ROCK FAILURE CRITERIA A KEY FOR PREDICTING SAND-FREE PRODUCTION RATES

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ABSTRACT

Rock strength determined by core testing is combined with Darcy equation for the radial flow and Kirsch solution of linear poro-elastic material and used to predict sand-free production rates for various states of rock strength. From this study it is seen that high production rate decreases the bottom hole flowing pressure causing the effective stresses on the rock at the borehole wall to increase leading to failure. Therefore a zone of failed rock exists around the borehole and destabilizes due to its remaining residual strength. If the high production rate continues, the rock loses its remaining residual strength and crushes under the effective load and detaches into the borehole leading to sand production. Sand-free Production rates in horizontal boreholes are found to be two folds lower than that of vertical boreholes.

1. INTRODUCTION

Sand Production from weak, but competent rock,as a result of high production rates is a growing concern. In unconsolidated sand, the decision of gravel pack is usually clear, however, the decision is harder in weak rock because the need for sand control often depends on the desired drawdown or production rate. Also, wells that do not initially require sand control may later become sand producers. The ability to predict at what point sand problems will occur is useful[1]. Sand Production is the production of small or large amounts of solids together with the reservoir fluid.

The amount can vary from few grams or less per ton of reservoir fluid to catastrophic amounts possibly leading to complete filling of the borehole (sand up)[2]. Sand production is essentially an economical problem. When the sand production risk is under-evaluated, the problems which have to be confronted are numerous: Safety, downhole and surface installation erosion, well cleaning which may require major workovers and impose production delays, etc. On the other hand, when the sand production risk is over-evaluated, the problem becomes the unnecessary reduction of the productivity of the wells which may require the drilling of further wells to achieve the targeted production for a given reservoir [3].

2.MECHANISMS CAUSING SAND PRODUCTION

Sand can be produced by excessive drawdown, which causes local failure around the borehole, or by depletion, which causes shear failure of the entire reservoir. Higher production rates require an enhancement of the reservoir permeability. Sand production is believed to enhance the reservoir permeability by increasing the effective area of inflow near the wellbore. There are at least three possible scenarios for enhanced permeability zone caused by sand production[4]:

i- The establishment of new flow channels.
ii- The extensive localized shear bands.
iii- Formation dilatancy over a large volume.

Whenever reservoir pressure (pore pressure) declines, both net vertical and horizontal stresses increase in such a way that the formation shear stresses increase. If the shear stress increases to the point that the formation generally fails in shear, then weakly cemented rock may become desegregated, leading to a low value of unit cohesive strength. The allowable drawdown for sand-free production with hole stability maintained then be very low. Therefore, the stress state of the reservoir must be checked to determine whether the stresses will be below the failure envelope (limit) throughout the life of the field[1]. Many predicting techniques of the critical conditions for sand production are currently available. They can be grouped into three main categories according to their complexity and to the way their equations are solved.

3. FORMULATION OF THE MODEL

When high production rates are maintained, the effective stresses in the rock around the wellbore will increase. If these effective stresses exceed the rock

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failure criteria, the rock will fail and sand will be produced. Laboratory experiments can be performed on cores to obtain the failure envelope at any desired depth. The failure envelope can be then used to estimate the maximum production rate with hole stability maintained. The direction and the magnitude of the in-situ stresses, pore pressure and the direction (inclination and azimuth) of the well, all affect the wellbore rock principal stresses. The near wellbore rock fluid is a function of production rate, bottomhole flowing pressure, and rock permeability. If the fluid pressure in the near wellbore rock is too low the rock will fail[5]. To avoid this, the maximum production rate without collapse (rock failure) must be estimated. Therefore, this paper presents a method for predicting sand production in vertical and inclined oil wells (when no sand control equipment is installed). The method developed here is based on Mohr-Coulomb failure criterion, Kirsch solution for linear poro-elastic materials, Darcy equation for the radial flow and rock strength data derived from laboratory core testing. By applying this method sand-free production rates can be predicted as well as the extent of the sand producing rock around the well (yielded rock radius).

