Lecture 6
Isospin

- What is Isospin?
- Rotations in Isospin space
- Reaction rates
- Quarks and Isospin
- Gell-Mann-Nishijima formula

SU(2) – Isospin

Isospin introduced based on the observation that:

$$m_p = 0.9383 \text{ GeV and } m_n = 0.9396 \text{ GeV}$$

Heisenberg proposed in 1932 the idea that the proton and neutrons are two states of the same particle, the nucleon.

Later we could observe several multiplets of particles with similar masses but differing charges:

$\{p, n\}$	$\{\pi^+, \pi^0, \pi^-\}$	$\{\Xi^-, \Xi^0\}$	$\{\Sigma^+, \Sigma^0, \Sigma^-\}$
$m \approx 0.94$ GeV	≈ 0.140 GeV	≈ 1.32 GeV	≈ 1.19 GeV

Similar with different spin states of the same particle, the nucleon, the pion, …

But the masses are NOT exactly the same within a mutiplet.

This lead to the idea that protons and neutrons would be the up and down states of the same particle.

A general nucleon = an arbitrary combination of up and down state of this new "internal" spin:

$$N = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

And the proton and neutron:

$$p = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad n = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

This new, abstract internal spin is called "Isospin" , behaves in the same way as spin

An isospin state is written as: $\left| I, I_3 \right\rangle$ where I is the total Isospin and I_3 is its 3rd component.

$$p = \left| \frac{1}{2} \frac{1}{2} \right\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad n = \left| \frac{1}{2} \frac{-1}{2} \right\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

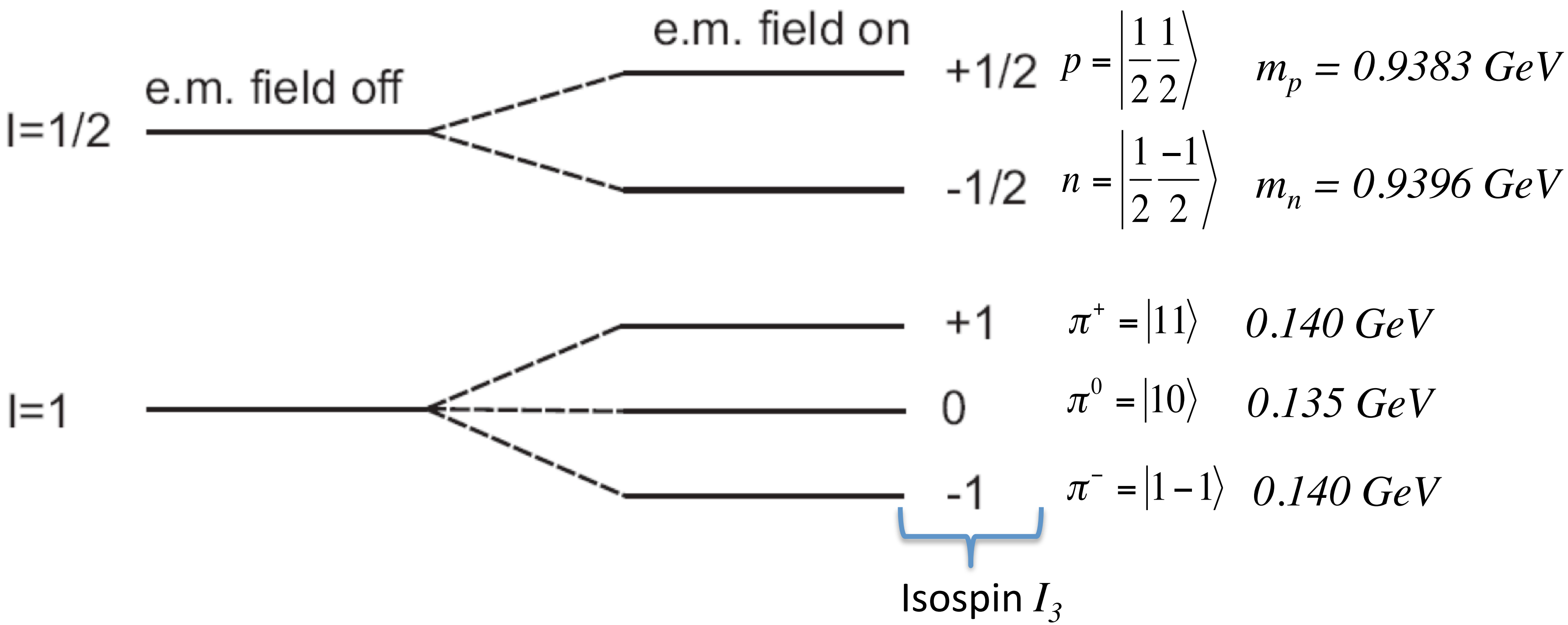
This is only notations, but the interesting physics arises from Heisenberg postulate:

