8.4 The Nucleon Isospin

Charge independence of nuclear forces leads to the introduction of a new conserved quantum number, isospin. As early as 1932, Heisenberg treated the neutron and the proton as two states of one particle, the nucleon N.[(6)] The two states presumably have the same mass, but the electromagnetic interaction makes the masses slightly different. (The mass difference of the u and d quarks also contributes, but we neglect this effect here and throughout this chapter.)

To describe the two states of the nucleon, an isospin space (internal charge space) is introduced, and the following analogy to the two spin states of a spin-$\frac{1}{2}$ particle is made:

	Spin-$\frac{1}{2}$ Particle in Ordinary Space	Nucleon in Isospin Space
Orientation	Up	Up, proton
	Down	Down, neutron

The two states of an ordinary spin-$\frac{1}{2}$ particle are not treated as two particles but as two states of one particle. Similarly, the proton and the neutron are considered as the *up* and the *down* state of the nucleon. Formally, the situation is described by introducing a new quantity, *isospin* $\vec{I}$.[(7)] The nucleon with isospin $\frac{1}{2}$ has $2I+1=2$ possible orientations in isospin space. The three components of the isospin vector $\vec{I}$ are denoted by I_1, I_2, and I_3. The value of I_3 distinguishes, *by definition*, between the proton and the neutron. $I_3 = +\frac{1}{2}$ is the proton and $I_3 = -\frac{1}{2}$ is the neutron.[(8)] The most convenient way to write the value of I and I_3 for a given state is by using a Dirac ket:

$$|I, I_3\rangle.$$

Then proton and neutron are

$$\text{proton } |\tfrac{1}{2}, \tfrac{1}{2}\rangle, \qquad \text{neutron } |\tfrac{1}{2}, -\tfrac{1}{2}\rangle. \tag{8.13}$$

The charge for the particle $|I, I_3\rangle$ is given by

$$q = e(I_3 + \tfrac{1}{2}). \tag{8.14}$$

With the values of the third component of I_3 given in Eq. (8.13), the proton has charge e, and the neutron charge 0.

[6] W. Heisenberg, *Z. Physik* **77**, 1 (1932). [Translated in D. M. Brink, *Nuclear Forces*, Pergamon, Elmsford, N.Y., 1965].

[7] To distinguish spin and isospin, we write isospin vectors with an arrow.

[8] In nuclear physics, isospin is sometimes called isobaric spin; it is often denoted by T, and the neutron is taken to have $I_3 = \frac{1}{2}$ and the proton $I_3 = -\frac{1}{2}$, because there are more neutrons than protons in stable nuclei and $I_3(T_3)$ is then positive for these cases.

8.5 Isospin Invariance

What have we gained with the introduction of isospin? So far, very little. Formally, the neutron and the proton can be described as two states of one particle. New aspects and new results appear when charge independence is introduced and when isospin is generalized to all hadrons.

Charge independence states that the hadronic forces do not distinguish between the proton and the neutron. As long as only the hadronic interaction is present, the isospin vector $\vec{I}$ can point in any direction. In other words, there exists rotational invariance in isospin space; the system is invariant under rotations about any direction. As in Eq. (8.10), this fact is expressed by

$$[H_h, \vec{I}] = 0. \tag{8.15}$$

With only H_h present, the $2I + 1$ states with different values of I_3 are degenerate; they have the same energy (mass). Said simply, with only the hadronic interaction present, neutron and proton would have the same mass. The electromagnetic interaction (and the up–down quark mass difference) destroy the isotropy of isospin space; it breaks the symmetry, and, as in Eq. (8.11), it gives

$$[H_h + H_{em}, \vec{I}] \neq 0. \tag{8.16}$$

However, we know from Section 7.1 that the electric charge is always conserved, even in the presence of H_{em}:

$$[H_h + H_{em}, Q] = 0. \tag{8.17}$$

Q is the operator corresponding to the electric charge q; it is connected to I_3 by Eq. (8.14): $Q = e(I_3 + \frac{1}{2})$. Introducing Q into the commutator, Eq. (8.17), gives

$$[H_h + H_{em}, I_3] = 0. \tag{8.18}$$

The third component of isospin is conserved even in the presence of the electromagnetic interaction. The analogy to the magnetic field case is evident; Eq. (8.18) is the isospin equivalent of Eq. (8.12).