3.1 Rock Failure Criteria

Two obvious mechanisms causing borehole failure are shear and tensile failure. One of the most used failure theories is the Mohr-Coulomb failure criterion. This criterion is defined as follows:

$$\tau_f = \tau_o + \sigma \tan \phi \qquad (1)$$

When a rock is loaded beyond its elastic limit it will fail. If the failed rock has a residual strength and supported by a confining pressure it will remain in place and a zone of yielded rock will be formed around the borehole as shown in **Fig.1A**. If there is no support for the yielded rock it will drop into the wellbore and part of it will be produced with the reservoir fluids and the rest will remain in the bottom of the well, requiring a cleaning process to be done. **Fig.1B** shows the distribution of the rocks around the wellbore. **Fig.2** shows the Mohr-Coulomb failure criteria for a hypothetical sandstone used to characterize the proposed methods.

3.2 Distribution Of Stresses Around The Wellbore

The ease with which the borehole stability can be computed is highly dependent on the stress-strain behavior commonly chosen for modelling the formation response to loading. The most common behavior assumed is that the formation is homogenous, isotropic, and linear poro-elastic (HILE). These assumptions

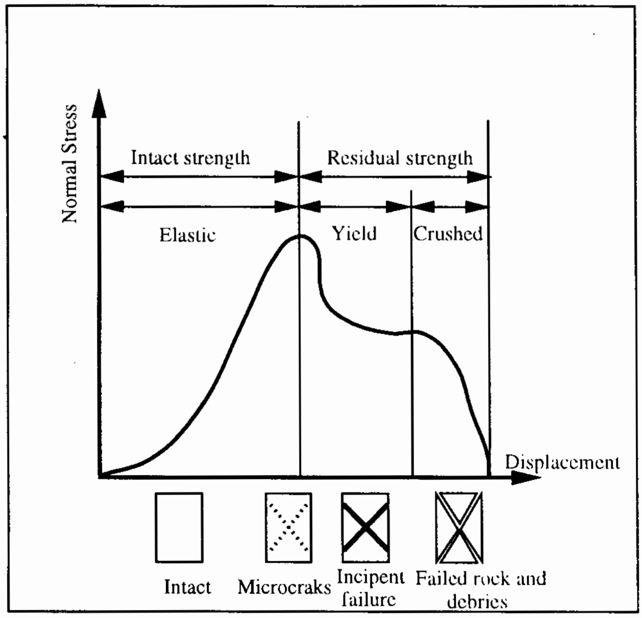


Fig. 1A- Typical load-displacement curve for brittle-ductile rocks.

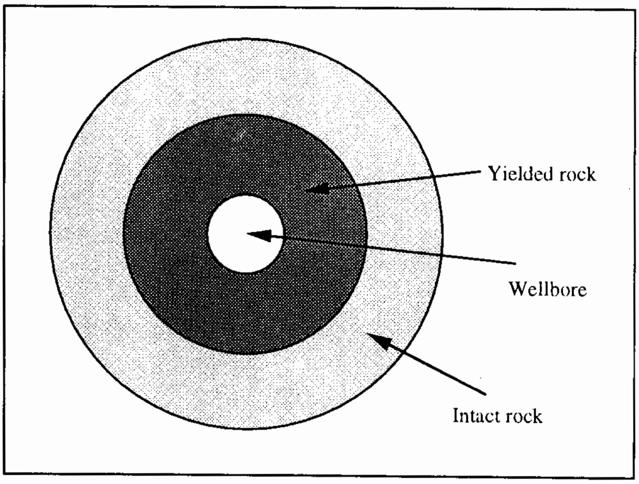
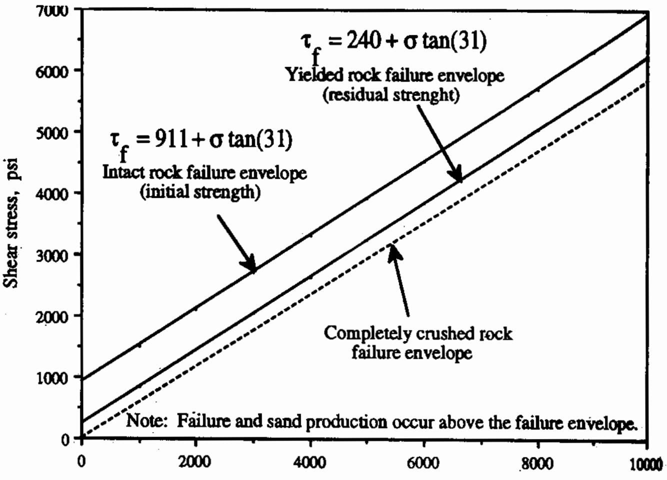


