Inconsistencies in Health Care Knowledge

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Outline

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Paraconsistent Logic

What is a Paraconsistent Logic?

A paraconsistent logic is a logic where the Principle of Non-Contradiction does not hold - this Principle claims that two contradictory propositions can not be both true at the same time.

Motivations

The primary motivation for paraconsistent logic is the conviction that it ought to be possible to reason with inconsistent information in a controlled and discriminating way.

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- various scientific theories
- constitutions and other legal documents
- descriptions of fictional (and other non-existent) objects
- descriptions of counterfactual situations
- health care diagnosis

Schools of Paraconsistent Logic

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- Dialetheism

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- Adaptive logics
- Logics of Formal Inconsistency

Applications

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- Linguistics

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- Law and Science

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- Automated Reasoning

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- Paraconsistent Artificial Neural Networks – PANNets

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Hybrid Logics

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with the ability to refer to worlds by considering a new class of atomic formulas, called nominals, and using a new operator, @, called satisfaction operator.

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- what happens at a specific world:
  - $@_i p$
  - $\neg @_i p$ (logically equivalent to $@_i \neg p$)

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- equality between worlds:
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- accessibility between worlds:
  - $@_i \Diamond j$
  - $\neg @_i \Diamond j$ (logically equivalent to $@_i \Box \neg j$)

Multimodal Hybrid Logic

Definition

$L = \langle \mathrm{Prop}, \mathrm{Nom}, \mathrm{Mod} \rangle$ – *hybrid similarity type.*

$\mathrm{Form}_{@}(L)$ – *set of well-formed formulas over L:*

$$WFF := i \mid p \mid \bot \mid \top \mid \neg\varphi \mid \varphi \vee \psi \mid \varphi \wedge \psi \mid \langle \pi \rangle \varphi \mid [\pi]\varphi \mid @_i \varphi$$

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Definition

Let $L = \langle \mathrm{Prop}, \mathrm{Nom} \rangle$ *be a hybrid similarity type.*

A hybrid structure $\mathcal{H}$ *over L is a tuple* $(W, (R_\pi)_{\pi \in \mathrm{Mod}}, N, V)$, *where:*

$W \neq \emptyset$ – *domain; whose elements are called states or worlds,*

$R_\pi \subseteq W \times W$ – *accessibility relation,*

$N : \mathrm{Nom} \to W$ – *hybrid nomination,*

$V : \mathrm{Prop} \to Pow(W)$ – *hybrid valuation.*

Definition

The local satisfaction relation $\models$ *between a hybrid structure* $\mathcal{H} = (W, (R_\pi)_{\pi \in \mathrm{Mod}}, N, V)$, *a state* $w \in W$ *and a hybrid formula:*

1. $\mathcal{H}, w \models i$ *iff* $w = N(i)$;
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For $\Delta \subseteq \mathrm{Form}_@(L)$, it is said that $\mathcal{H}$ is a model of Δ iff for all $\theta \in \Delta$, $\mathcal{H} \models \theta$.

Hybrid Diagrams

Definition

For a hybrid similarity type $L = \langle \mathrm{Prop}, \mathrm{Nom}, \mathrm{Mod} \rangle$, *we define*

1. *Hybrid atoms over* L:
   $\mathrm{HAt}(L) = \{@_i p,\ @_i j,\ @_i \langle \pi \rangle j \mid i, j \in \mathrm{Nom}, p \in \mathrm{Prop}, \pi \in \mathrm{Mod}\}$;
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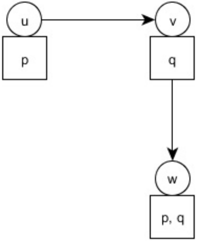
L is expanded by adding new nominals for the elements of the domain W: $L(W) = \langle \mathrm{Prop}, \mathrm{Nom} \cup W, \mathrm{Mod} \rangle$.

The diagram of a hybrid structure $\mathcal{H}$ over L is the set of literals over $L(W)$ that are valid in $\mathcal{H}(W)$.

