PHYS 111
1ST semester 1439-1440

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Lecture 9

Chapter 26
Capacitance and Dielectrics

Parallel-Plate Capacitors

Quick Quiz 26.2 Many computer keyboard buttons are constructed of capacitors, as shown in Figure 26.5. When a key is pushed down, the soft insulator between the movable plate and the fixed plate is compressed. When the key is pressed, the capacitance (a) increases, (b) decreases, or (c) changes in a way that we cannot determine because the complicated electric circuit connected to the keyboard button may cause a change in ΔV.

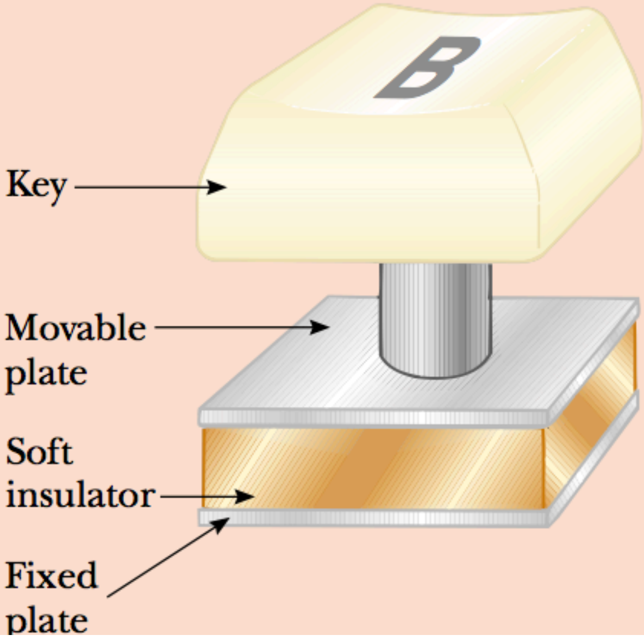
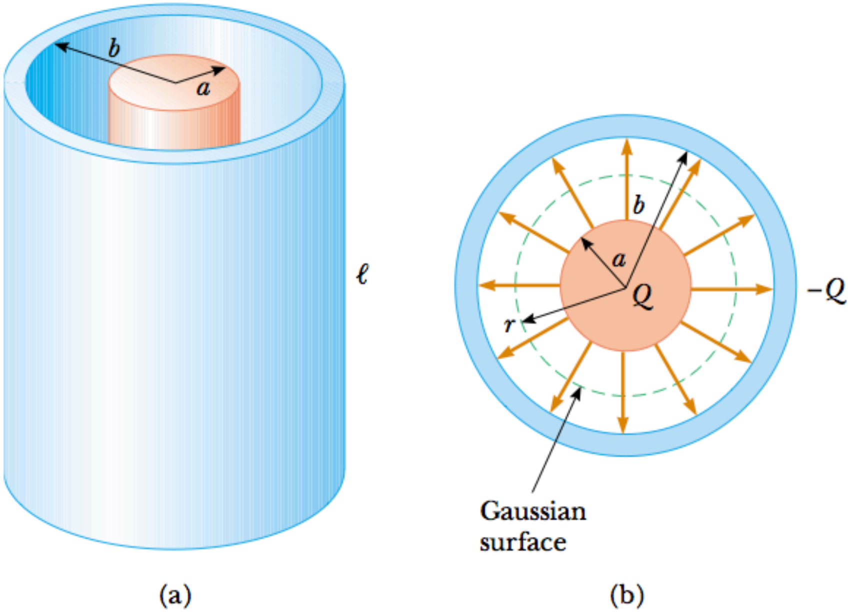


Figure 26.5 (Quick Quiz 26.2) One type of computer keyboard button.

Example 26.2 **The Cylindrical Capacitor**

A solid cylindrical conductor of radius a and charge Q is coaxial with a cylindrical shell of negligible thickness, radius $b > a$, and charge $-Q$ (Fig. 26.6a). Find the capacitance of this cylindrical capacitor if its length is ℓ.



