PHYS 111
1ST semester 1439-1440

Dr. Nadyah Alanazi

Lecture 27

Chapter 44

Nuclear Structure

- **44.1** Some Properties of Nuclei
- **44.2** Nuclear Binding Energy
- **44.4** Radioactivity
- **44.5** The Decay Processes
- **44.6** Natural Radioactivity
- **44.7** Nuclear Reactions

44.4 Radioactivity

- In 1896, Becquerel accidentally discovered that uranyl potassium sulfate crystals emit an invisible radiation that can darken a photographic plate even though the plate is covered to exclude light.
- This process of spontaneous emission of radiation by uranium was soon to be called **radioactivity.**
- Additional experiments, including Rutherford's famous work on alpha-particle scattering, suggested that radioactivity is the result of the ***decay***, or disintegration, of unstable nuclei.

44.4 Radioactivity

- Three types of radioactive decay occur in radioactive substances:
  - alpha (α) decay, in which the emitted particles are ^{4}He nuclei;
  - beta (β) decay, in which the emitted particles are either electrons or positrons;
  - and gamma (γ) decay, in which the emitted particles are high-energy photons.
- A **positron** is a particle like the electron in all respects except that the positron has a charge of +*e.* (The positron is the *antiparticle* of the electron) The symbol e^- is used to designate an electron, and e^+ designates a positron.
- The three types of radiation have quite different penetrating powers.
  - Alpha particles barely penetrate a sheet of paper,
  - beta particles can penetrate a few millimeters of aluminum,
  - and gamma rays can penetrate several centimeters of lead.

44.4 Radioactivity

- If N is the number of undecayed radioactive nuclei present at some instant, the rate of change of N with time is

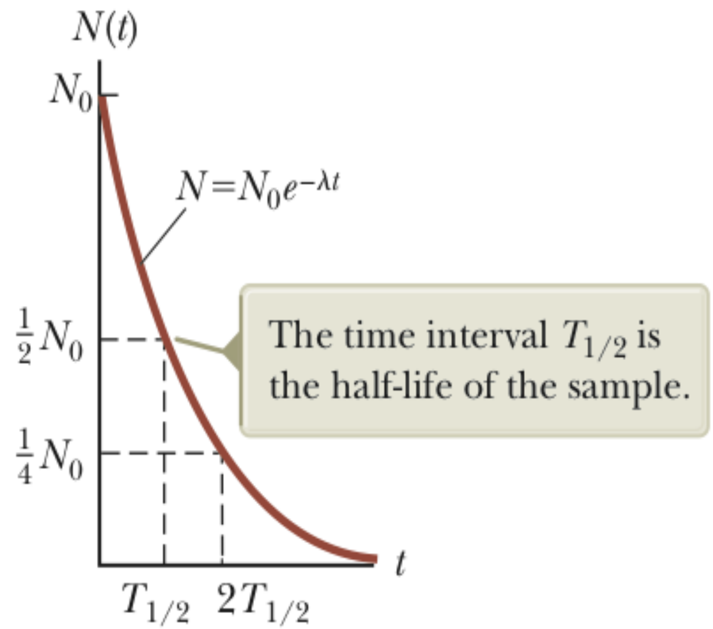
$$\frac{dN}{dt} = -\lambda N$$

- where λ, called the **decay constant,** is the probability of decay per nucleus per second. The negative sign indicates that dN/dt is negative; that is, N decreases in time.

$$\frac{dN}{N} = -\lambda\, dt$$

$$N = N_0 e^{-\lambda t}$$

- where the constant N_0 represents the number of undecayed radioactive nuclei at t = 0.
- The number of undecayed radioactive nuclei in a sample decreases exponentially with time.



44.4 Radioactivity

- The decay rate, which is the number of decays per second.

$$R = \left|\frac{dN}{dt}\right| = \lambda N = \lambda N_0 e^{-\lambda t} = R_0 e^{-\lambda t}$$

- where $R_0 = \lambda N_0$ is the decay rate at $t = 0$.
- The decay rate R of a sample is often referred to as its **activity.**
- Note that both N and R decrease exponentially with time. Another parameter useful in characterizing nuclear decay is the **half-life** $T_{1/2}$:

The **half-life** of a radioactive substance is the time interval during which half of a given number of radioactive nuclei decay.

44.4 Radioactivity

- To find an expression for the half-life, we first set $N = N_0/2$ and $t = T_{1/2}$

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$$

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

- After a time interval equal to one half-life, there are $N_0/2$ radioactive nuclei remaining; after two half-lives, half of these remaining nuclei have decayed and $N_0/4$ radioactive nuclei are left; after three half-lives, $N_0/8$ are left; and so on.
- In general, after *n* half-lives, the number of undecayed radioactive nuclei remaining is

$$N = N_0\left(\tfrac{1}{2}\right)^n$$

- where *n* can be an integer or a noninteger.

