Problem Set (1)

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PHYS 453: Quantum mechanics

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Problem 1.1. Show that the momentum operator is hermitian

Problem 1.2. Let $\hat{L}_1$,$\hat{L}_2$,$\hat{L}_3$ and $\hat{M}$be linear operators. Using the definition of the commutator, prove the following identities

(i) $[\hat{L}_1\hat{L}_2, \hat{M}] = [\hat{L}_1, \hat{M}]\,\hat{L}_2 \,+ \hat{L}_1\,[\hat{L}_2, \hat{M}]$

(ii) $[\hat{L}_1, [\hat{L}_2, \hat{L}_3]] + [\hat{L}_3, [\hat{L}_2, \hat{L}_1]] + [\hat{L}_2, [\hat{L}_1, \hat{L}_3]] = 0$
(The Jacobi identity)

Problem 1.3. Find the momentum operator $\hat{p}$ eigenfunctions in the x-space representation.

Problem 1.4. Given the following (normalised) wavefunctions

$$\psi_{1s}(\varrho) = A_{1s}e^{-\varrho} \tag{1}$$
$$\psi_{2s}(\varrho) = A_{2s}\left(1 - \frac{\varrho}{2}\right)e^{-\varrho/2} \tag{2}$$

where $\varrho = r/a$ a dimensionless ‘ distance’ defined over $[0, \infty[$ with the integration measure $d\mu = \varrho^2 d\varrho$.

(i) Show that ψ_{1s} and ψ_{2s} are orthogonal

(ii) Find the normalisation coefficient for ψ_{2s}.

Hint; You may find the following integral formula useful

$$\int_0^\infty x^{\nu-1}e^{-\mu x}dx = \left(\frac{1}{\mu}\right)^\nu \Gamma(\nu)$$

You may set $a = 1$ in order to simplify the calculations

Problem 1.5. Let $\hat{f}$ be an operator function of $\hat{x}$. Show that the following commutation relation with the momentum operator $\hat{p}$ holds

$$[\hat{p}, \hat{f}] = -i\hbar\frac{d\hat{f}}{d\hat{x}}$$

Hint; You may need to use Taylor expansion of a function

Problem 1.6. (Challenging problem)
What is the p-space representation of the operator $\hat{\frac{1}{r}}$? *Hint; use the Fourier transform of the function* $1/r$

$$\mathcal{F}(1/r) = \frac{1}{2\pi^2}\int \frac{e^{i\vec{k}\cdot\vec{r}}}{k^2}d^3k$$