# The Difference Between Fermions and Bosons

$n_1 := 1 \qquad n_2 := 2 \qquad \Psi(x) := \sqrt{2}\cdot\sin(n_1\cdot\pi\cdot x) \qquad \Phi(x) := \sqrt{2}\cdot\sin(n_2\cdot\pi\cdot x)$

Calculate the average separation, $|x_1 - x_2|$, for two fermions and two bosons in a 1D box of unit length.

Fermions have antisymmetric wave functions:

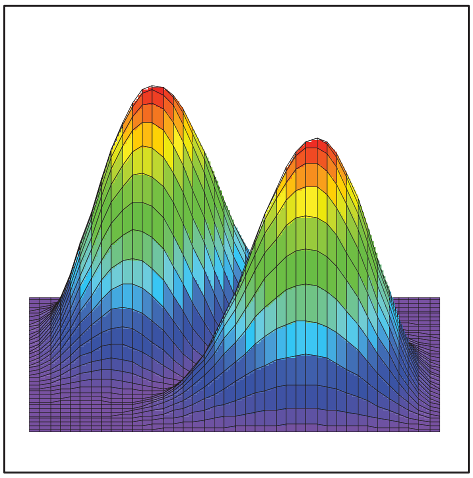
$$\Psi_f(x_1, x_2) := \frac{\Psi(x_1)\cdot\Phi(x_2) - \Psi(x_2)\cdot\Phi(x_1)}{\sqrt{2}}$$

The average particle separation for indistinquishable fermions:

$$\int_0^1 \int_0^1 \Psi_f(x_1, x_2)\cdot|x_1 - x_2|\cdot\Psi_f(x_1, x_2)\, dx_1\, dx_2 = 0.383$$

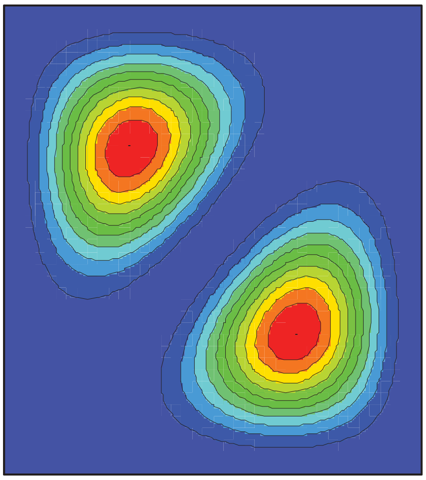
The particles are correlated so as to keep them apart.

$N := 40 \quad i := 0..N \qquad x1_i := \frac{i}{N} \quad j := 0..N \qquad x2_j := \frac{j}{N} \qquad \Psi f_{i,j} := \Psi_f(x1_i, x2_j)^2$



$\Psi f$

$x_1$



$x_2$

$\Psi f$

Bosons have symmetric wave functions:

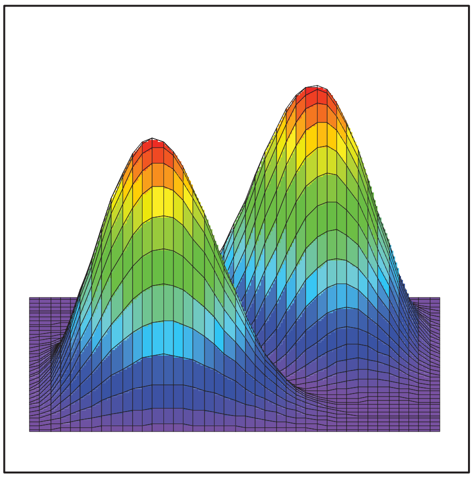
$$\Psi_b(x_1, x_2) := \frac{\Psi(x_1)\cdot\Phi(x_2) + \Psi(x_2)\cdot\Phi(x_1)}{\sqrt{2}}$$

The average particle separation for indistinquishable bosons:

$$\int_0^1 \int_0^1 \Psi_b(x_1, x_2)\cdot|x_1 - x_2|\cdot\Psi_b(x_1, x_2)\, dx_1\, dx_2 = 0.157$$

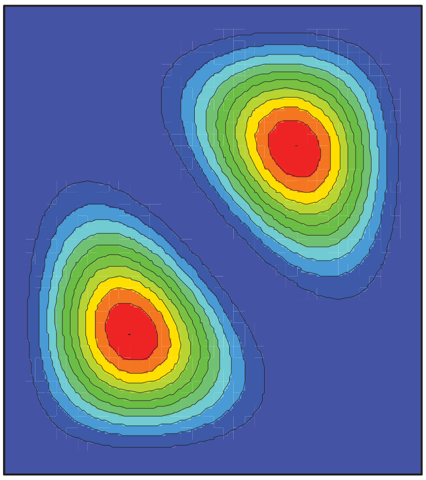
The particles are correlated so as to bring them closer together.

$$N := 40 \quad i := 0..N \quad x1_i := \frac{i}{N} \quad j := 0..N \quad x2_j := \frac{j}{N} \quad \Psi b_{i,j} := \Psi b\left(x1_i, x2_j\right)^2$$



$\Psi b$

$x_1$



$x_2$

$\Psi b$

All fundamental particles (electrons, neutrons, protons, photons, etc.) are either bosons or fermions. Composite entities such as the elements also fall into these two categories. The fundamental distinction is spin: bosons have integer spin (0, 1, 2, ...) while fermions have half-integer spin (1/2, 3/2, ....).

The dramatic difference in behavior between bosons and fermions has led to a sociology of fundamental particles. Bosons are social and gregarious, while fermions are antisocial and aloof.