PHYS 111
1^{ST} semester 1439-1440

Dr. Nadyah Alanazi

Lecture 21

Chapter 40

Introduction to Quantum Physics

40.1 Blackbody Radiation and Planck's Hypothesis

40.2 The Photoelectric Effect

40.1 Blackbody radiation and Planck's hypothesis

An object at any temperature emits electromagnetic waves in the form of **thermal radiation** from its surface as discussed in Section 20.7. The characteristics of this radiation depend on the temperature and properties of the object's surface. Careful study shows that the radiation consists of a continuous distribution of wavelengths from all portions of the electromagnetic spectrum. If the object is at room temperature, the wavelengths of thermal radiation are mainly in the infrared region and hence the radiation is not detected by the human eye. As the surface temperature of the object increases, the object eventually begins to glow visibly red, like the coils of a toaster. At sufficiently high temperatures, the glowing object appears white, as in the hot tungsten filament of an incandescent lightbulb.

40.1 Blackbody radiation and Planck's hypothesis

a **black body** is an ideal system that absorbs all radiation incident on it. The electromagnetic radiation emitted by the black body is called **blackbody radiation.**

- A good approximation of a black body is a small hole leading to the inside of a hollow object.
- The hole acts as a perfect absorber.
- The nature of the radiation leaving the cavity through the hole depends only on the temperature of the cavity.

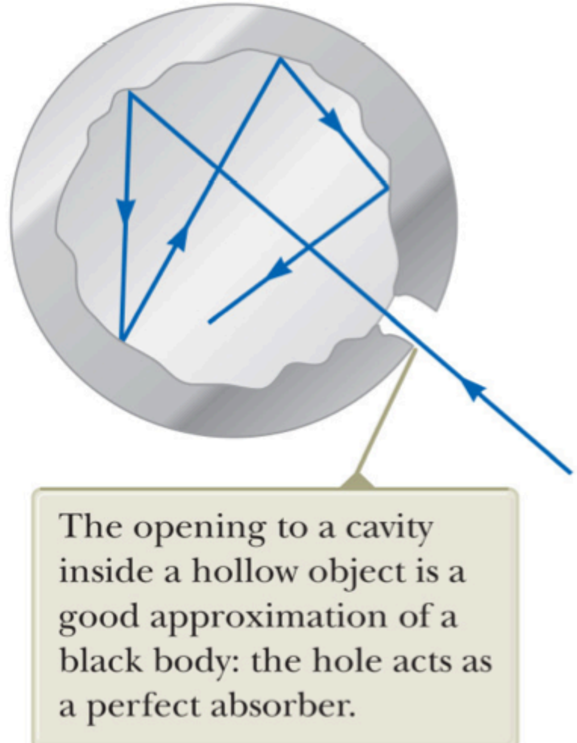


Figure 40.1 A physical model of a black body.

Two Experimental Findings:

- The total power of the emitted radiation increases with temperature.
  - **Stefan's law:** $$P = \sigma A e T^4$$

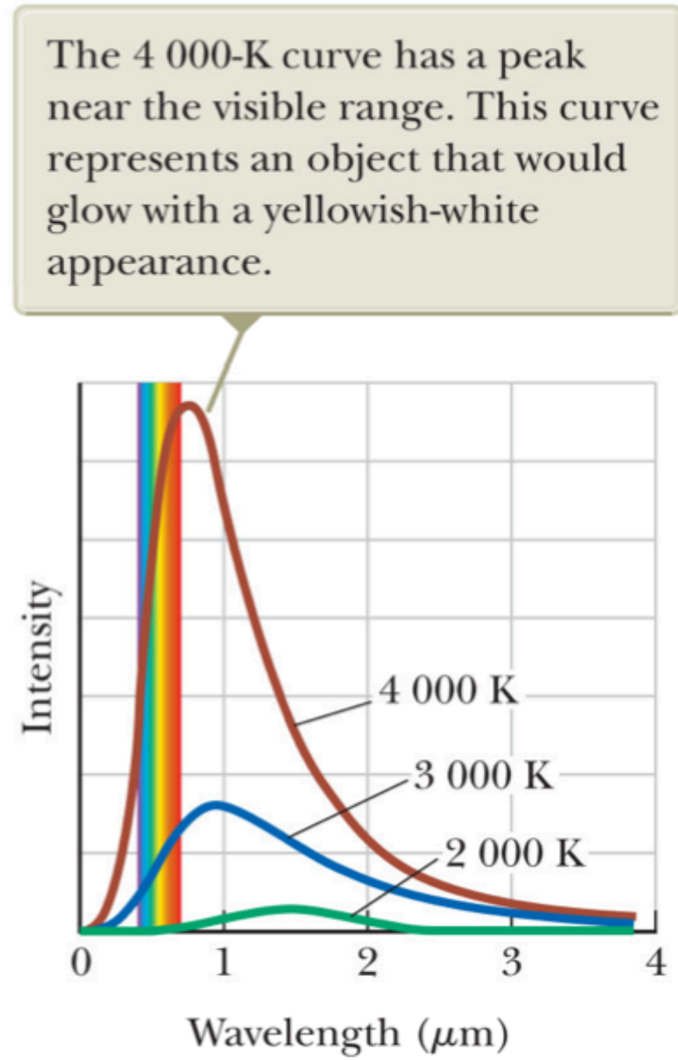
where P is the power in watts radiated at all wavelengths from the surface of an object, $\sigma = 5.670 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4}$ is the Stefan–Boltzmann constant, A is the surface area of the object in square meters, e is the emissivity of the surface, and T is the surface temperature in kelvins. For a black body, the emissivity is $e = 1$ exactly.

