King Saud University
Quizz 2-Math 151-Section 348
Time Limit: 40 Minutes

Name : Answer

Assistant prof: *Dr Borhen Halouani*

(1) Use a direct proof to show that: *if n is an even number then $(3n+5)$ is odd.* (2 marks)

0,5

We suppose that n is even then $n = 2k$ with $k \in \mathbb{Z}$

Then $3n+5 = 3(2k)+5 = 6k+5 = 6k+4+1$

$= 2(3k+2)+1$

1,5

$= 2 \quad M \quad +1$ with $M = (3k+2) \in \mathbb{Z}$

So $(3n+5)$ is odd

(2) Let a, b, c be real numbers. Prove by contraposition that: *if $5a-2b+3c > 7$ then $a > 2$ or $b < 3$ or $c > 1$.* (3 marks)

The contraposition of our statement is:

1

" if $a \leq 2$ and $b \geq 3$ and $c \leq 1$ then $5a-2b+3c \leq 7$ "

As $a \leq 2$ then $5a \leq 10$

1

Also $b \geq 3$ then $-2b \leq -6$

1

$c \leq 1$ then $3c \leq 3$

by summation, we get

$5a-2b+3c \leq 10-6+3=7$.

(3) Assume that $\sqrt{5}$ is irrational number. Give a proof by ~~contraposition~~ Contradiction to show that $\frac{3-\sqrt{5}}{3}$ is an irrational number. (2 marks)

We suppose that $x = \frac{3-\sqrt{5}}{3}$ is rational number.

1

Then $3x = 3-\sqrt{5} \Leftrightarrow \sqrt{5} = 3-3x$

As the sum of 2 rational numbers is rational number

We get $\sqrt{5}$ is rational number $\lightning$

It is a contradiction with the assumption $\sqrt{5} \in \mathbb{R} \setminus \mathbb{Q}$

1

So $\frac{3-\sqrt{5}}{3} \in \mathbb{R} \setminus \mathbb{Q}$.

(4) Using proof by induction, show that:

$$\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+\ldots+\frac{n}{2^n}=2-\frac{n+2}{2^n} \text{ for } n\geq 1. \quad \text{(3 marks)}$$

Put $P(n): \frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+\cdots+\frac{n}{2^n} = 2-\frac{n+2}{2^n}$

- Basis step: $n=1$

$$\frac{1}{2} \stackrel{?}{=} 2-\frac{1+2}{2} = 2-\frac{3}{2} = \frac{1}{2} \checkmark$$

0,5

So $P(1)$ is true.

- Inductive step: Let $k \geq 2$, we assume that $P(k)$ is true

0,5 (we have: $\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+\cdots+\frac{k}{2^k} = 2-\frac{k+2}{2^k}$)

Now we prove that $P(k+1)$ remains true.

$$\frac{1}{2}+\frac{2}{2^2}+\cdots+\frac{k+1}{2^{k+1}} \stackrel{?}{=} 2-\frac{(k+1)+2}{2^{k+1}}$$

$$\underbrace{\frac{1}{2}+\frac{2}{2^2}+\cdots+\frac{k}{2^k}}+\frac{k+1}{2^{k+1}} = 2-\frac{k+2}{2^k}+\frac{k+1}{2^{k+1}}$$

2

$$= 2-\frac{2k+4}{2^{k+1}}+\frac{k+1}{2^{k+1}}$$

$$= 2-\frac{k+3}{2^{k+1}}$$

$$-(2k+4)+k+1 = -k-3 = -(k+3)$$

We deduce that $P(n)$ is true for every $n \geq 1$.

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