THU 28 March 2019 *Quiz 3*

Name : SOLUTION **Student ID :**

Question 1

You are given :

$$\boldsymbol{v}_1 = (2,0)\,, \qquad \boldsymbol{v}_2 = (0,2)$$

a) Detrmine whether $\boldsymbol{v}_1$ *and* $\boldsymbol{v}_2$ *span* R^2.

We must determine whether an arbitrary vector $\boldsymbol{b} = (b_1, b_2)$ *in* R^2 *can be expressed as a linear combination of the vectors* $\boldsymbol{v}_1, \boldsymbol{v}_2$

$\boldsymbol{b} = k_1\boldsymbol{v}_1 + k_2\boldsymbol{v}_2$

$(b_1, b_2) = k_1(2,0) + k_2(0,2)$

$(b_1, b_2) = (2k_1, 0) + (0, 2k_2)$

$b_1 = 2k_1$

$b_2 = \qquad 2k_2$

The system is consistent if and only if its coefficient matrix has a nonzero determinant.

$A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$\det(A) = 4 \neq 0$

$\therefore\ \boldsymbol{v}_1, \boldsymbol{v}_2$ *span* R^2

b) Detrmine whether v_1 *and* v_2 *are linearly independent .*

The following vector equation must be satisfied with coefficients that are all zero for vectors to be Linearly Independent

$\boldsymbol{0} = k_1\boldsymbol{v}_1 + k_2\boldsymbol{v}_2$

$(0,0) = k_1(2,0) + k_2(0,2)$

$(0,0) = (2k_1, 0) + (0, 2k_2)$

$0 = 2k_1$

$0 = \qquad 2k_2$

The homogenuous system has only the trivial solution if and only if its coefficient matrix has a nonzero determinant.

$A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$\det(A) = 4 \neq 0$

$k_1 = 0\,, \qquad k_2 = 0$

$\therefore\ \boldsymbol{v}_1, \boldsymbol{v}_2$ *are Linearly Independent*

c) Detrmine whether $S = \{v_1, v_2\}$ *form a basis for* R^2.

Since v_1, v_2 *are Linearly Independent and Span* R^2 *then they form a basis for* R^2.