It was pointed out in Section 8.4 that charge independence holds not only for nucleons but for all hadrons. Before generalizing the isospin concept to all hadrons and exploring the consequences of such an assumption, a few preliminary remarks are in order concerning isospin space. We stress that $\vec{I}$ is a vector in isospin space, not in ordinary space. The direction in isospin space has nothing to do with any direction in ordinary space, and the value of the operator $\vec{I}$ or I_3 in isospin space has nothing to do with ordinary space. So far, we have related only the third component of $\vec{I}$ to an observable, the electric charge q (Eq. (8.14)). What is the physical significance of I_1 and I_2? These two quantities cannot be connected directly to a physically measurable quantity. The reason is nature: in the laboratory, two magnetic fields can be set up. The first can point in the z direction, and the second in the x direction. The effect of such a combination on the spin of the particle can be computed, and the measurement along any direction is meaningful (within the limits of the uncertainty relations). The electromagnetic field in the isospin space, however, cannot be switched on and off. The charge is always related to one component of $\vec{I}$, and this component is traditionally taken to be I_3. Renaming the components and connecting the charge, for instance, to I_2 does not change the situation.

We now assume the general existence of an isospin space, with its third component connected to the charge of the particle by a linear relation of the form

$$q = aI_3 + b. \tag{8.19}$$

With such a relationship, conservation of the electric charge implies conservation of I_3. I_3 is therefore a good quantum number, even in the presence of the electromagnetic interaction. The unitary operator for a rotation in isospin space by an angle ω about the direction $\hat{\alpha}$ is

$$U_{\hat{\alpha}}(\omega) = \exp(-i\omega\hat{\alpha} \cdot \vec{I}), \tag{8.20}$$

where $\vec{I}$ is the Hermitian generator associated with the unitary operator U, and we expect $\vec{I}$ to be an observable. As in the case of the angular momentum operator $\boldsymbol{J}$, the arguments follow the general steps outlined in Section 7.1. To study the physical properties of $\vec{I}$, we assume first that only the hadronic interaction is present. Then the electric charge is zero for all systems, and Eq. (8.19) does not determine the direction of I_3. Charge independence thus implies that a hadronic system without electromagnetic interaction is invariant under any rotation in isospin space. We know from Section 7.1, Eq. (7.9), that U then commutes with H_h:

$$[H_h, U_{\hat{\alpha}}(\omega)] = 0. \tag{8.21}$$

As in Eq. (7.15), conservation of isospin follows immediately,

$$[H_h, \vec{I}\,] = 0.$$

Charge independence of the hadronic forces leads to conservation of isospin.

In the case of the ordinary angular momentum, the commutation relations for $\boldsymbol{J}$ follow from the unitary operator (8.7) by straightforward algebraic steps. No further assumptions are involved. The same argument can be applied to $U_{\hat{\alpha}}(\omega)$, and the three components of the isospin vector must satisfy the commutation relations

$$[I_1, I_2] = iI_3, \quad [I_2, I_3] = iI_1, \quad [I_3, I_1] = iI_2. \tag{8.22}$$

The eigenvalues and eigenfunctions of the isospin operators do not have to be computed because they are analogous to the corresponding quantities for ordinary spin.

The steps from Eq. (5.6) to Eqs. (5.7) and (5.8) are independent of the physical interpretation of the operators. All results for ordinary angular momentum can be taken over. In particular, I^2 and I_3 obey the eigenvalue equations

$$I^2_{\text{op}}|I, I_3\rangle = I(I+1)|I, I_3\rangle \tag{8.23}$$

$$I_{3,\text{op}}|I, I_3\rangle = I_3|I, I_3\rangle. \tag{8.24}$$

Here I^2_{op} and $I_{3,\text{op}}$ on the left-hand side are operators, and I and I_3 on the right-hand side are quantum numbers. The symbol $|I, I_3\rangle$ denotes the eigenfunction ψ_{I,I_3}. (In a situation where no confusion can arise, the subscripts "op" will be omitted.) The allowed values of I are the same as for J, Eq. (5.9), and they are

$$I = 0, \tfrac{1}{2}, 1, \tfrac{3}{2}, 2, \ldots. \tag{8.25}$$

For each value of I, I_3 can assume the $2I+1$ values from $-I$ to I.

In the following sections, the results expressed by Eqs. (8.22)–(8.25) will be applied to nuclei and to particles. It will turn out that isospin is essential for understanding and classifying subatomic particles.