The Strong interaction is invariant under rotation in Isospin space.

From Noether's theorem, it means that isospin is conserved by the strong interaction.

Isospin Multiplets

Assume that if the EM interaction could be turned off the masses of the particles inside the same multiplet would have the same mass.



The number of states in a multiplet is given by $2I+1$

So we can deduce the total isospin from the number of states in the multiplet.

If we know the Isospin we can deduce the number of states in the multiplet.

Usefulness of Isospin

Invariance of the strong force under isospin transformation

If a reaction/decay is governed by the strong interaction, all other reactions/decays that are the results of its **rotation in isospin space** also exist and have rate/lifetime that can be deduced from it.

If one particle is found in Nature within a certain multiplet, then all other members of the multiplet must exist must exist too, with very close mass.

Conservation of Isospin in reactions governed by the strong interaction

The total isospin before a reaction/decay is equal to the isospin after the reaction/decay (applies to both I and I_3)

Usefulness of Isospin (2)

Isospin is only useful when dealing with the **strong force**.

The theory of strong force works very well for high energy interactions (q^2>> 1GeV)

At low energy it is too strong to allow the use of perturbation theory

Much more difficult to compute hadron masses from first principles

Or to compute reaction rates from first principles.

Easier to make predictions at **very high energies** when working with **quarks**

When working with hadrons, often phenomenological models

Isospin is a powerful tool to understand particle masses and rates

Rotation in Isospin State

Consider a rotation in isospin space around the y-axis

It transforms isospin up ($I_3=+1$) $\begin{pmatrix}1\\0\end{pmatrix}$ into isospin down ($I_3=-1$): $\begin{pmatrix}0\\1\end{pmatrix}$

$\Rightarrow$ **transforms a proton into a neutron and vice-versa.**

Generally transforms $\left|I \quad I_3\right\rangle \Rightarrow \left|I \quad -I_3\right\rangle$

$\pi^+ = |11\rangle \quad \pi^0 = |10\rangle \quad \pi^- = |1-1\rangle$ **=> π^+ transforms into π^- and vice versa.**

$p = \left|\frac{1}{2}\frac{1}{2}\right\rangle \qquad n = \left|\frac{1}{2}\frac{-1}{2}\right\rangle$ **=> p transforms into n and vice versa.**

What is the **Isospin rotated equivalent of reaction:** $(a) \quad p+p \rightarrow d+\pi^+$?

What is the representation of Deuteron in Isospin Space?