44.4 Radioactivity

A frequently used unit of activity is the **curie** (Ci), defined as

$$1 \text{ Ci} \equiv 3.7 \times 10^{10} \text{ decays/s}$$

This value was originally selected because it is the approximate activity of 1 g of radium. The SI unit of activity is the **becquerel** (Bq):

$$1 \text{ Bq} \equiv 1 \text{ decay/s}$$

Therefore, $1 \text{ Ci} = 3.7 \times 10^{10}$ Bq. The curie is a rather large unit, and the more frequently used activity units are the millicurie and the microcurie.

Example 44.4 How Many Nuclei Are Left?

The isotope carbon-14, ${}^{14}_{6}\text{C}$, is radioactive and has a half-life of 5 730 years. If you start with a sample of 1 000 carbon-14 nuclei, how many nuclei will still be undecayed in 25 000 years?

SOLUTION

Conceptualize The time interval of 25 000 years is much longer than the half-life, so only a small fraction of the originally undecayed nuclei will remain.

Categorize The text of the problem allows us to categorize this example as a substitution problem involving radioactive decay.

Analyze Divide the time interval by the half-life to determine the number of half-lives:

$$n = \frac{25\,000 \text{ yr}}{5\,730 \text{ yr}} = 4.363$$

Determine how many undecayed nuclei are left after this many half-lives using Equation 44.9:

$$N = N_0\left(\tfrac{1}{2}\right)^n = 1\,000\left(\tfrac{1}{2}\right)^{4.363} = \boxed{49}$$

Example 44.5 The Activity of Carbon

At time $t = 0$, a radioactive sample contains 3.50 μg of pure $^{11}_{6}C$, which has a half-life of 20.4 min.

(A) Determine the number N_0 of nuclei in the sample at $t = 0$.

SOLUTION

Conceptualize The half-life is relatively short, so the number of undecayed nuclei drops rapidly. The molar mass of $^{11}_{6}C$ is approximately 11.0 g/mol.

Categorize We evaluate results using equations developed in this section, so we categorize this example as a substitution problem.

Find the number of moles in 3.50 μg of pure $^{11}_{6}C$:

$$n = \frac{3.50 \times 10^{-6}\ \text{g}}{11.0\ \text{g/mol}} = 3.18 \times 10^{-7}\ \text{mol}$$

Find the number of undecayed nuclei in this amount of pure $^{11}_{6}C$:

$$N_0 = (3.18 \times 10^{-7}\ \text{mol})(6.02 \times 10^{23}\ \text{nuclei/mol}) = \boxed{1.92 \times 10^{17}\ \text{nuclei}}$$

(B) What is the activity of the sample initially and after 8.00 h?

SOLUTION

Find the initial activity of the sample using Equations 44.7 and 44.8:

$$R_0 = \lambda N_0 = \frac{0.693}{T_{1/2}} N_0 = \frac{0.693}{20.4\ \text{min}} \left(\frac{1\ \text{min}}{60\ \text{s}}\right)(1.92 \times 10^{17})$$

$$= (5.66 \times 10^{-4}\ \text{s}^{-1})(1.92 \times 10^{17}) = \boxed{1.09 \times 10^{14}\ \text{Bq}}$$

Use Equation 44.7 to find the activity at $t = 8.00\ \text{h} = 2.88 \times 10^4\ \text{s}$:

$$R = R_0 e^{-\lambda t} = (1.09 \times 10^{14}\ \text{Bq}) e^{-(5.66 \times 10^{-4}\ \text{s}^{-1})(2.88 \times 10^4\ \text{s})} = \boxed{8.96 \times 10^6\ \text{Bq}}$$

Example 44.6 A Radioactive Isotope of Iodine

A sample of the isotope ^{131}I, which has a half-life of 8.04 days, has an activity of 5.0 mCi at the time of shipment. Upon receipt of the sample at a medical laboratory, the activity is 2.1 mCi. How much time has elapsed between the two measurements?

SOLUTION

Conceptualize The sample is continuously decaying as it is in transit. The decrease in the activity is 58% during the time interval between shipment and receipt, so we expect the elapsed time to be greater than the half-life of 8.04 d.

Categorize The stated activity corresponds to many decays per second, so N is large and we can categorize this problem as one in which we can use our statistical analysis of radioactivity.

Analyze Solve Equation 44.7 for the ratio of the final activity to the initial activity:

$$\frac{R}{R_0} = e^{-\lambda t}$$

Take the natural logarithm of both sides:

$$\ln\left(\frac{R}{R_0}\right) = -\lambda t$$

Solve for the time t:

$$(1)\quad t = -\frac{1}{\lambda}\ln\left(\frac{R}{R_0}\right)$$

Use Equation 44.8 to substitute for λ:

$$t = -\frac{T_{1/2}}{\ln 2}\ln\left(\frac{R}{R_0}\right)$$

Substitute numerical values:

$$t = -\frac{8.04\text{ d}}{0.693}\ln\left(\frac{2.1\text{ mCi}}{5.0\text{ mCi}}\right) = 10\text{ d}$$