Physics 201

Problem Set (2)

Problem (1)

1. In each part, determine whether the matrix is in row echelon form, reduced row echelon form, both, or neither.

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 2 & 4 \end{bmatrix}$

(e) $\begin{bmatrix} 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

(f) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

(g) $\begin{bmatrix} 1 & -7 & 5 & 5 \\ 0 & 1 & 3 & 2 \end{bmatrix}$

Solution:

(a) Both

(b) Both

(c) Both

(d) Both

(e) Both

(f) Both

(g) Row echelon

Problem (2)

Use Gauss-Jordan elimination to solve the linear system

$$\begin{array}{rcrcrcr} x & + & y & + & z & = & 3 \\ 2x & + & 3y & + & 4z & = & 11 \\ 4x & + & 9y & + & 16z & = & 41 \end{array}$$

Solution:

Solution: Denote the augmented matrix

$$A = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 2 & 3 & 4 & 11 \\ 4 & 9 & 16 & 41 \end{array}\right].$$

We will compute the reduced row-echelon form for A. Subtract twice the first equation from the second, and subtract 4 times the first equation from the third:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & 5 \\ 0 & 5 & 12 & 29 \end{array}\right]$$

Now subtract the second row from the first, and subtract 5 times the second row from the third. Divide the resulting row by 2:

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

Finally, add the third row to the first, and subtract twice the third row from the second:

$$\text{rref}(A) = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

We conclude that the solution is $\boxed{x = 0,\ y = 1, \text{ and } z = 2.}$

Problem (3)

Use Gaussian elimination to solve the system of linear equations

$$\begin{aligned} x_1 - 2x_2 - 6x_3 &= 12 \\ 2x_1 + 4x_2 + 12x_3 &= -17 \\ x_1 - 4x_2 - 12x_3 &= 22. \end{aligned}$$

Solution:

Solution: In this case, we convert the system to its corresponding augmented matrix, perform the necessary row operations on the matrix alone, and then convert back to equations at the end to identify the solution.

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & - & 6x_3 & = & 12 \\ 2x_1 & + & 4x_2 & + & 12x_3 & = & -17 \\ x_1 & - & 4x_2 & - & 12x_3 & = & 22 \end{array} \qquad \begin{pmatrix} 1 & -2 & -6 & 12 \\ 2 & 4 & 12 & -17 \\ 1 & -4 & -12 & 22 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -2 & -6 & 12 \\ 0 & 8 & 24 & -41 \\ 0 & -2 & -6 & 10 \end{pmatrix}$$

Add -2 times Row 1 to Row 2.
Add -1 times Row 1 to Row 3.

$$\begin{pmatrix} 1 & -2 & -6 & 12 \\ 0 & -2 & -6 & 10 \\ 0 & 8 & 24 & -41 \end{pmatrix}$$

Swap Row 2 and Row 3.

$$\begin{pmatrix} 1 & -2 & -6 & 12 \\ 0 & -2 & -6 & 10 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Add 4 times Row 2 to Row 3.

$$\begin{pmatrix} 1 & -2 & -6 & 12 \\ 0 & -2 & -6 & 10 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad \begin{array}{rcrcrcr} x_1 & - & 2x_2 & - & 6x_3 & = & 12 \\ & - & 2x_2 & - & 6x_3 & = & 10 \\ & & & & 0 & = & -1 \end{array}$$

Since the final equation

$$0 = -1$$

cannot be satisfied, this system has no solutions.

Problem (4)

Solve the system of linear equations.

$$\begin{aligned} 2x_1 + 4x_2 - 2x_3 &= 0 \\ 3x_1 + 5x_2 &= 1 \end{aligned}$$

Solution:

LUTION The augmented matrix of the system of linear equations is

$$\begin{bmatrix} 2 & 4 & -2 & 0 \\ 3 & 5 & 0 & 1 \end{bmatrix}.$$

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f Linear Equations

Using a graphing utility, a computer software program, or Gauss-Jordan elimination, you can verify that the reduced row-echelon form of the matrix is

$$\begin{bmatrix} 1 & 0 & 5 & 2 \\ 0 & 1 & -3 & -1 \end{bmatrix}.$$

The corresponding system of equations is

$$\begin{aligned} x_1 + 5x_3 &= 2 \\ x_2 - 3x_3 &= -1. \end{aligned}$$

Now, using the parameter t to represent the *nonleading* variable x_3, you have

$$x_1 = 2 - 5t, \qquad x_2 = -1 + 3t, \qquad x_3 = t, \qquad \text{where } t \text{ is any real number.}$$

Problem (5)

Use Gauss-Jordan elimination to solve the homogeneous linear system

$$\begin{aligned} x_1 + 3x_2 - 2x_3 \quad\quad + 2x_5 \quad\quad &= 0 \\ 2x_1 + 6x_2 - 5x_3 - 2x_4 + 4x_5 - 3x_6 &= 0 \\ 5x_3 + 10x_4 \quad\quad + 15x_6 &= 0 \\ 2x_1 + 6x_2 \quad\quad + 8x_4 + 4x_5 + 18x_6 &= 0 \end{aligned}$$

Solution:

Solution Observe first that the coefficients of the unknowns in this system are the same as those in Example 5; that is, the two systems differ only in the constants on the right side. The augmented matrix for the given homogeneous system is

$$\begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 2 & 6 & -5 & -2 & 4 & -3 & 0 \\ 0 & 0 & 5 & 10 & 0 & 15 & 0 \\ 2 & 6 & 0 & 8 & 4 & 18 & 0 \end{bmatrix} \tag{5}$$

which is the same as the augmented matrix for the system in Example 5, except for zeros in the last column. Thus, the reduced row echelon form of this matrix will be the same as that of the augmented matrix in Example 5, except for the last column. However, a moment's reflection will make it evident that a column of zeros is not changed by an elementary row operation, so the reduced row echelon form of 5 is

$$\begin{bmatrix} 1 & 3 & 0 & 4 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{6}$$

The corresponding system of equations is

$$\begin{aligned} x_1 + 3x_2 \quad + 4x_4 + 2x_5 \quad &= 0 \\ x_3 + 2x_4 \quad\quad &= 0 \\ x_6 &= 0 \end{aligned}$$

Solving for the leading variables we obtain

$$\begin{aligned} x_1 &= -3x_2 - 4x_4 - 2x_5 \\ x_3 &= -2x_4 \\ x_6 &= 0 \end{aligned} \tag{7}$$

If we now assign the free variables x_2, x_4, and x_5 arbitrary values r, s, and t, respectively, then we can

express the solution set parametrically as

$$x_1 = -3r - 4s - 2t, \quad x_2 = r, \quad x_3 = -2s, \quad x_4 = s, \quad x_5 = t, \quad x_6 = 0$$

Note that the trivial solution results when $r = s = t = 0$.