Consider a state made of two nucleons $n+p$, total isospin is either $I=0$ or 1

I=0
Iso-singlet

$$|00\rangle = \frac{1}{\sqrt{2}}\left(\left|\frac{1}{2}\,\frac{1}{2}\right\rangle\left|\frac{1}{2}\,-\frac{1}{2}\right\rangle - \left|\frac{1}{2}\,-\frac{1}{2}\right\rangle\left|\frac{1}{2}\,\frac{1}{2}\right\rangle\right)$$

$$|00\rangle = \frac{1}{\sqrt{2}}\left(pn - np\right)$$

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I=1
Iso-triplet

$$|11\rangle = \left|\frac{1}{2}\frac{1}{2}\right\rangle\left|\frac{1}{2}\frac{1}{2}\right\rangle \qquad |10\rangle = \frac{1}{\sqrt{2}}\left(\left|\frac{1}{2}\frac{1}{2}\right\rangle\left|\frac{1}{2}-\frac{1}{2}\right\rangle + \left|\frac{1}{2}-\frac{1}{2}\right\rangle\left|\frac{1}{2}\frac{1}{2}\right\rangle\right) \qquad |1-1\rangle = \left|\frac{1}{2}-\frac{1}{2}\right\rangle\left|\frac{1}{2}-\frac{1}{2}\right\rangle$$

$$|11\rangle = pp \qquad |10\rangle = \frac{1}{\sqrt{2}}(pn + np) \qquad |1-1\rangle = nn$$

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Which one represents the deuteron?

If the deuteron has $I=1$ then we expect additional pp and nn states,
We do not observe these states in Nature => $I=0$ for the deuteron

Rotation in Isospin State

Consider a rotation in isospin space around the y-axis

It transforms isospin up ($I_3=+1$) $\begin{pmatrix}1\\0\end{pmatrix}$ into isospin down ($I_3=-1$): $\begin{pmatrix}0\\1\end{pmatrix}$

$\Rightarrow$ **transforms a proton into a neutron and vice-versa.**

Generally transforms $\left|I \quad I_3\right\rangle \Rightarrow \left|I \quad -I_3\right\rangle$

$\pi^+ = |11\rangle \quad \pi^0 = |10\rangle \quad \pi^- = |1-1\rangle$ => π^+ transforms into π^- and vice versa.

$p = \left|\frac{1}{2}\frac{1}{2}\right\rangle \qquad n = \left|\frac{1}{2}\frac{-1}{2}\right\rangle$ => p transforms into n and vice versa.

What is the **Isospin rotated equivalent of reaction:** $(a) \quad p + p \rightarrow d + \pi^+$?

$$(b) \quad n + n \rightarrow d + \pi^-$$

From the point of view these reactions (a) and (b) are the same => Same reaction rates!
Well verified experimentally.

Isospin and Reaction Rate

Consider the 3 reactions:

$$(a) p + p \to d + \pi^{+} \quad (b) n + n \to d + \pi^{-} \quad (c) p + n \to d + \pi^{0}$$

1) Verify that I_3 is conserved in each reaction.
2) Estimate the ratio of the rates of $(a):(b):(c)$ for identical experimental conditions, same energies momenta, etc…, the **only difference being the particles interacting**.

- Sum I_3 for each side of (a): ½ + ½ = 0 + 1

  (b): - ½ - ½ = 0 – 1

  (c): ½ - ½ = 0 + 0

- What happens with the rates?

(LHS-a) $\left|\frac{1}{2}\frac{1}{2}\right\rangle\left|\frac{1}{2}\frac{1}{2}\right\rangle = |11\rangle$ (RHS-a) $|00\rangle|11\rangle = |11\rangle$

(LHS-b) $\left|\frac{1}{2}-\frac{1}{2}\right\rangle\left|\frac{1}{2}-\frac{1}{2}\right\rangle = |1-1\rangle$ (RHS-b) $|00\rangle|1-1\rangle = |1-1\rangle$

(LHS-c) $\left|\frac{1}{2}\frac{1}{2}\right\rangle\left|\frac{1}{2}-\frac{1}{2}\right\rangle = \frac{1}{\sqrt{2}}\left[|10\rangle + |00\rangle\right]$ (RHS-c) $|00\rangle|10\rangle = |10\rangle$

Isospin and Reaction Rate (cont'd)

$$(a)\, p + p \rightarrow d + \pi^{+} \quad (b)\, n + n \rightarrow d + \pi^{-} \quad (c)\, p + n \rightarrow d + \pi^{0}$$

- What happens with the rates?

