

### Exercise 1

$$f(x) = \begin{cases} -1 & -\pi < x < 0 \\ 1 & 0 < x < \pi \end{cases}$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx = \frac{1}{\pi} \left\{ \int_{-\pi}^0 (-1) dx + \int_0^{\pi} dx \right\} = \frac{1}{\pi} \left\{ -x \Big|_{-\pi}^0 + x \Big|_0^{\pi} \right\} = \frac{1}{\pi} \{-\pi + \pi\} = 0$$

$$a_n = \frac{2}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx) dx = \frac{1}{\pi} \left\{ \int_{-\pi}^0 (-1) \cdot \cos(nx) dx + \int_0^{\pi} \cos(nx) dx \right\} \\ = 0$$

$$b_n = \frac{1}{\pi} \left\{ \int_{-\pi}^0 (-1) \sin(nx) dx + \int_0^{\pi} \sin(nx) dx \right\} = \frac{1}{\pi} \left\{ \frac{(+1)}{n} \cos(nx) \Big|_{-\pi}^0 + \frac{1}{n} \cos(nx) \Big|_0^{\pi} \right\} \\ = \frac{1}{n\pi} \left\{ 1 - \cos(n\pi) - \cos(n\pi) + 1 \right\} = \frac{1}{n\pi} \{ 2 - 2\cos(n\pi) \}$$

If  $n = \text{even}$   $b_n = 0$  if  $n = \text{odd}$   $b_n = 4/n\pi$

$$f(x) = \frac{4}{\pi} \sum_{n=\text{odd}}^{\infty} \frac{\sin(nx)}{n}$$

$$f(0) = 0 \quad f_+(0) = 1 \quad f_-(0) = -1$$

$$f(0) = \frac{1}{2} [1 - 1] = 0 \quad \text{OK}$$

## Exercise 2

This is a periodic function with period  $2\pi$ .

$$\alpha_n = \frac{1}{\pi} \int_0^\pi (t/\pi) \cos(nt) dt = \frac{1}{\pi^2} \int_0^\pi t \cos(nt) dt = \frac{1}{n^2 \pi^2} \int_0^\pi (nt) \cos(at) dt$$

$$= \frac{1}{n^2 \pi^2} \left\{ nt \sin(nt) \Big|_0^\pi - \int_0^\pi \cos(nt) (nt)' dt \right\} = \frac{1}{n^2 \pi^2} \left\{ n\pi \sin(n\pi) - n \sin(nt) \Big|_0^\pi \right\}$$

$$= \frac{1}{n^2 \pi^2} \left\{ n\pi \sin(n\pi) - n \sin(n\pi) \right\} \text{ let } u=nt$$

$$= \frac{1}{n^2 \pi^2} \left\{ \int_0^{t=\pi} u \cos(u) du \right\} = \frac{1}{n^2 \pi^2} \left\{ u \sin u - \int_0^{t=\pi} \sin u du \right\} =$$

$$= \frac{1}{n^2 \pi^2} \left\{ u \sin u \Big|_0^{t=\pi} + \sin u \Big|_0^{t=\pi} \right\} = \frac{1}{n^2 \pi^2} \left\{ n\pi \sin(n\pi) + \sin(n\pi) \right\}$$

$$= \frac{1}{n^2 \pi^2} \left\{ \cos(n\pi) - 1 \right\} = \begin{cases} 0 & n = \text{even} \\ -\frac{2}{n^2 \pi^2} & n = \text{odd} \end{cases}$$

For  $n=0$

$$\alpha_0 = \frac{1}{\pi} \int_0^\pi t dt = \frac{1}{\pi^2} \int_0^\pi t dt = \frac{\pi^2}{2\pi^2} = \frac{1}{2}$$

$$b_n = \frac{1}{\pi} \int_0^{\pi} \left(\frac{t}{\pi}\right) \sin(nt) dt = \frac{1}{\pi^2 n^2} \int_0^{\pi} (nt) \sin(nt) d(nt) =$$

3

$$= \frac{1}{n^2 \pi^2} \left\{ \int_0^{\pi} u \sin u du \right\} = \frac{-1}{n^2 \pi^2} \left\{ \int_0^{\pi} u \cos u du \right\} = -\frac{1}{n^2 \pi^2} \left\{ u \cos u \Big|_0^{\pi} - \int_0^{\pi} \cos u du \right\}$$

$$= \frac{-1}{n^2 \pi^2} \left\{ u \cos u \Big|_0^{\pi} - \sin u \Big|_0^{\pi} \right\} = -\frac{1}{n^2 \pi^2} \left\{ n\pi \cos(n\pi) - n\pi \right\} =$$

$$= -\frac{1}{(n\pi)} \left\{ \cos(n\pi) - 1 \right\} = \frac{-\cos n\pi}{(n\pi)} =$$

$$= \begin{cases} \frac{1}{n\pi} & \text{if } n = \text{odd} \\ -\frac{1}{n\pi} & \text{if } n = \text{even} \end{cases}$$

