Thermodynamics of Quantum-Corrected Black Holes

Phys 499 Graduation Project

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Outline

Introduction

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- It was shown by Jacobson [1] that Einstein equations of general relativity (GR) can be retrieved from the second law of thermodynamics $dE = dS/T$, using Raychaudhuri equation (RE).
- Inspired by Jacobson's formalism, Das [2] and followed by Alsaleh *et. al* [3] thought of RE as a fundamental equation of gravity instead of Einstein equations.

Later, RE was canonically quantized. Leading to a discovery of a new dynamical system for gravity called **Geometric flows** [3].
Studying quantum geometric flows lead to proving rigorously that singularities only exist as a classical limit of the quantum space-time [4].

Outline

1. Introduction
2. Geometric flows
3. Statistical mechanics of geometric flows
4. Quantum-corrected black holes
5. Conclusion

Geometric flows

We start by Studying the congruence of test particles moving on an $n+1$ dimensional space-time $\mathcal{M}$. We can use the proper time for the test particles λ as a dynamical foliation parameter, such that we foliate the space-time

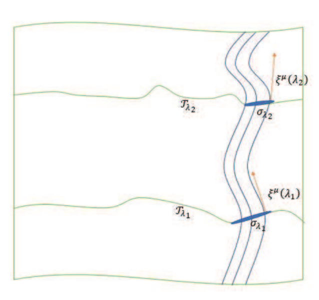

Figure: The dynamic foliation of the space-time $\mathcal{M}$ by the flow of geodesic congruences.

Geometric flows

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The dynamical variable χ posses the Lagrangian:

$$L = \frac{n}{2}\dot{\chi}^2 - \frac{1}{2}\mathcal{R}\chi^2 - \Sigma, \tag{1}$$

with $\mathcal{R}$ being the Raychaudhuri scalar and Σ is the shear potential

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Raychaudhuri equation can be recovered from Hamilton's equation

$$\{\theta\,, H\} = -\dot{\theta} = -\frac{1}{n}\hat{\theta}^2 - \mathcal{R} - 2\sigma^2. \tag{3}$$

Quantisation

We can now quantise the system by introducing the operators $\hat{\chi}$ and $\hat{\varpi}$, that obey the CCR:

$$[\hat{\chi}, \hat{\varpi}] = i\hbar^{-1/2}\hat{I}. \tag{4}$$

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In the χ representation, we define the geometric flow wavefunctionals $\Psi[\chi; \lambda]$ that obey the Schrödinger-Raychudhri-Das (SRD) equation:

$$\left(\frac{-\hbar^2}{2n} \frac{\delta^2}{\delta \chi^2} + \frac{1}{2}\mathcal{R}\chi^2 + \Sigma \right) \Psi[\chi; \lambda] = i\hbar \frac{\partial}{\partial \lambda} \Psi[\chi; \lambda] \tag{5}$$

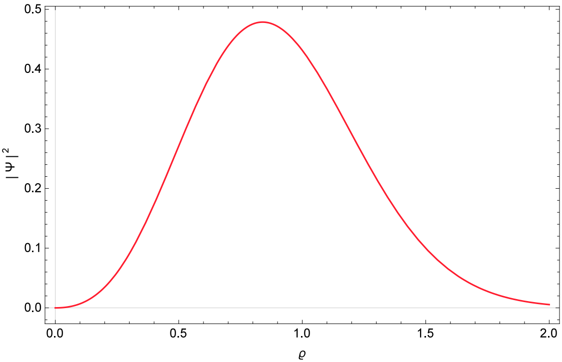

Figure: A plot of the probability density function $|\Psi|^2$ vs $\rho = \chi^2$ obtained by solving (5) with $\mathcal{R} = \boldsymbol{\Sigma} = 0$. The plot indicates that the probability density function rapidly decreases as $\rho \to 0$ and vanishes identically at the singularity.

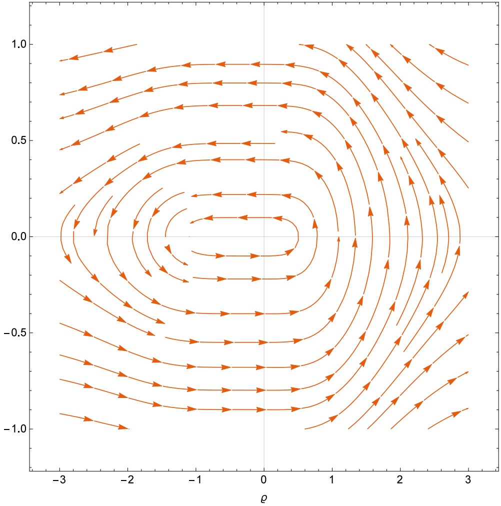

Figure: Analytically continued probability current density showing the flow of probability around the singularity. It can bee seen that there is no probability flow at the singularity.