Fig. 1B- Distribution of rocks around a borehole in yielded formation.



Note: Failure and sand production occur above the failure envelope.

Fig. 2. Peak and residual strength determined by the mohr-coulomb failure criteria for hypothetical sandstone

allow the stresses to be determined from a set of fairly simple equations. More complex models suffer from an extensive list of input parameters, many of which cannot be realistically determined. The equation required to compute the stresses around vertical, inclined, and horizontal wells is called Kirsch solution [6-7]. The in-situ principal stresses can be transformed parallel to the wellbore axis by the application of the following matrices:

$$\begin{bmatrix}\sigma_x \\ \sigma_y \\ \sigma_{zz}\end{bmatrix} = \begin{bmatrix}\mathrm{Cos}^2\beta\,\mathrm{Cos}^2\alpha & \mathrm{Sin}^2\beta\,\mathrm{Cos}^2\alpha & \mathrm{Sin}^2\alpha \\ \mathrm{Sin}^2\beta & \mathrm{Cos}^2\beta & 0 \\ \mathrm{Cos}^2\beta\,\mathrm{Sin}^2\alpha & \mathrm{Sin}^2\beta\,\mathrm{Sin}^2\alpha & \mathrm{Cos}^2\alpha\end{bmatrix}\begin{bmatrix}\sigma_H \\ \sigma_h \\ \sigma_v\end{bmatrix} \tag{2}$$

$$\begin{bmatrix}\tau_{yz} \\ \tau_{xz} \\ \tau_{xy}\end{bmatrix} = \frac{1}{2}\begin{bmatrix}\mathrm{Sin}2\beta\,\mathrm{Sin}\alpha & -\mathrm{Sin}2\beta\,\mathrm{Sin}\alpha & 0 \\ \mathrm{Sin}2\alpha\,\mathrm{Cos}\beta & \mathrm{Sin}^2\beta\,\mathrm{Sin}2\alpha & -\mathrm{Sin}2\alpha \\ \mathrm{Cos}^2\beta\,\mathrm{Sin}^2\alpha & -\mathrm{Sin}2\beta\,\mathrm{Cos}\alpha & 0\end{bmatrix}\begin{bmatrix}\sigma_H \\ \sigma_h \\ \sigma_v\end{bmatrix} \tag{3}$$

The stresses acting on the wall of a borehole then can be calculated as follows:

$$\begin{aligned}
&\sigma_r = P_{wc} \\
&\sigma_\theta = \left(\sigma_x + \sigma_y - P_{wc}\right) - 2\left(\sigma_x - \sigma_y\right)\mathrm{Cos}2\theta - 4\tau_{xy}\mathrm{Sin}2\theta \\
&\sigma_z = \sigma_{zz} - 2\nu\left(\sigma_x - \sigma_y\right)\mathrm{Cos}2\theta - 4\nu\tau_{xy}\mathrm{Sin}2\theta \\
&\tau_{r\theta} = \tau_{rz} = 0 \\
&\tau_{\theta z} = 2\left[-\tau_{zx}\,\mathrm{Sin}\theta + \tau_{yz}\,\mathrm{Cos}\theta\right]
\end{aligned} \tag{4}$$