An Example



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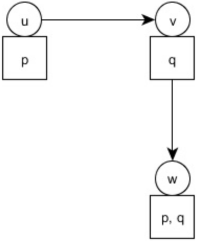

$$diag(\mathcal{H}) = \{@_u p, @_u \neg q, @_v \neg p, @_v q, @_w p, @_w q$$

$$@_u \neg v, @_u \neg w, @_v \neg u, @_v \neg w, @_w \neg u, @_w \neg v$$

$$@_u \Diamond v, @_u \Box \neg u, @_u \Box \neg w, @_v \Diamond w, @_v \Box \neg u$$

$$@_v \Box \neg w, @_w \Box \neg u, @_w \Box \neg v, @_w \Box \neg w\}$$

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The study of Paraconsistency in Hybrid Logic follows the approach of Grant and Hunter in *Measuring inconsistency in knowledgebases, (2006)*.

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The negation normal form of a formula, for short NNF, is defined just as in propositional logic: a formula is said to be in NNF if negation only appears directly before propositional variables and/or nominals.

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Proposition

Every hybrid formula is logically equivalent to one that is in NNF.

In order to accommodate inconsistencies in a model, we consider two valuations for propositions: V^+ and V^-.

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Definition

A hybrid bistructure is a tuple $(W, (R_\pi)_{\pi \in \mathrm{Mod}}, N, V^+, V^-)$ *where* $(W, (R_\pi)_{\pi \in \mathrm{Mod}}, N, V^+)$ *and* $(W, (R_\pi)_{\pi \in \mathrm{Mod}}, N, V^-)$ *are hybrid structures.*

Definition

Let $E = (W, (R_\pi)_{\pi \in \mathrm{Mod}}, N, V^+, V^-)$ *and* $w \in W$

1. $E, w \models_d p$ *iff* $w \in V^+(p)$;
2. $E, w \models_d i$ *iff* $w = N(i)$;
3. $E, w \models_d \neg p$ *iff* $w \in V^-(p)$;
4. $E, w \models_d \neg i$ *iff* $w \neq N(i)$;

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The $\sim$ operator:

Definition

Let θ *be a formula in* NNF *and let* $\sim$ *be a complementation operation such that* $\sim \theta = nnf(\neg\theta)$.

Definition

Strong satisfaction $\models_s$:

1. $E, w \models_s \top$ *always;*
2. $E, w \models_s \bot$ *never;*
3. $E, w \models_s \alpha$ *iff* $E, w \models_d \alpha$, $\alpha \in \mathrm{Prop} \cup \mathrm{Nom}$;
4. $E, w \models_s \theta_1 \vee \theta_2$ *iff* $[E, w \models_s \theta_1$ *or* $E, w \models_s \theta_2]$ *and* $[E, w \models_s \sim \theta_1 \Rightarrow E, w \models_s \theta_2]$ *and* $[E, w \models_s \sim \theta_2 \Rightarrow E, w \models_s \theta_1]$;
5. $E, w \models_s \theta_1 \wedge \theta_2$ *iff* $E, w \models_s \theta_1$ *and* $E, w \models_s \theta_2$;
6. $E, w \models_s \langle \pi \rangle \theta$ *iff* $\exists w' (w R_\pi w' \;\&\; E, w' \models_s \theta)$;
7. $E, w \models_s [\pi] \theta$ *iff* $\forall w' (w R_\pi w' \Rightarrow E, w' \models_s \theta)$;
8. $E, w \models_s @_i \theta$ *iff* $E, w' \models_s \theta$ *where* $w' = N(i)$;

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5. $E, w \models_s \theta_1 \wedge \theta_2$ *iff* $E, w \models_s \theta_1$ *and* $E, w \models_s \theta_2$;
6. $E, w \models_s \langle\pi\rangle\theta$ *iff* $\exists w'(wR_\pi w'$ & $E, w' \models_s \theta)$;
7. $E, w \models_s [\pi]\theta$ *iff* $\forall w'(wR_\pi w' \Rightarrow E, w' \models_s \theta)$;
8. $E, w \models_s @_i\theta$ *iff* $E, w' \models_s \theta$ *where* $w' = N(i)$;

Strong validity:

$$E \models_s \theta \text{ iff for all } w \in W, E, w \models_s \theta.$$

An Example

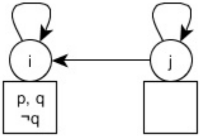

Figure : A quasi-hybrid model $\mathcal{H}$.