• We have noted above that the components I_1 and I_2 are not directly connected to observables. However, the linear combinations

$$I_\pm = I_1 \pm iI_2 \tag{8.26}$$

have a physical meaning. Applied to a state $|I, I_3\rangle$, I_+ raises and I_- lowers the value of I_3 by one unit:

$$I_\pm|I, I_3\rangle = [(I \mp I_3)(I \pm I_3 + 1)]^{1/2}|I, I_3 \pm 1\rangle. \tag{8.27}$$

Equation (8.27) can be derived with the help of Eqs. (8.22) to (8.24).[9] •

8.6 Isospin of Particles

The isospin concept was first applied to nuclei, but it is easier to see its salient features in connection with particles. As stated in the previous section, isospin is presumably a good quantum number as long as only the hadronic interaction is present. The electromagnetic interaction destroys the isotropy of isospin space, just as a magnetic field destroys the isotropy of ordinary space. Isospin and its manifestations should consequently appear most clearly in situations where the electromagnetic interaction is small. For nuclei, the total electric charge number Z can be as high as 100, whereas for particles it is usually 0 or 1. Isospin should therefore be a better and more easily recognized quantum number in particle physics.

If isospin is an observable that is realized in nature, then Eqs. (8.15) and (8.23)–(8.25) predict the following characteristics: The quantum number I can take on

[9] Merzbacher, Section 16.2; Messiah, Section XIII.I.

the values $0, \frac{1}{2}, 1, \frac{3}{2}, \ldots$. For a given particle, I is an immutable property. In the absence of the electromagnetic interaction, a particle with isospin I is $(2I+1)$-fold degenerate, and the $2I+1$ *subparticles* all have the same mass.

Since H_h and $\vec{I}$ commute, all subparticles have the same hadronic properties and are differentiated only by the value of I_3. The electromagnetic interaction partially or completely lifts the degeneracy, as shown in Fig. 8.2, and it thus gives rise to the isospin analog of the Zeeman effect. The $2I+1$ subparticles belonging to a given state with isospin I are said to form an *isospin multiplet.* The electric charge of each member is related to I_3 by Eq. (8.19). Quantum numbers that are conserved by the electromagnetic interaction are unaffected by the switching on of H_{em}. Since most quantum numbers have this property, the members of an isospin multiplet have very nearly identical properties; they have, for instance, the same spin, baryon number, hypercharge, and intrinsic parity. (Intrinsic parity will be discussed in Section 9.2.)

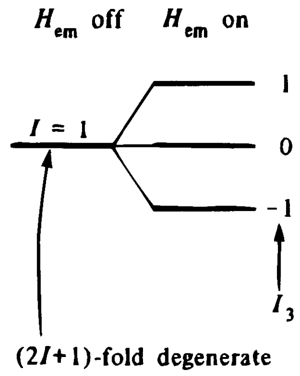

Figure 8.2: A particle with isospin I is $(2I+1)$-fold degenerate in the absence of the electromagnetic interaction. H_{em} lifts the degeneracy, and the resulting subparticles are labeled by I_3.

The different members of an isospin multiplet are in essence the same particle appearing with different orientations in isospin space, just as the various Zeeman levels are states of the same particle with different orientations of its spin with respect to the applied magnetic field. The determination of the quantum number I for a given state is straightforward if all subparticles belonging to the multiplet can be found: Their number is $2I+1$ and thus yields I. Sometimes counting is not possible, and it is then necessary to resort to other approaches, such as the use of selection rules.

The arguments given so far can be applied most easily to the pion. The possible values of the isospin of the pion can be found by looking at Fig. 5.19: If virtual pions are exchanged between nucleons, the basic Yukawa reaction

$$N \longrightarrow N' + \pi$$

should conserve isospin. Nucleons have isospin $\frac{1}{2}$; isospins add vectorially like angular momenta, and the pion consequently must have isospin 0 or 1. If I were 0, only one pion would exist. The assignment $I = 1$, on the other hand, implies the existence of three pions.[10] Indeed, three and only three hadrons with mass of

[10]N. Kemmer, *Proc. Cambridge Phil. Soc.* **34**, 354 (1938).

about 140 MeV/c^2 are known, and the three form an *isovector* with the assignment

$$I_3 = \begin{cases} +1 & \pi^+, \quad m = 139.569 \text{ MeV}/c^2, \\ 0 & \pi^0, \quad m = 134.964 \text{ MeV}/c^2, \\ -1 & \pi^-, \quad m = 139.569 \text{ MeV}/c^2. \end{cases}$$

The charge is connected to I_3 by the relation

$$q = eI_3, \tag{8.28}$$

which is a special case of Eq. (8.19). The pion shows particularly clearly that the properties in ordinary and in isospin space are not related because it is a vector in isospin space but a scalar (spin 0) in ordinary space.

In the ordinary Zeeman effect, it is easy to demonstrate that the various sublevels are members of one Zeeman multiplet: if the applied magnetic field is reduced to zero, they coalesce into one degenerate level. This method cannot be applied to an isospin multiplet because the electromagnetic interaction cannot be switched off. It is necessary to resort to calculations to show that the observed splitting can be blamed solely on H_{em}. Comparison of the pion and the nucleon shows that the problem is not straight forward: the proton is lighter than the neutron, whereas the charged pions are heavier than the neutral one. Nevertheless, the computations performed up to the present time account for the mass splitting by the electromagnetic interaction and the mass difference between the *up* and *down* quarks.[(11)]

After having spent considerable time on the isospin of the pion, the other hadrons can be discussed more concisely.

