

$$1::\langle u, v \rangle = v^T A^T A u = (v^T A^T A u)^T = u^T A^T A v = \langle v, u \rangle$$

$$2::\langle u + v, w \rangle = w^T A^T A (u + v) = w^T A^T A u + w^T A^T A v \\ = \langle u, w \rangle + \langle v, w \rangle$$

$$3::\langle ku, v \rangle = v^T A^T A (ku) = k (v^T A^T A u) = k \langle u, v \rangle$$

$$4::\langle u, u \rangle = u^T A^T A u = (Au)^T Au$$

$$Au = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \Rightarrow (Au)^T Au = \begin{bmatrix} y_1 & y_2 & \cdots & y_n \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

$$= (y_1)^2 + (y_2)^2 + \cdots + (y_n)^2 \geq 0$$

$$\&(y_1)^2 + (y_2)^2 + \cdots + (y_n)^2 = 0 \Leftrightarrow y_i = 0 \forall i$$

$$\Leftrightarrow Au = 0 \Leftrightarrow u = 0$$

since  $A$  is invertible