

- (1) Use a direct proof to show that: if n is an odd number then $(5n + 6)$ is odd. (2 marks)

0,5

We suppose that n is odd then $n = 2k + 1$ with k integer

$$\text{then } 5n + 6 = 5(2k + 1) + 6 = 10k + 11 = 10k + 10 + 1$$

1,5

$$= 2[5k + 5] + 1$$

$$= 2M + 1 \quad \text{with } M = (5k + 5) \in \mathbb{Z}$$

So $(5n + 6)$ is odd.

- (2) Let a, b, c be real numbers. Prove by contraposition that: if $a + 2b - 3c > 7$ then $a > 4$ or $b > 3$ or $c < 1$. (3 marks)

The contraposition of our statement is:

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if $a \leq 4$ and $b \leq 3$ and $c \geq 1$ then $a + 2b - 3c \leq 7$

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As $b \leq 3$ then $2b \leq 6$

Also $c \geq 1$ then $-3c \leq -3$

$$\text{then } a + 2b - 3c \leq 4 + 6 - 3 \leq 7.$$

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- (3) Assume that $\sqrt{3}$ is irrational number. Give a proof by ^{contradiction} ~~contraposition~~ to show that $\frac{2 + \sqrt{3}}{5}$ is an irrational number. (2 marks)

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We assume that $x = \frac{2 + \sqrt{3}}{5}$ is rational number

$$\text{then } 5x = 2 + \sqrt{3} \Leftrightarrow \sqrt{3} = (5x - 2) \in \mathbb{Q} \quad \text{by}$$

(Sum of 2 rational numbers is rational number)

It is impossible because $\sqrt{3}$ is irrational

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So $\frac{2 + \sqrt{3}}{5}$ is an irrational number.