(LHS-a) $\left|\frac{1}{2}\frac{1}{2}\right\rangle\left|\frac{1}{2}\frac{1}{2}\right\rangle = |11\rangle$

(RHS-a) $|00\rangle|11\rangle = |11\rangle$

(LHS-b) $\left|\frac{1}{2}-\frac{1}{2}\right\rangle\left|\frac{1}{2}-\frac{1}{2}\right\rangle = |1-1\rangle$

(RHS-b) $|00\rangle|1-1\rangle = |1-1\rangle$

(LHS-c) $\left|\frac{1}{2}\frac{1}{2}\right\rangle\left|\frac{1}{2}-\frac{1}{2}\right\rangle = \frac{1}{\sqrt{2}}\left[|10\rangle + |00\rangle\right]$

(RHS-c) $|00\rangle|10\rangle = |10\rangle$

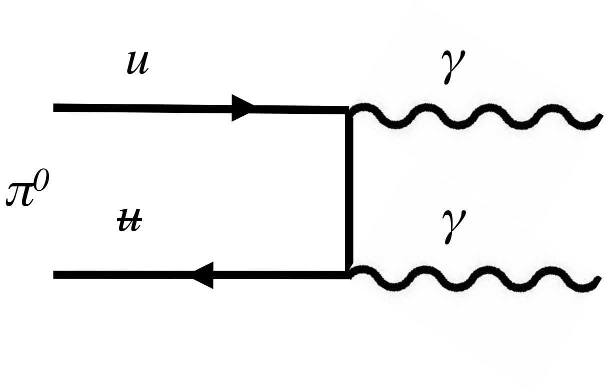
Isospin is conserved in the strong interaction so for the reaction (c)
the transition from $|00>$ to $|10>$ is not allowed.

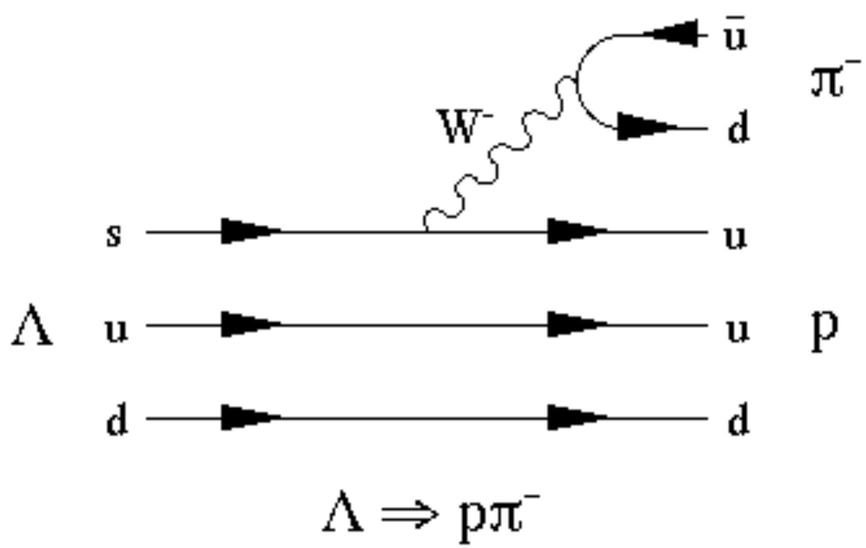
Only the initial states ($|10>$) can lead to the $d+\pi^0$ state

⇒ This yields the ratio of amplitudes: $M_a{:}M_b{:}M_c=1{:}1{:}1/\sqrt{2}$

⇒ Ratios of rates: $|M_a|{:}|M_b|{:}|M_c|=1{:}1{:}½$

Non Conservation of Isospin with the EM and Weak Interactions





EM Decay of π^0

Initial state: $|I\ I_3\rangle = |1\ 0\rangle$

Final state: $|I\ I_3\rangle = |0\ 0\rangle$

Weak Decay of Λ

Initial state: $|I\ I_3\rangle = |0\ 0\rangle$

Final state: $|I\ I_3\rangle = |\tfrac{1}{2}\ \ \tfrac{1}{2}\rangle + |1\ \ -1\rangle$

The EM and Weak Interactions **do not conserve Isospin**.