Valid formulas in $\mathcal{H}$:

$@_i(p \wedge \neg q)$

$@_i \Box p$

$@_j \Diamond p$

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For a hybrid similarity type $L = \langle \mathrm{Prop}, \mathrm{Nom} \rangle$,

1. Quasi-hybrid atoms over L:
   $\mathrm{QHAt}(L) = \{@_i p,\ @_i\langle\pi\rangle j \mid i, j \in \mathrm{Nom},\ p \in \mathrm{Prop}, \pi \in \mathrm{Mod}\}$;
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In order to build the paraconsistent diagram, new nominals are added for the elements of W which are not named yet, and this expanded similarity type is denoted by $L(W)$, *i.e.*, $L(W) = \langle \mathrm{Prop}, W, \mathrm{Mod} \rangle$.

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Let $L = \langle \mathrm{Prop}, \mathrm{Nom}, \mathrm{Mod} \rangle$ *be a hybrid similarity type, and* $E = (W, (R_\pi)_{\pi \in \mathrm{Mod}}, N, V^+, V^-)$*. The elementary paraconsistent diagram of* E *is*

$$Pdiag(E) = \{\alpha \in \mathrm{QHLit}(L(W)) \mid E(W) \models_s \alpha\}$$

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The paraconsistent diagram of a bistructure is unique.
Therefore, a bistructure $E = (W, (R_\pi)_{\pi \in \mathrm{Mod}}, N, V^+, V^-)$ will be represented by its (finite) paraconsistent diagram $Pdiag(E)$.

Definition

Let L be a hybrid similarity type, $\Delta \subseteq \mathrm{Form}_{\mathrm{NNF}(@)}(\mathsf{L})$ *and W a non-empty set. The set of minimal* QH *models of* Δ *with domain W is:*

$$\begin{aligned}\mathrm{MQH}(L, \Delta, W) \;=\; & \{\mathcal{M} \in \mathrm{QH}(L, \Delta, W) \mid \textit{if } \mathcal{M}' \subset \mathcal{M} \\ & \textit{then } \mathcal{M}' \notin \mathrm{QH}(L, \Delta, W)\}\end{aligned}$$

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The minimal QH models are just models with no irrelevant and useless information.

Example

Let $L = \langle \{p, q\}, \{i\}, \{\pi\} \rangle$, $W = \{i\}$, *and* $\Delta = \{@_i(p \wedge q), @_i \neg p\}$.

There are exactly two minimal QH *models of* Δ*:*

$\mathcal{M}_1 = \{@_i \neg p, @_i q, @_i p, @_i [\pi] \neg i\}$;
$\mathcal{M}_2 = \{@_i \neg p, @_i q, @_i p, @_i \langle \pi \rangle i\}$.

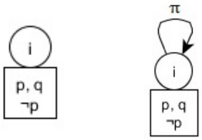

Figure : Minimal models $\mathcal{M}_1$ and $\mathcal{M}_2$.

The set of inconsistency literals over L and W

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Definition

For a QH *model* $\mathcal{M}$,

$$Conflictbase(\mathcal{M}) = \{@_i p \in \mathrm{IL}(L, W) \mid @_i p \in \mathcal{M} \;\&\; @_i \neg p \in \mathcal{M}\}$$

Definition

The measure of inconsistency for a model $\mathcal{M}$ *in the context of a hybrid similarity type* L *and domain* W *is given by the ModelInc function giving a value between 0 and 1 as follows:*

$$ModelInc(\mathcal{M}, L, W) = \frac{|Conflictbase(\mathcal{M})|}{|\mathrm{IL}(L, W)|}$$

Example

From the previous example,

Conflictbase$(\mathcal{M}_1) = \{@_i p\}$. *Then,*

$$ModelInc(\mathcal{M}_1, L, W) = \frac{|Conflictbase(\mathcal{M}_1)|}{|\mathrm{IL}(L, W)|} = \frac{1}{2}$$

The same happens for $\mathcal{M}_2$.