The principal stresses to be used in the Mohr-Coulomb failure criterion is as follows:

$$\begin{aligned}
&\sigma_1 = \sigma_r = P_{wc} \\
&\sigma_2 = \frac{1}{2}\left(\sigma_\theta + \sigma_z\right) - \frac{1}{2}\sqrt{\left(\sigma_\theta - \sigma_z\right)^2 + 4\tau_{\theta z}} \\
&\sigma_3 = \frac{1}{2}\left(\sigma_\theta + \sigma_z\right) + \frac{1}{2}\sqrt{\left(\sigma_\theta - \sigma_z\right)^2 + 4\tau_{\theta z}}
\end{aligned} \tag{5}$$

The effective principal stress acting on the borehole wall can be computed by applying the following relationship:

$$\sigma_{eff} = \sigma - P_p \tag{6}$$

Knowing the stress distribution around the borehole, the normal stress acting at each point and the corresponding maximum shear stress can be calculated as follows:

$$\tau_f = \tau_o + \left[\frac{\sigma_1 + \sigma_3}{2}\right]\tan\phi \tag{7.a}$$

$$\tau_{Max} = \left[\frac{\sigma_1 - \sigma_3}{2}\right] \tag{7.b}$$

Therefore, one can predict whether or not the rock will fail by comparing the maximum shear stress with the limit value given by the Mohr-Coulomb failure envelope (see Fig.1).

4. CALCULATION PROCEDURE

In porous rock, pore pressure is the failure controlling parameter. The formation fluid carries part of the load applied to the system, therefore relieving the rock matrix from part of the load. Darcy's law can be used to estimate the maximum allowable production rate without borehole collapse. Assuming the formation fluids is flowing in the radial direction through a horizontal porous media the maximum allowable production rate can be calculated as follows:

$$q_c = \frac{7.082\,k\,h\left(P_e - P_{wc}\right)}{\mu\ln\left(\frac{r_e}{r_w}\right)} \tag{8}$$

If the reservoir produces with a higher than the critical flow rate (q_c) the rock surrounding the well will fail and the borehole may collapse and a new well diameter will exist. The new wellbore radius due to rock failure and detachment can be calculated as follows:

$$\left(r_w\right)_{new} = \left[\frac{\left(r_w\right)_{initial}}{\mathrm{Exp}\left(\frac{7.082\,k\,h\left(P_e - P_{wc}\right)}{\mu\,q_c}\right)}\right] \tag{9}$$

The volume of the produced sand then can be calculated as follows:

$$V_{sand} = \pi\,h\,\omega\left(r_{new}^2 - r_{initial}^2\right) \tag{10}$$

5. RESULTS AND DISCUSSION

Sand production analysis was conducted based on the hypothetical in-situ data shown in Table 1. The critical wellbore pressure (P_{wc}) changes for both intact and yielded rock states for different well inclinations are presented in Fig.3 and the corresponding maximum production rate without collapse is shown in Fig.4. It is seen that as the angle of inclination increases the critical wellbore pressure at which the wellbore collapse increases for both rock and stress states. Also, these figures show that once the rock around the borehole is loaded beyond its elastic limit, the production rate must be maintained very low to avoid exceeding the failure limit of the yielded rock and causing the hole to collapse leading to sand up case. The maximum production rate at which the borehole is stable decreases with the increase of the inclination angle. It is noted that for this specific case the maximum production rate is two times higher for a vertical wellbore as compared to horizontal wellbore. Fig.5 shows the effect of production rate on borehole stability. It is clear that as the production rate is increased above the critical allowable production rate the wellbore will fail and sand will be produced. The new wellbore radius (or the radius of the yielded rock) and sand volume as a function of production rate exceeding the wellbore stability failure criteria are shown in Fig.6. Therefore the dependency between the depth of the failed zone, sand production volume, and production rate is clear and directly depends on rock strength and hole inclination (stress state)[8]. Finally it should be noted that for anisotropic stress state boreholes drilled parallel to the minimum horizontal principal insitu stress are more stable than those drilled parallel to the maximum horizontal principal insitu stress.

Table 1: Hypothetical parameters Required to study sand production problem.