Outline

Horizons as ensembles of geometric flows

We can assign a single particle geodesic to a geometric flow degree of freedom. Hence, geometric flows play the role of subsystems in Jacobson's formalism [1].

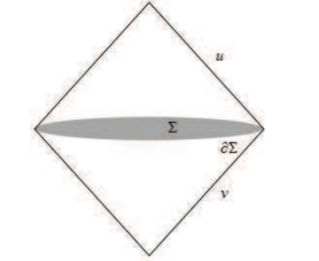

Figure: A region of space-time, enclosed by a horizon generated by null geodesics.

The null generators correspond to an ensemble of N geometric flows. The number N is bounded by the Bekenstein limit [6] for the number of events or objects enclosing an area A.

$$N \sim \frac{A}{4\hbar} \tag{6}$$

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With this assumption, the SRD equation becomes an equation of SHO with half potential, with angular frequency $\omega = (\mathcal{R}/n)^{1/2}$.

Statistical mechanics of geometric flows

Thus, we may calculate the partition function of the quantum geometric flows ensemble

$$Z = \frac{e^{-\frac{3}{2}\beta N\hbar\omega}}{\left(1 - e^{-\beta\hbar\omega}\right)^N}, \tag{7}$$

from which we can fully characterise the statistical mechanics of geometric flows.

Statistical mechanics of geometric flows

We can calculate the entropy of the horizon using (7)

$$S = N\left(1 + \ln(\frac{T\hbar}{\omega}) + \frac{\omega^2\hbar^2}{24T^2} + \dots\right). \tag{8}$$

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Moreover, we define the BH mass as its average energy $M = \langle E \rangle$, that is given by

$$\langle E \rangle = M = N\left(2T + \frac{\omega^2\hbar^2}{6T} + \dots\right). \tag{9}$$

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Recovering black hole thermodynamics

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By taking the leading term, we also recover the relation between the BH mass and its temperature

$$T = \frac{\hbar}{8\pi M} \tag{11}$$

Quantum-corrected black hole

Using the sub-leading terms in (8) and (9), implementing the results into the standard calculations of black hole thermodynamics [7], we find the quantum-corrected Schwarzschild metric to be

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{4\pi^2\sqrt{\mathcal{R}}\hbar}{r^2}\right)dt^2 + \left(1 - \frac{2M}{r} + \frac{4\pi^2\sqrt{\mathcal{R}}\hbar}{r^2}\right)^{-1} dr^2 + r^2 d\Omega^2 \quad (12)$$

Quantum-corrected temperature

From the above metric we find the quantum-corrected thermodynamics

- Temperature

$$T = \frac{\sqrt{M^2 - 4\pi^2\sqrt{\mathcal{R}}\hbar}}{2\pi\left(\sqrt{M^2 - 4\pi^2\sqrt{\mathcal{R}}\hbar} + M\right)^2}, \tag{13}$$

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- Entropy, using the relation $S = \int dM/T$.

$$S = 2\pi\left(M\sqrt{M^2 - 4\pi^2\sqrt{\mathcal{R}}\hbar} + M^2\right). \tag{14}$$

Quantum-corrected temperature

- Heat capacity, using $C = T\dfrac{\partial S}{\partial T}$

$$C = -\frac{2\pi\sqrt{M^2 - 4\pi^2\sqrt{\mathcal{R}}\hbar}\left(M + \sqrt{M^2 - 4\pi^2\sqrt{\mathcal{R}}\hbar}\right)^2}{2\sqrt{M^2 - 4\pi^2\sqrt{\mathcal{R}}\hbar} - M} \qquad (15)$$

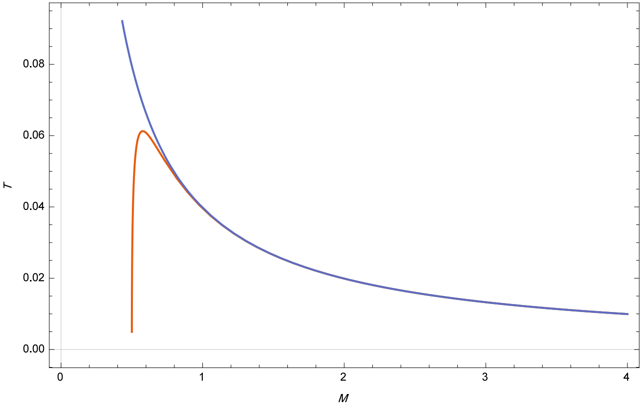

Figure: A graph of M vs T for classical (blue) and quantum (red) black holes. We assumed area fluctuation of Plankian order $\omega \sim 1$. The graph indicates the existence of remnant for the quantum black hole.

