PHYS 502
HANDOUT 7

1. A uniform beam with insulated surface has length equal to 3 units and coefficient of thermal conductivity equal to 2 units. If both ends of the beam are at zero temperature. If the initial temperature was 25 degrees Celsius find the temperature of the beam $u(x,t)$.

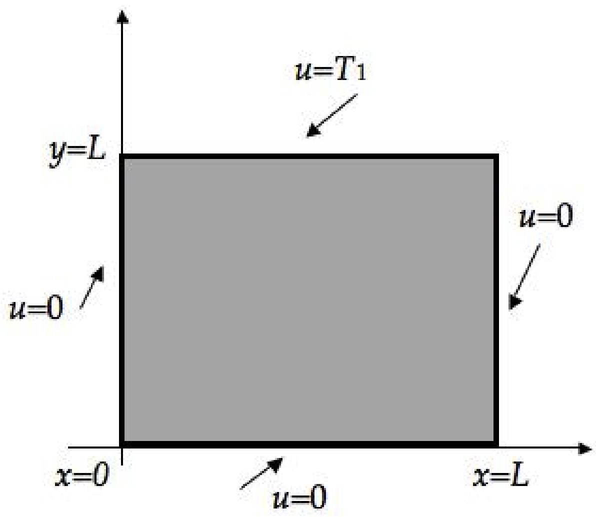
(Sch. p. 38)

2. A circular disc of radius a has its surface insulated. The upper half of the disk has a constant temperature T_1 and the lower has a constant temperature T_2. Find the steady state temperature of the disk.

(Sch. p.39)

3. The three sides of the following plate are kept at zero temperature, the other one is kept at a constant temperature T_1. Find the temperature of the plate at the steady state.

(Sch. p. 42)

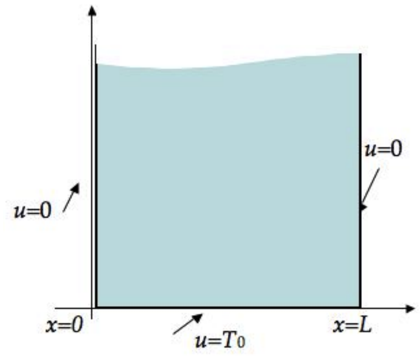


4. If in problem 3, all the sides are kept at constants temperatures T_1, T_2, T_3 and T_4 respectively, could you suggest a way to find the temperature of the plate at the steady state?

(Sch. p. 42)

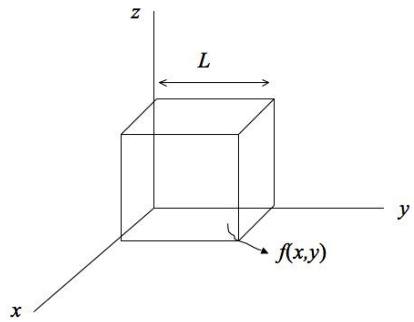
5. A plate of infinite length and width L has its parallel sides at zero temperature and the lower side at temperature T_0 as shown in figure below. Find the steady state temperature of the plate.

(Sch. p. 49)



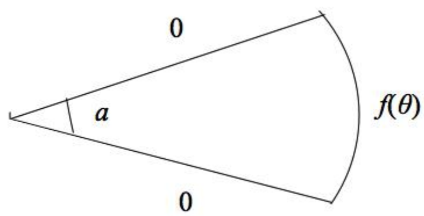
6. Calculate the steady state temperature in a compact cube in which the side xy is kept at temperature $u = f(x,y)$ while the rest are kept at zero temperature.

(Sch. p. 50)



7. Find the steady state temperature of the following wedge-like plate with the boundary conditions show in the figure.

(Sch. p. 51)



8. Sound waves in a pipe are described by the following wave equation:

$$u_{xx} - \frac{1}{c^2} u_{tt} = 0$$

where $u(x,t)$ the displacement from the equilibrium position of the air molecules which at time t are found in the cross section at point x, while $c = \sqrt{p_0 / p}$ is the propagation speed of the sound waves in the pipe (p_0 is the normal air pressure and ρ its density).

a) Assuming that the air inside the pipe behaves like an ideal gas where pV = constant, show that *pressure variation* Δp which is created by the sound wave is related to the molecules displacement by $\Delta p = -p_0 u_x$.

b) Denoting, for simplicity reasons, that $\Delta p \equiv p$ show that the pressure variation satisfies the wave equation:

$$p_{xx} - \frac{1}{c^2} p_{tt} = 0$$

c) Calculate the eigenfrequencies of a pipe of length *L:* i) closed at both ends, ii) closed at one end and iii) open at both ends

9. An infinitely long metallic beam of square cross section, with side *L*, and initial temperature T_0 in all its bulk, is immersed in a cooling liquid of zero temperature. Show that, after time *t*, the temperature distribution in any cross section of the beam will be given by

$$u(x,y,t) = \frac{16T_0}{\pi^2} \sum_{\substack{n.m \\ \text{odd}}} \frac{1}{nm} \sin\frac{n\pi x}{L} \sin\frac{m\pi y}{L} e^{-(n^2+m^2)\pi^2 t/L^2}$$

10. Show that the solution of the two-dimensional Laplace equation in theinterior of a semi-circular disk of radius *a* with the following boundary conditions: $0 < \theta < \pi$, $u(a,\theta) = 1$, $u(\rho,0) = u(\rho,\pi) = 0$, is given by

$$u(\rho,\theta) = \frac{4}{\pi} \sum_{n\ \text{odd}} \frac{1}{n} \left(\frac{\rho}{a}\right)^n \sin n\theta \frac{n\pi x}{L}$$