The $W^{\pm}$, Z^0 and γ and the leptons are assigned Isospin state= $|00\rangle$

Quarks and Isospin

The initial symmetry between the proton and the neutron is related to their quark contents and the similar masses between the u and d quarks.

The Isospin states for the quarks are:

$$u \equiv \left|\frac{1}{2}\frac{1}{2}\right\rangle \qquad d \equiv \left|\frac{1}{2}-\frac{1}{2}\right\rangle$$

For the anti-quarks:

$$\bar{u} \equiv \left|\frac{1}{2}-\frac{1}{2}\right\rangle \quad \bar{d} \equiv -\left|\frac{1}{2}\frac{1}{2}\right\rangle$$

The (-) sign in front of đ will not matter for us in this course

And the other quarks carry no isospin: $|00>$

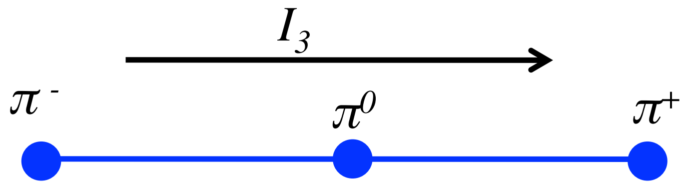
Quarks and Isospin (2)

Let's reformulate some Isospin states from the quarks:

$$|\pi^+\rangle = u\bar{d} = -\left|\frac{1}{2}\frac{1}{2}\right\rangle\left|\frac{1}{2}\frac{1}{2}\right\rangle = -|11\rangle$$

$$|\pi^-\rangle = d\bar{u} = \left|\frac{1}{2}-\frac{1}{2}\right\rangle\left|\frac{1}{2}-\frac{1}{2}\right\rangle = |1-1\rangle$$

$$|\pi^0\rangle = \frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d}) = |10\rangle = \frac{1}{\sqrt{2}}\left(\left|\frac{1}{2}\frac{1}{2}\right\rangle\left|\frac{1}{2}-\frac{1}{2}\right\rangle + \left|\frac{1}{2}-\frac{1}{2}\right\rangle\left|\frac{1}{2}\frac{1}{2}\right\rangle\right)$$



3 pions with approximately the same mass of 140 MeV

What about the following state:

$$\frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d}) = |00\rangle = \frac{1}{\sqrt{2}}\left(\left|\frac{1}{2}\frac{1}{2}\right\rangle\left|\frac{1}{2}-\frac{1}{2}\right\rangle - \left|\frac{1}{2}-\frac{1}{2}\right\rangle\left|\frac{1}{2}\frac{1}{2}\right\rangle\right)$$

This corresponds to another particle with I_3=0 with mass 548 MeV and Strangeness=0 η

Why does Isospin work?

Not a stupid question!

Think in terms of quarks: $m_u \approx 3$ MeV and $m_d \approx 5$ MeV to be compared with the masses of : π^- π^0 π^+ m ≈ 135-140 MeV

$$|\pi^+\rangle = u\bar{d} \qquad |\pi^-\rangle = d\bar{u} \qquad |\pi^0\rangle = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$$

The quark masses are essentially neglible compared to the pion mass itself.

The pion masses are dominated by **binding energy** of the quarks and **quark motion**

If $m_u > 1\ GeV >> m_d$ for example then the Isospin symmetry would break down.

