temperatures of different rotating K-K black holes(fixed Q, P and J) as a function of their mass M. We set $E_p = 5$, $\eta = 1$ and $\nu = 2$. The remnant can be observed at the same point for all deformed black holes.

Similarly, the deformed entropy is calculated from the integral $S = \int \frac{dM}{T}$, it is found to be given by the Hypergeometric functions ${}_2F_1(a, b; c; d)$,

$$S(M) = \frac{2\pi}{\mu\hbar}\left(\mu M\left(\frac{M\,{}_2F_1\left(\frac{1}{2}, -\frac{2}{\nu}; \frac{\nu-2}{\nu}; \left(\frac{1}{ME_p}\right)^\nu \eta\right)}{\sqrt{1-j^2}} + \mu\,{}_2F_1\left(\frac{1}{2}, -\frac{1}{\nu}; \frac{\nu-1}{\nu}; \left(\frac{1}{ME_p}\right)^\nu \eta\right)\right) + \frac{1}{3}M^3\,{}_2F_1\left(\frac{1}{2}, -\frac{3}{\nu}; \frac{\nu-3}{\nu}; \left(\frac{1}{ME_p}\right)^\nu \eta\right)\right) \tag{9}$$

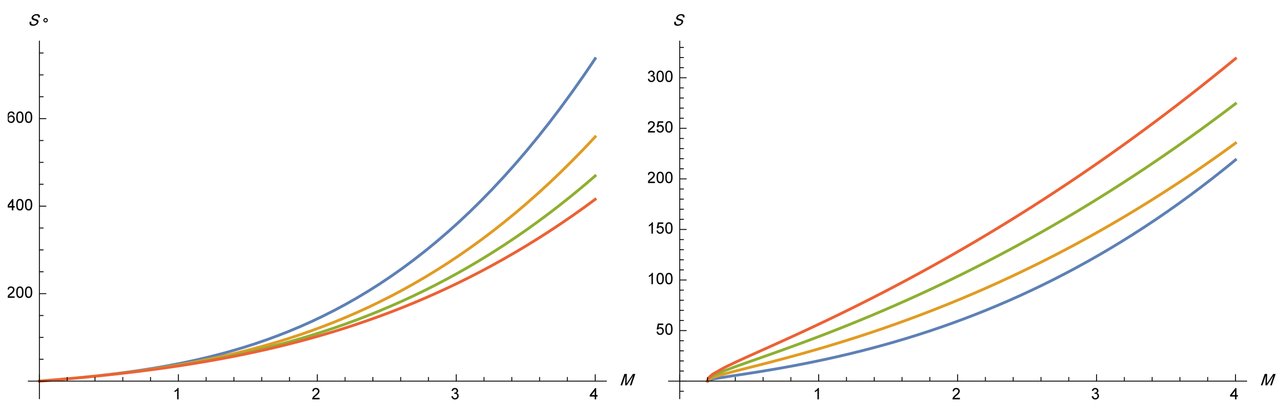

Figure 2: The deformed entropy of different rotating K-K black holes(fixed Q, P and J) as a function of their mass M.We set $E_p = 5$, $\eta = 1$ and $\nu = 2$. The remnant can be observed at the same point for all deformed black holes

It is interesting to look at the criticality of rotating K-K black holes and their rainbow deformation, this can be done by studying the Gibbs free energy of this black hole. The Gibbs free energy is generally given by $G(M, J, Q, P) = M - TS$ For the ordinary rotating K-K black hole it is found to be

$$G_0(M) = \frac{M^2\left(2\sqrt{1-j^2}M + 3\mu\right)}{3\left(\sqrt{1-j^2}\mu^2 + \sqrt{1-j^2}M^2 + 2\mu M\right)}. \tag{10}$$

