Physics 201

Problem Set (5)

Problem (1)

Find the determinant of the following matrix using *Arrow Method*

$$\begin{vmatrix} 1 & 1 & 2 \\ 2 & 0 & -1 \\ 3 & 0 & 1 \end{vmatrix}.$$

Solution

$$\begin{vmatrix} 1 & 1 & 2 \\ 2 & 0 & -1 \\ 3 & 0 & 1 \end{vmatrix}\begin{matrix} 1 & 1 \\ 2 & 0 \\ 3 & 0 \end{matrix} = \begin{matrix} 1\cdot 0\cdot 1 + 1\cdot(-1)\cdot 3 + 2\cdot 2\cdot 0 \\ -2\cdot 0\cdot 3 - 1\cdot(-1)\cdot 0 - 1\cdot 2\cdot 1 \end{matrix}$$

$$= 0 - 3 + 0 - 0 - 0 - 2 = -5$$

Problem (2)

Find the determinant of following matrix using *Cofactor Expansion*

$$A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \\ 2 & 1 & -3 \end{bmatrix}$$

Solution

$$\begin{aligned} det(A) &= a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31} \\ &= (-1)^{1+1}\begin{vmatrix} 0 & 1 \\ 1 & -3 \end{vmatrix} + (-1)^{2+1}2\begin{vmatrix} -1 & 1 \\ 2 & -3 \end{vmatrix} + 0 \\ &= -1 + (-2) = -3 \end{aligned}$$

Problem (3)

Evaluate the determinant of following matrix using *Cofactor Expansion*

$$\begin{vmatrix} 1 & 2 & -3 & 4 \\ -4 & 2 & 1 & 3 \\ 3 & 0 & 0 & 0 \\ 2 & 0 & -2 & 3 \end{vmatrix}.$$

Solution

Solution. For cofactor expansion, we select the row or the column with maximal number of zeros. Expanding about the third row, we obtain

$$\begin{vmatrix} 1 & 2 & -3 & 4 \\ -4 & 2 & 1 & 3 \\ 3 & 0 & 0 & 0 \\ 2 & 0 & -2 & 3 \end{vmatrix} = (-1)^{3+1}3\begin{vmatrix} 2 & -3 & 4 \\ 2 & 1 & 3 \\ 0 & -2 & 3 \end{vmatrix}$$

Expanding about the first column,

$$= 3\left((-1)^{1+1}2\begin{vmatrix} 1 & 3 \\ -2 & 3 \end{vmatrix} + (-1)^{2+1}2\begin{vmatrix} -3 & 4 \\ -2 & 3 \end{vmatrix}\right) = 3(2\cdot 9 + (-1)\cdot 2\cdot(-1)) = 60$$

Remark. The determinant

$$\begin{vmatrix} 2 & -3 & 4 \\ 2 & 1 & 3 \\ 0 & -2 & 3 \end{vmatrix}$$

can also be computed using the formula for 3×3 determinant.

Problem (4)

Using properties of determinants, compute the determinants of the following matrices

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 & -2 \\ 3 & 4 & 0 \\ 1 & 2 & -2 \end{bmatrix}, \quad C = \begin{bmatrix} -1 & 12 & 0 \\ 3 & 4 & 0 \\ 5 & 5 & 0 \end{bmatrix}.$$

Solution

A is a diagonal matrix, so

$$det(A) = 1 \cdot (-4) \cdot (-3) = 12.$$

B has two equal rows, so

$$det(B) = 0.$$

C has a zero column, so

$$det(C) = 0.$$

Problem (5)

Using properties of determinants, Evaluate the determinant of following matrix

$$\begin{vmatrix} 3 & 3 & 3 \\ 5 & 1 & 1 \\ 3 & 4 & 3 \end{vmatrix}$$

Solution

Solution.

$$\begin{vmatrix} 3 & 3 & 3 \\ 5 & 1 & 1 \\ 3 & 4 & 3 \end{vmatrix} = 3 \begin{vmatrix} 1 & 1 & 1 \\ 5 & 1 & 1 \\ 3 & 4 & 3 \end{vmatrix}$$

$Row_2 - Row_1$

$$= 3 \begin{vmatrix} 1 & 1 & 1 \\ 4 & 0 & 0 \\ 3 & 4 & 3 \end{vmatrix}$$

expansion about the second row

$$= 3(-1)^{2+1} 4 \begin{vmatrix} 1 & 1 \\ 4 & 3 \end{vmatrix} = 3(-4)(-1) = 12$$

Problem (6)

Suppose the determinant of matrix A equals 5,

$$A = \begin{bmatrix} 1 & 0 & 0 \\ x & y & z \\ 1 & 2 & 3 \end{bmatrix}$$

Suppose that we know

$$det(A) = 5.$$

Compute the following determinants

$$\textbf{a)} \begin{vmatrix} x & y & z \\ 1 & 0 & 0 \\ 1 & 2 & 3 \end{vmatrix}, \quad \textbf{b)} \begin{vmatrix} 1 & 0 & 0 \\ x & y & z \\ 5 & 10 & 15 \end{vmatrix}, \quad \textbf{c)} \begin{vmatrix} 1 & 0 & 0 \\ x+1 & y+2 & z+3 \\ 1 & 2 & 3 \end{vmatrix}.$$

Solution

Solution. **a)** The determinant is obtained from $det(A)$ by interchanging two rows, so

$$\begin{vmatrix} x & y & z \\ 1 & 0 & 0 \\ 1 & 2 & 3 \end{vmatrix} = -det(A) = -5.$$

b) The determinant is obtained from $det(A)$ by multiplying the third ro 5 and therefore

$$\begin{vmatrix} 1 & 0 & 0 \\ x & y & z \\ 5 & 10 & 15 \end{vmatrix} = 5det(A) = 25.$$

b) The determinant is obtained from $det(A)$ by adding Row_3 to Row_2,

$$\begin{vmatrix} 1 & 0 & 0 \\ x+1 & y+2 & z+3 \\ 1 & 2 & 3 \end{vmatrix} = det(A) = 5$$