Definition

Let L be a hybrid similarity type, $\Delta \subseteq \mathrm{Form}_{\mathrm{NNF}(@)}(\mathsf{L})$ and W a non-empty set. The set of ***preferred QH models*** *for Δ with domain W is:*

$$\begin{aligned}\mathrm{PQH}(L,\Delta,W) \;=\; \{&\mathcal{M} \in \mathrm{MQH}(L,\Delta,W) \mid \textit{for all } \mathcal{M}' \in \mathrm{MQH}(L,\Delta,W), \\ &|\mathit{Conflictbase}(\mathcal{M})| \leq |\mathit{Conflictbase}(\mathcal{M}')|\}\end{aligned}$$

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Preferred models are minimal models with the least number of inconsistencies.

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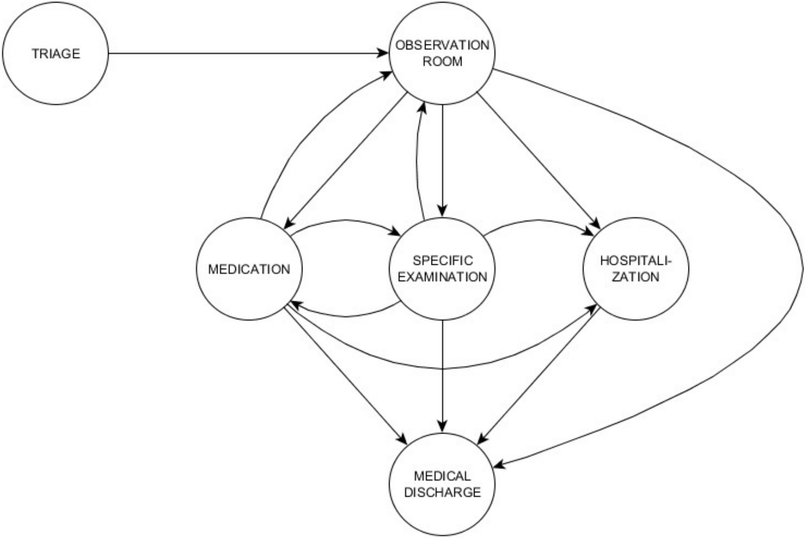

Figure : The care delivery process.

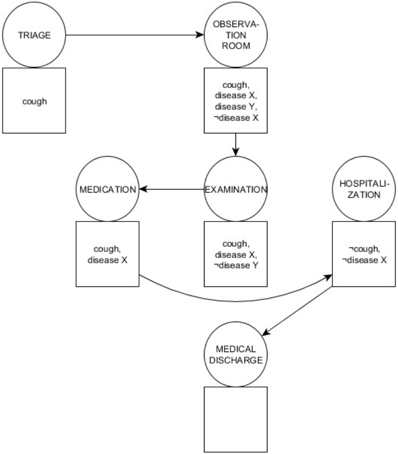

Figure : The QH model $\mathcal{M}$ for a patient A.

Assuming that the propositional variable *cough* can not be paraconsistent, and that the paraconsistency relies only on the medical diagnoses, and also that at triage there is no paraconsistency as well as at medical discharge because there are not diagnoses to make, the measure of inconsistency for this model is:

$$ModelInc(\mathcal{M}, L, W) = \frac{|Conflictbase(\mathcal{M})|}{|IL(L, W)|} = \frac{1}{8}$$

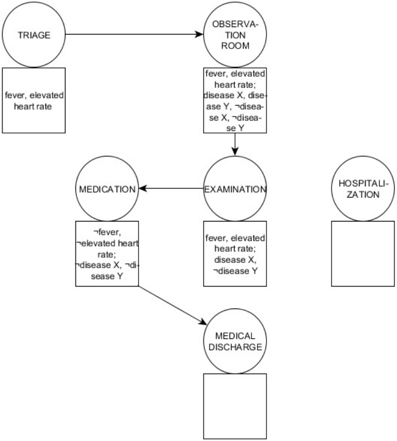

Figure : A QH model $\mathcal{M}'$ of the patient B.

Conclusions and Further Work

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Future work:

- one can investigate inconsistency in nominals and in events
- studying paraconsistency in the context of strong Priorean logic

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