Example 26.2 **The Cylindrical Capacitor**

- We must first calculate the potential difference between the two cylinders, which is given in general by

$$V_b - V_a = -\int_a^b \mathbf{E} \cdot d\mathbf{s}$$

- where **E** is the electric field in the region between the cylinders.

using Gauss's law that the magnitude of the electric field of a cylindrical charge distribution having linear charge density λ is $E = 2k_e\lambda/r$

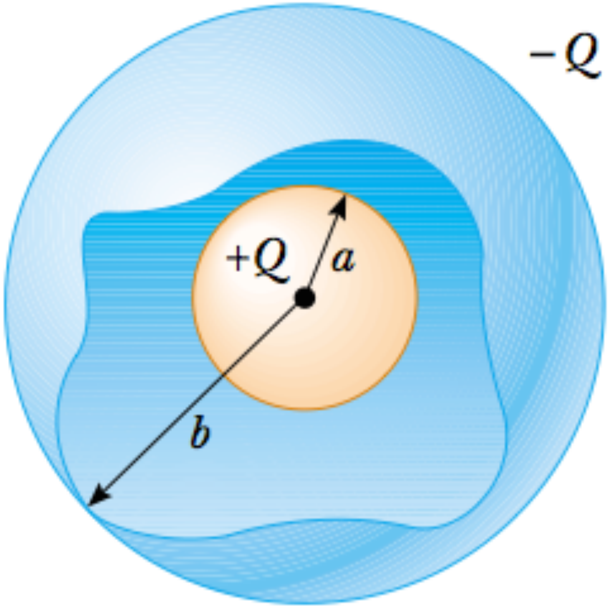
$$V_b - V_a = -\int_a^b E_r\, dr = -2k_e\lambda \int_a^b \frac{dr}{r} = -2k_e\lambda \ln\left(\frac{b}{a}\right)$$

Substituting this result into Equation 26.1 and using the fact that $\lambda = Q/\ell$, we obtain

$$C = \frac{Q}{\Delta V} = \frac{Q}{(2k_e Q/\ell)\ln(b/a)} = \frac{\ell}{2k_e \ln(b/a)} \qquad (26.4)$$

Example 26.3 **The Spherical Capacitor**

A spherical capacitor consists of a spherical conducting shell of radius b and charge $-Q$ concentric with a smaller conducting sphere of radius a and charge Q (Fig. 26.7). Find the capacitance of this device.



Example 26.3 **The Spherical Capacitor**

- As we showed in Chapter 24, the field outside a spherically symmetric charge distribution is radial and given by the expression $k_e Q/r^2$.

$$V_b - V_a = -\int_a^b E_r\,dr = -k_e Q \int_a^b \frac{dr}{r^2} = k_e Q \left[\frac{1}{r}\right]_a^b$$

$$= k_e Q \left(\frac{1}{b} - \frac{1}{a}\right)$$

The magnitude of the potential difference is

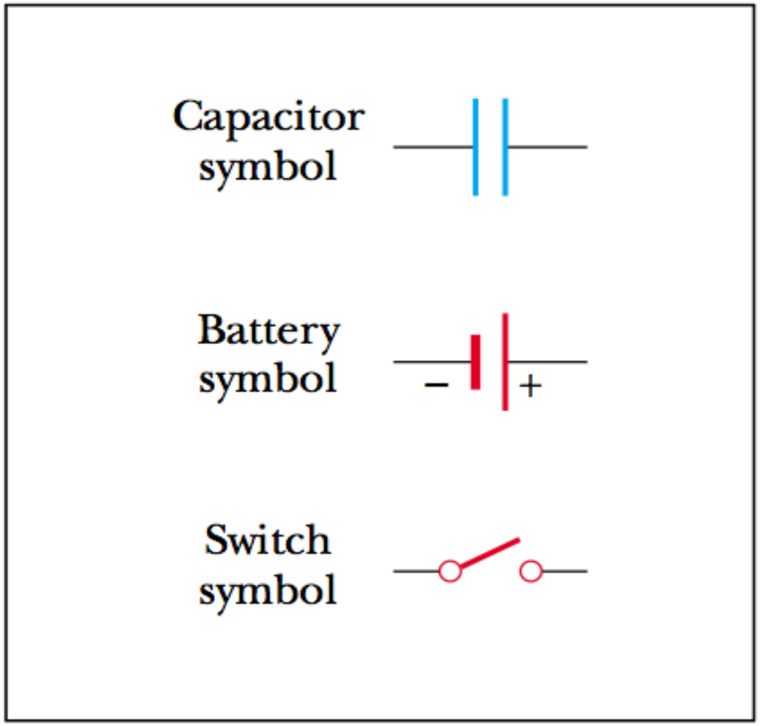
$$\Delta V = |V_b - V_a| = k_e Q \frac{(b - a)}{ab}$$