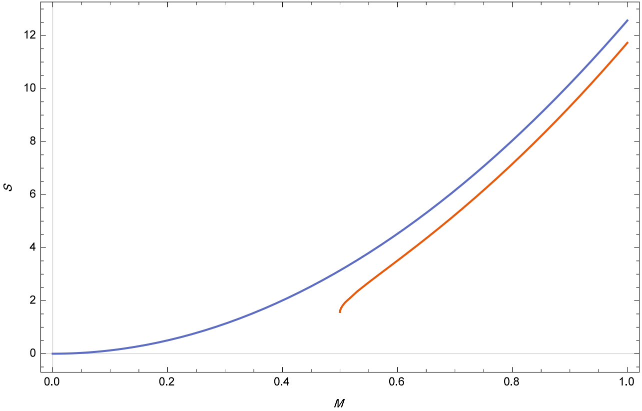

Figure: Graph M vs S, for classical (blue) and quantum (red) black holes. We assumed area fluctuation of Planckian order $\omega \sim 1$. The graph indicates the existence of remnant for the quantum black hole.

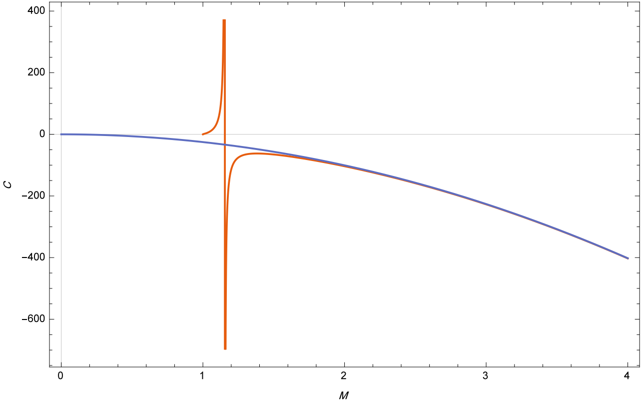

Figure: A graph of heat capacity C vs mass M, obtained from the relation $C = T\partial S/\partial T$. For classical (blue) and quantum (red) black hole. Remnant can also be observed at a critical mass

Outline

Conclusion

We can see from the previous graphs.

- The quantum correction to Schwarzschild geometry using the statistical mechanics of geometric flows reproduces the same quantum corrections obtained in [5], by considering the the quantum Raychaudhuri equation (QRE) [2].
- Indicating that the initial conjecture about the space-time being an ensemble of geometric flows is true. Moreover, inducting a logarithmic correction to entropy, as seen in all quantum gravity programs.

Conclusion

- Recovering the standard formulae for BH temperature and entropy is a great test for creditability of quantum geometric flows as a potential approach for quantum gravity.

However, there is a lot to be done with geometric flows, it would be interesting to consider a more realistic model for quantum black holes with time-dependent area fluctuations, for example.

Thank You !

References

[1] T. Jacobson,
"Thermodynamics of space-time: The Einstein equation of state,"
Phys. Rev. Lett. **75** (1995) 1260

[2] S. Das,
"Quantum Raychaudhuri equation,"
Phys. Rev. D **89** (2014) no.8, 084068

[3] S. Alsaleh, L. Alasfar, M. Faizal, and A. Farag Ali,
"Geometric Flows and implication to quantum gravity"
Accepted in Phys. Lett. B

[4] S. Alsaleh, L. Alasfar, M. Faizal and A. F. Ali,
"Quantum no-singularity theorem from geometric flows,"
arXiv:1705.00977 [hep-th].
Accepted in Phys. Rev. Lett.

[5] A. F. Ali and M. M. Khalil,
"Black Hole with Quantum Potential,"
Nucl. Phys. B **909** (2016) 173

[6] J. D. Bekenstein,
"A Universal Upper Bound on the Entropy to Energy Ratio for Bounded Systems,"
Phys. Rev. D **23** (1981) 287.

[7] A. Dabholkar and S. Nampuri,
"Quantum black holes,"
Lect. Notes Phys. **851** (2012) 165