PARAMETERS	DATA	SOURCE
In-Situ Data		
σ_v	1.0 psi/ft	Field
σ_h	0.75 psi/ft	Field
σ_H	0.85 psi/ft	Field
α	zero to 90 degree	Field
β	zero to 90 degree	Field
θ	zero to 90 degree	Variable
ν	0.21	Laboratory (assumed)
Pp	0.44 psi/ft	Field
Rock Strength Data		
Angle of Internal Friction Intact Residual	31 degree 31 degree	Laboratory (assumed)
Apparent Cohesion Intact Residual	911 psi 240 psi	Laboratory (assumed)
Reservoir and Well Data		
k	75 mD	Laboratory (assumed)
q	variable	Field
rw	4.5 in. (0.375 ft)	Field
re	450	Field
Pe	6200 psi	Field
μ_o	API Gravity = 35° Reservoir Temp. = 190 °F $\mu_o = 1.6$ cp	Laboratory (assumed)
h	40 ft	Filed
P_{wf}	variable	Field
Total Vertical Depth	9000 ft	Field

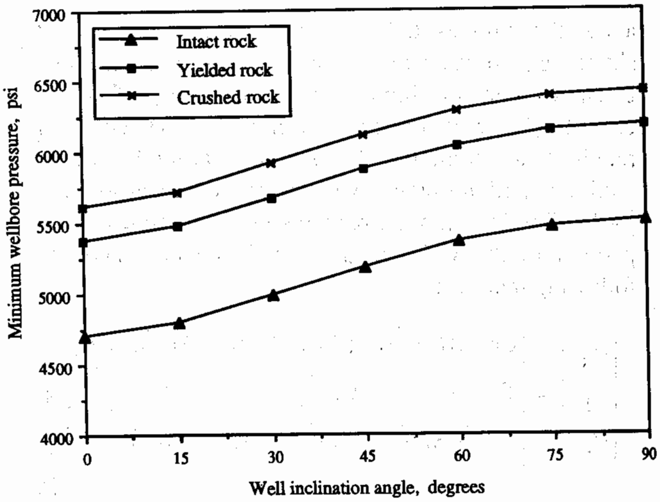


Fig. 3 Minimum wellbore pressure for stable (no sand production) borehole.

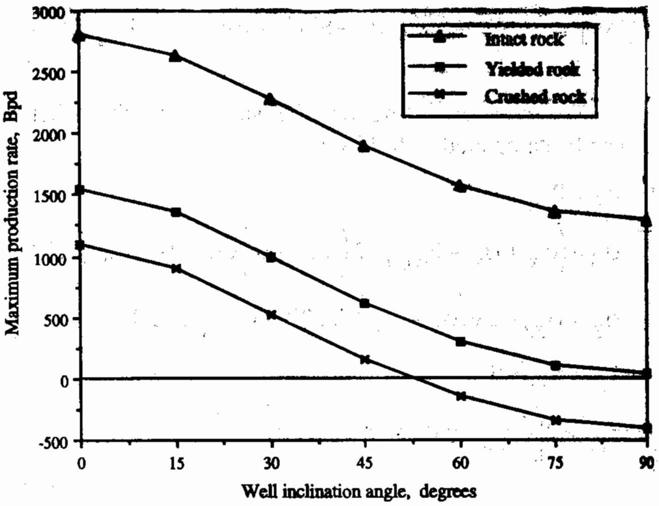


Fig. 4 Minimum production rate foe a stable borehole (i.e. no sand production).

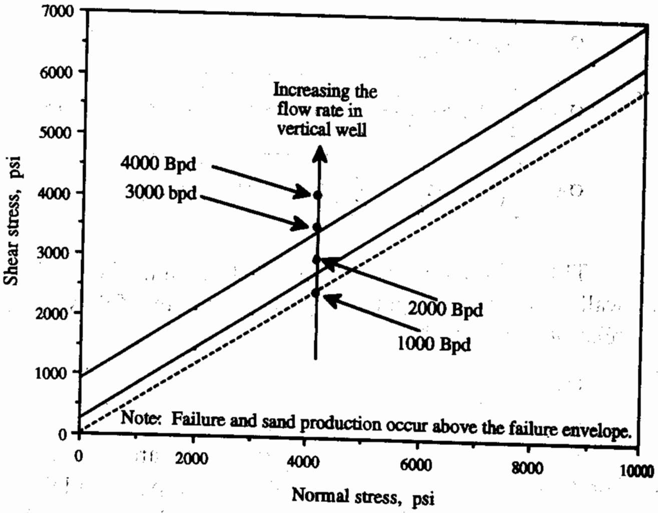


Fig. 5- The effect of flow rate on borehole stability (i.e. sand production).