The proton *(uud)* and *(udd)* would have very different masses and the rates of:

$$(a) p + p \rightarrow d + \pi^+ \qquad (b) n + n \rightarrow d + \pi^-$$

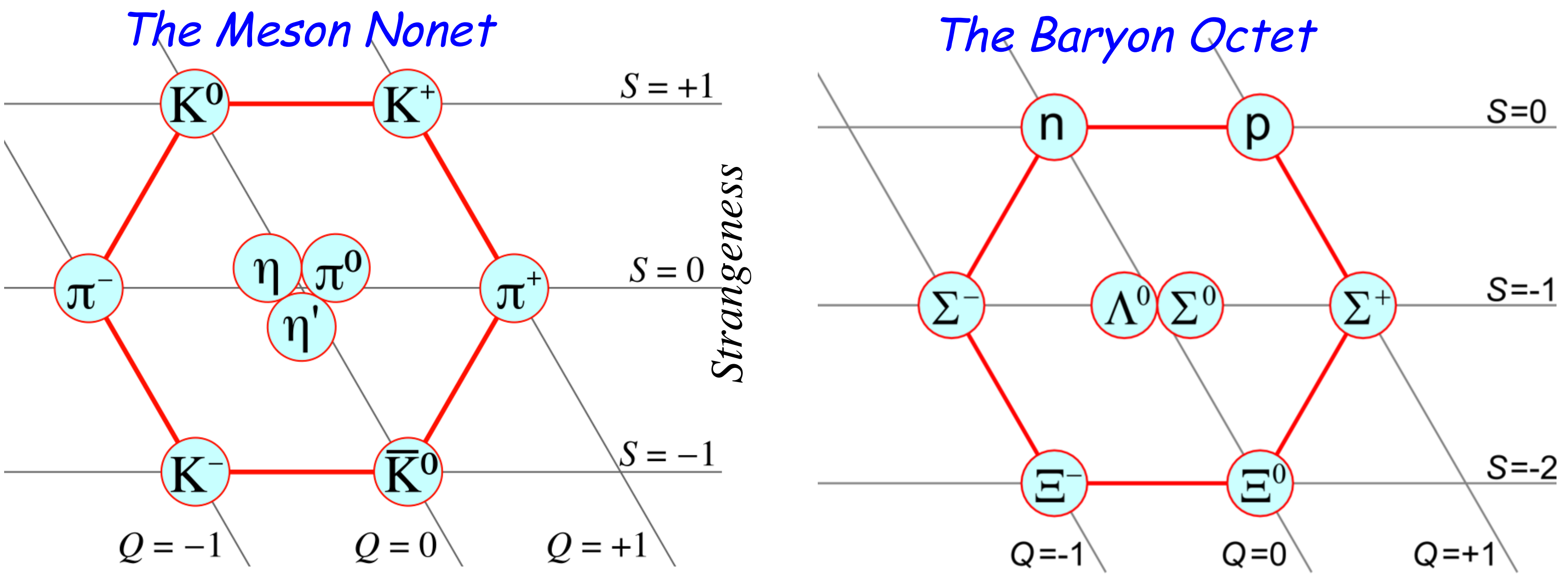
would be very different.

Isospin symmetry is essentially a symmetry of interchange of u and d quarks.

Gell-Mann-Nishijima Formula

Empirically we observe that: $$Q = I_3 + \frac{1}{2}(B + S + C + \overline{B})$$

where Q is the charge, I_3 is the 3rd component of the Isospin, B Baryon number, S, C, $\overline{B}$ are the Strangeness, Charmness and Beauty of the hadron.



Summary

- Isospin is a good symmetry of the strong interaction
  - Hadron masses, reaction/decay rates respect the Isospin symmetry
  - The strong interaction is invariant under SU(2) transformations in isospin space.
  - This is due to the fact that the mass difference between u and d quarks is negligible compared to the binding energy of all hadrons.

- The other quarks are much heavier
  - Observables are no longer invariant under the interchange with a u or a d quark.
  - Still SU(3) symmetry can be useful to enumerate the right list of expected hadrons
  - The strong interaction is not invariant under u,d,s SU(3) transformations