Substituting this value for ΔV into Equation 26.1, we obtain

$$C = \frac{Q}{\Delta V} = \frac{ab}{k_e(b - a)} \qquad (26.6)$$

26.3 Combinations of Capacitors

- The **circuit diagram** uses **circuit symbols** to represent various circuit elements.



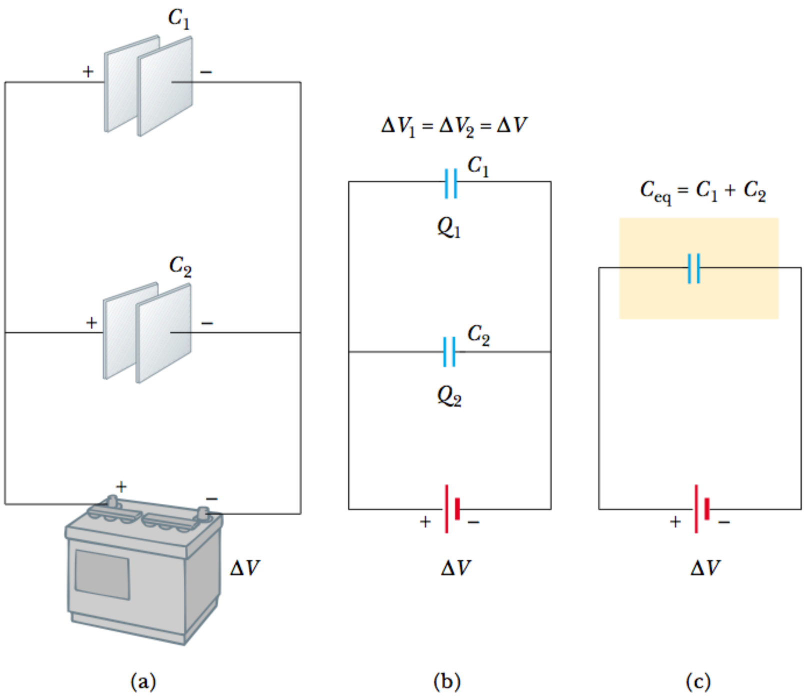
Parallel Combination

- The individual potential differences across capacitors connected in **parallel** are the same and are equal to the potential difference applied across the combination.
- The *total charge Q* stored by the two capacitors is

$$Q=Q_1+Q_2$$

- The total charge on capacitors connected in parallel is the sum of the charges on the individual **capacitors.**