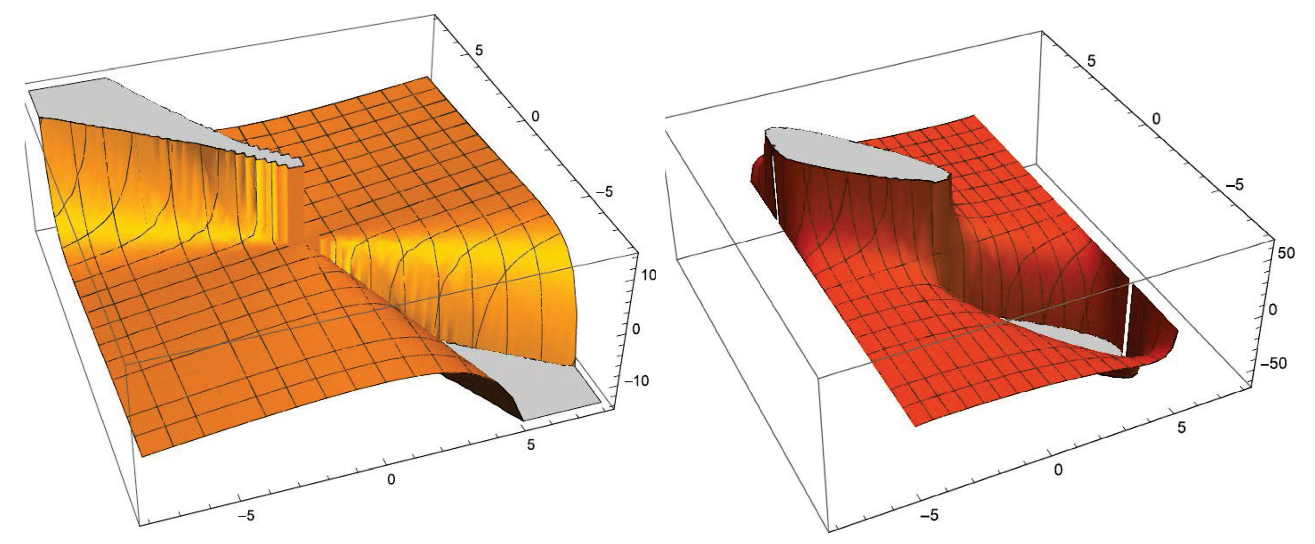

Figure 3: A plot of the ordinary(left) and deformed (right) Gibbs free energy $G(T, J, , Q, P)$ rotating K-K black hole with fixed Q, P. Showing similar critical phenomena. We have set $\eta = 1$, $E_p = 5$ and $\nu = 2$ for the deformed one.

The deformed Gibbs free energy is calculated:

$$G = M - \frac{\sqrt{1 - \eta\left(\frac{M}{E_p}\right)^\nu}}{\frac{2\mu M}{\sqrt{1-j^2}} + \mu^2 + M^2}\left(\frac{\mu M^2\,{}_2F_1\left(\frac{1}{2}, \frac{2}{\nu}; \frac{\nu+2}{\nu}; \left(\frac{M}{E_p}\right)^\nu \eta\right)}{\sqrt{1-j^2}} + \frac{1}{3}M^3\,{}_2F_1\left(\frac{1}{2}, \frac{3}{\nu}; \frac{\nu+3}{\nu}; \left(\frac{M}{E_p}\right)^\nu \eta\right) + \mu^2 M\,{}_2F_1\left(\frac{1}{2}, \frac{1}{\nu}; 1 + \frac{1}{\nu}; \left(\frac{M}{E_p}\right)^\nu \eta\right)\right) \tag{11}$$

Both ordinary and deformed rotating K-K black holes show critical behaviour as the study of Gibbs free energy, if $G > 0$ the black hole is said to be ' critical' and when $G < 0$ it is said that the black hole is uncritical.

Conclusions

In this research, the geometry of 5-D rotating Kaluza Klein black holes with electric and magnetic charges was deformed by the rainbow functions f, g motivated by loop quantum gravity and non-commutative geometry. Resulting a deformation on the thermodynamics of the 4D rotating K-K black hole. The deformed temperature and entropy indicate the existence of a remnant after the decay of the black hole to a ' Planckkian' scale. Moreover, the critical behaviour of this black hole was studied via calculating its Gibbs free energy, the ordinary and the deformed black holes appear to show the same critical behaviour.

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