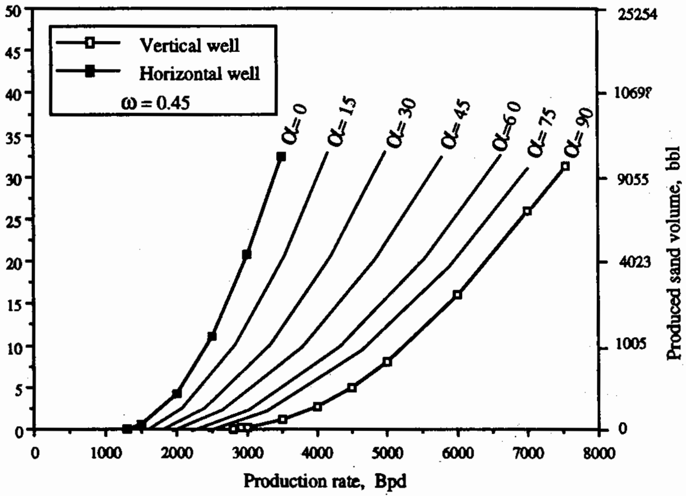


Fig. 6- Predicted yield-zone radius and sand volume as a function of production rate.

6. CONCLUSION

Based on the analysis performed in this study the following conclusions are drawn:

i- A method for predicting sand-free production rates has been developed based on Darcy law in its radial form, Kirsch solution for linear poro-elastic materials and rock failure criterion.

ii- When the production rate is too high, rock around the borehole will fail. The flow rate must be decreased to destabilize the failed rock and avoid its detachment into the borehole.

iii- Sand-free production rate is a function of rock strength, in-situ stress state, pore pressure and borehole inclination and orientation.

iv- Sand-free production rates in horizontal boreholes are two folds lower than that of vertical boreholes.

v- Sand production is believed to be a kind of self stimulation caused by the high drawdown, i.e. sand production will enhance the permeability of the inflow area near the wellbore.

vi- The amount of sand produced from a failed rock depends on rock strength. Strong rock may not be produced even after failure due to the remaining residual strength. If excessive production rate is maintained the residual strength will be destroyed and the rock will be crushed leading to the production of huge amount of sand or may lead to complete destruction of the borehole (sand-up).

7. NOMENCLATURE

h = Reservoir thickness.

k = Reservoir rock permeability.

P_{wc} = Critical wellbore pressure.

P_e = Reservoir pressure.

q_c = Critical production rate.

r_w = Wellbroe radius.

r_e = Reservoir radius.

V_{sand} = Produced sand volume.

ϕ = Rock angle of internal friction.

α = Wellbore inclination.

β = Wellbore orientation.

θ = Angular position around the borehole.

ω =Factor to be determined experimentally (between 0 and 1) and depends on rock type.

μ = Reservoir fluid viscosity.

σ = Normal stress at failure.

σ_{eff} = Effective Stress.

$\sigma_x, \sigma_y, \sigma_{zz}$ = Transformed in-situ stress in cartesian.

$\sigma_H, \sigma_h, \sigma_v$ = In-situ principal stresses.

$\sigma_r, \sigma_\theta, \sigma_z$ = Induced stresses in polar form.

$\sigma_1, \sigma_2, \sigma_3$ = Principal stresses acting on borehole wall.

τ_f = Shear stress at failure.

τ_{Max} = Calculated shear.

τ_o = Apparent cohesion of the reservoir rock.

$\tau_{xy}, \tau_{xz}, \tau_{yz}$ = Induced shear stresses acting on borehole.

$\tau_{r\theta}, \tau_{rz}, \tau_{\theta z}$ = Induced stresses acting on borehole wall.

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