$$Q_1=C_1\Delta V \text{ and } Q_2=C_2\Delta V$$



Parallel Combination

- The equivalent capacitor,

$$Q = C_{eq}\,\Delta V$$

$$C_{eq}\Delta V = C_1\Delta V + C_2\Delta V$$

$$C_{eq} = C_1 + C_2 \text{ (parallel combination)}$$

- $$C_{eq} = C_1 + C_2 + C_3 + \cdots \qquad \text{(parallel combination)}$$

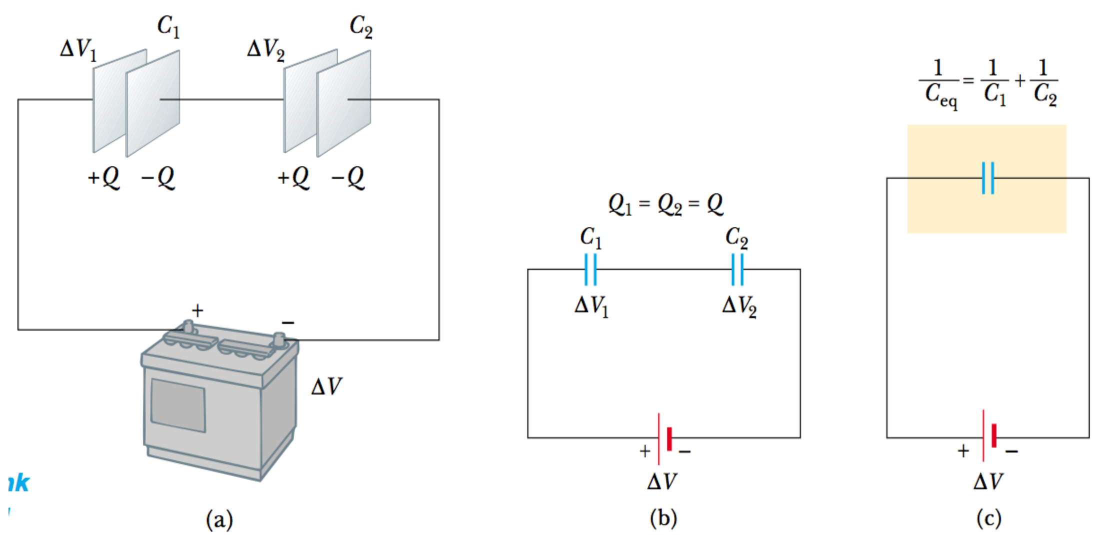
- The equivalent capacitance of a parallel combination of capacitors is the algebraic sum of the individual capacitances and is **greater** than any of the individual capacitances.

Series Combination

- The charges on capacitors connected in series are the same.
- The voltage ΔV across the battery terminals is split between the two capacitors:

$$\Delta V = \Delta V_1 + \Delta V_2$$

- The total potential difference across any number of capacitors connected in series is the sum of the potential differences across the individual capacitors.



Series Combination

- The equivalent capacitor must have a charge of -*Q* on its right plate and a charge of +*Q* on its left plate.

$$\Delta V = Q/C_{eq}$$

- The potential differences

$$\Delta V_1 = Q/C_1 \text{ and } \Delta V_2 = Q/C_2$$

$$\Delta V = \Delta V_1 + \Delta V_2$$

$$Q/C_{eq} = Q/C_1 + Q/C_2$$

$$1/C_{eq} = 1/C_1 + 1/C_2 \qquad \text{(series combination)}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \qquad \text{(series combination)}$$

- The inverse of the equivalent capacitance is the algebraic sum of the inverses of the individual capacitances and the equivalent capacitance of a series combination is always less than any individual capacitance in the combination.

26.3 Combinations of Capacitors

Quick Quiz 26.3 Two capacitors are identical. They can be connected in series or in parallel. If you want the *smallest* equivalent capacitance for the combination, do you connect them in (a) series, in (b) parallel, or (c) do the combinations have the same capacitance?

Quick Quiz 26.4 Consider the two capacitors in Quick Quiz 26.3 again. Each capacitor is charged to a voltage of 10 V. If you want the largest combined potential difference across the combination, do you connect them in (a) series, in (b) parallel, or (c) do the combinations have the same potential difference?

Example 26.4 Equivalent Capacitance

Interactive

Find the equivalent capacitance between *a* and *b* for the combination of capacitors shown in Figure 26.11a. All capacitances are in microfarads.

Solution Using Equations 26.8 and 26.10, we reduce the combination step by step as indicated in the figure. The 1.0-μF and 3.0-μF capacitors are in parallel and combine according to the expression $C_{eq} = C_1 + C_2 = 4.0\ \mu$F. The 2.0-$\mu$F and 6.0-$\mu$F capacitors also are in parallel and have an equivalent capacitance of 8.0 μF. Thus, the upper branch in Figure 26.11b consists of two 4.0-μF capacitors in series, which combine as follows:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{4.0\ \mu F} + \frac{1}{4.0\ \mu F} = \frac{1}{2.0\ \mu F}$$

$$C_{eq} = 2.0\ \mu\text{F}$$

The lower branch in Figure 26.11b consists of two 8.0-μF capacitors in series, which combine to yield an equivalent capacitance of 4.0 μF. Finally, the 2.0-μF and 4.0-μF capacitors in Figure 26.11c are in parallel and thus have an equivalent capacitance of 6.0 μF.