The *kaon* appears in two particle and two antiparticle states. The assignment $I = \frac{1}{2}$ is in agreement with all known facts.

The assignment of I to *hyperons* is also straightforward. It is assumed that hyperons with approximately equal masses form isospin multiplets. The lambda occurs alone, and it is a singlet. The sigma shows three charge states, and it is an isovector. The cascade particle is a doublet, and the omega is a singlet.

The hadrons encountered so far can all be characterized by a set of additive quantum numbers, A, q, Y, and I_3. For pions, charge and I_3 are connected by Eq. (8.28). Gell-Mann and Nishijima showed how this relation can be generalized to apply also to strange particles. They assumed charge and I_3 to be connected by a linear relation as in Eq. (8.19). The constant a in Eq. (8.19) is determined from Eq. (8.28) as e. To find the constant b, we note that I_3 ranges from $-I$ to $+I$. The average charge of a multiplet is therefore equal to b:

$$\langle q \rangle = b.$$

[11] See e.g., A. De Rújula, H. Georgi, and S. L. Glashow, *Phys. Rev.* **D12**, 147 (1975); N. Isgur and G. Karl, *Phys. Rev. D* **20**, 1191 (1979); J. Gaisser and H. Leutwyler, *Phys. Repts.* **87**, 77 (1982); E.M. Henley and G.A. Miller, *Nucl. Phys.* **A518**, 207 (1990).

The average charge of a multiplet has already been determined in Eq. (7.49):

$$\langle q \rangle = \tfrac{1}{2} eY. \tag{8.29}$$

Only particles with zero hypercharge have the center of charge of the multiplet at $q = 0$; for all others, it is displaced. Consequently the generalization of Eqs. (8.14) and (8.28) is

$$q = e(I_3 + \tfrac{1}{2}Y) = e(I_3 + \tfrac{1}{2}A + \tfrac{1}{2}S). \tag{8.30}$$

This equation is called the Gell-Mann–Nishijima relation. If q is considered to be an operator, it can be said that the electric charge operator is composed of an isoscalar ($\tfrac{1}{2}eY$) and the third component of an isovector (eI_3). For particles with charm, bottom, or top quantum numbers, Y in Eq. (8.30) is replaced by Y_{gen}, Eq. (7.50).

The Gell-Mann–Nishijima relation can be visualized in a Y versus q/e diagram, shown in Fig. 8.3. A few isospin multiplets are plotted. The multiplets with $Y \neq 0$ are *displaced*: Their center of charge is not at zero but, as expressed by Eq. (8.29), at $\tfrac{1}{2}eY$.

The considerations in the present section have shown that isospin is a useful quantum number in particle physics. The value of I for a given particle determines the number of subparticles belonging to this particular isospin multiplet. The third component, I_3, is conserved in hadronic and electromagnetic interactions, whereas $\vec{I}$ is conserved only by the hadronic force.

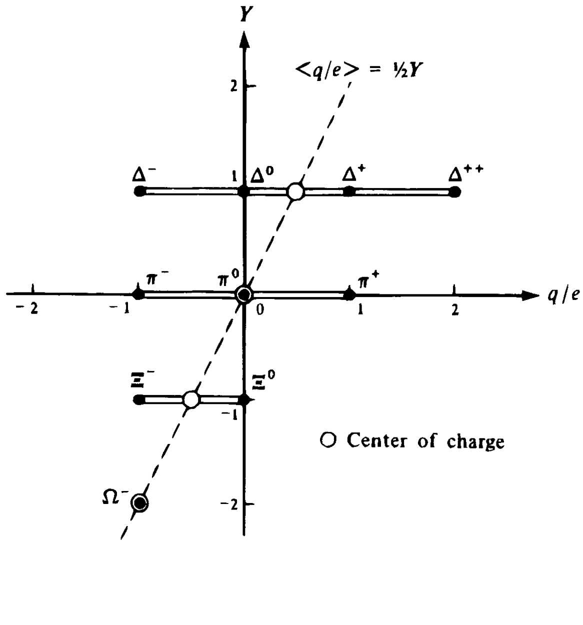

Figure 8.3: Isospin multiplets with $Y \neq 0$ are displaced: Their center of charge (average charge) is at $\tfrac{1}{2}eY$. A few representative multiplets are shown, but many more exist.

In the following section we shall demonstrate that isospin is also a valuable concept in nuclear physics.