- The peak of the wavelength distribution shifts to shorter wavelengths as the temperature increases.
  - **Wien's displacement law:** $$\lambda_{\mathrm{max}} T = 2.898 \times 10^{-3}\ \mathrm{m \cdot K}$$

where λ_{max} is the wavelength at which the curve peaks and T is the absolute temperature of the surface of the object emitting the radiation. The wavelength at the curve's peak is inversely proportional to the absolute temperature; that is, as the temperature increases, the peak is "displaced" to shorter wavelengths

Intensity of blackbody radiation

- The intensity increases with increasing temperature.
- The amount of radiation emitted increases with increasing temperature.
  - The area under the curve.
- The peak wavelength decreases with increasing temperature.



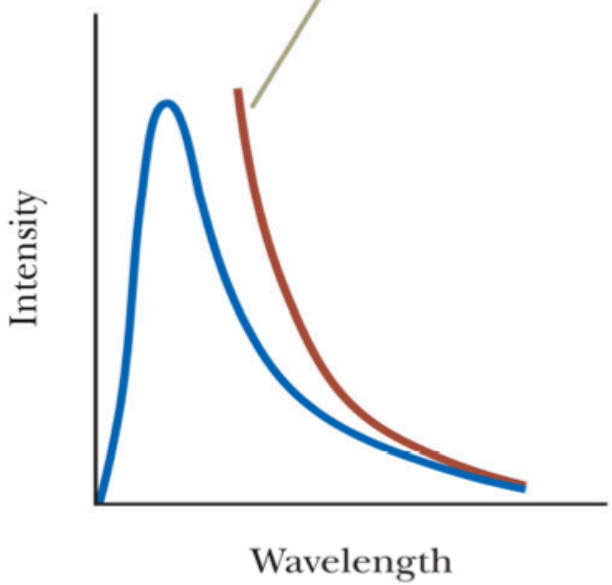
Rayleigh-Jeans law

- An early classical attempt to explain blackbody radiation was the **Rayleigh-Jeans law.**

$$I(\lambda, T) = \frac{2\pi c k_B T}{\lambda^4}$$

  - where k_B is Boltzmann's constant.

The classical theory (red-brown curve) shows intensity growing without bound for short wavelengths, unlike the experimental data (blue curve).



An experimental plot of the blackbody radiation spectrum, together with the theoretical prediction of the Rayleigh–Jeans law, is shown in Figure 40.5. At long wavelengths, the Rayleigh–Jeans law is in reasonable agreement with experimental data, but at short wavelengths, major disagreement is apparent.

Ultraviolet catastrophe

As λ approaches zero, the function $I(\lambda,T)$ given by Equation 40.3 approaches infinity. Hence, according to classical theory, not only should short wavelengths predominate in a blackbody spectrum, but also the energy emitted by any black body should become infinite in the limit of zero wavelength. In contrast to this prediction, the experimental data plotted in Figure 40.5 show that as λ approaches zero, $I(\lambda,T)$ also approaches zero. This mismatch of theory and experiment was so disconcerting that scientists called it the *ultraviolet catastrophe.*

Max Planck

In 1900, Max Planck developed a theory of blackbody radiation that leads to an equation for $I(\lambda, T)$ that is in complete agreement with experimental results at all wavelengths.

1. *Physical components:*
   Planck assumed the cavity radiation came from atomic oscillators in the cavity walls in Figure 40.1.
2. *Behavior of the components:*
   (a) The energy of an oscillator can have only certain *discrete* values E_n:

$$E_n = nhf \tag{40.4}$$

where n is a positive integer called a **quantum number,**[1] f is the oscillator's frequency, and h is a parameter Planck introduced that is now called **Planck's constant.** Because the energy of each oscillator can have only discrete values given by Equation 40.4, we say the energy is **quantized.** Each discrete energy value corresponds to a different **quantum state,** represented by the quantum number n. When the oscillator is in the $n = 1$ quantum state, its energy is hf; when it is in the $n = 2$ quantum state, its energy is $2hf$; and so on.

Max Planck

(b) The oscillators emit or absorb energy when making a transition from one quantum state to another.

- the amount of energy emitted by the oscillator and carried by the quantum of radiation is

$$E = hf \tag{40.5}$$

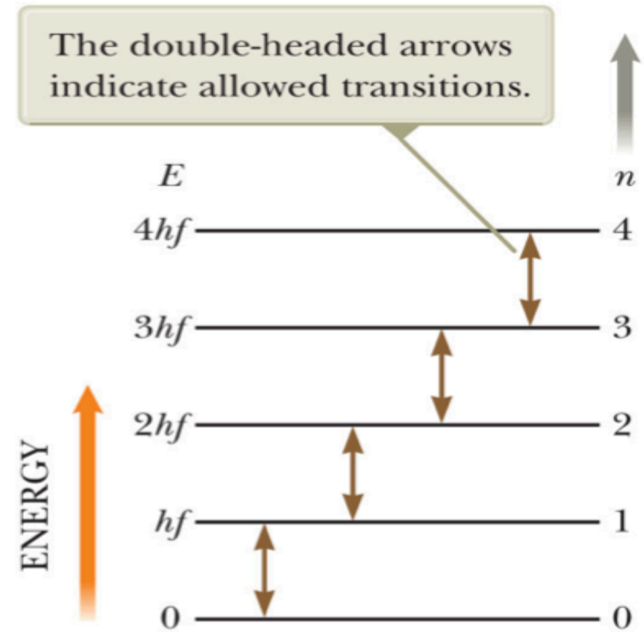


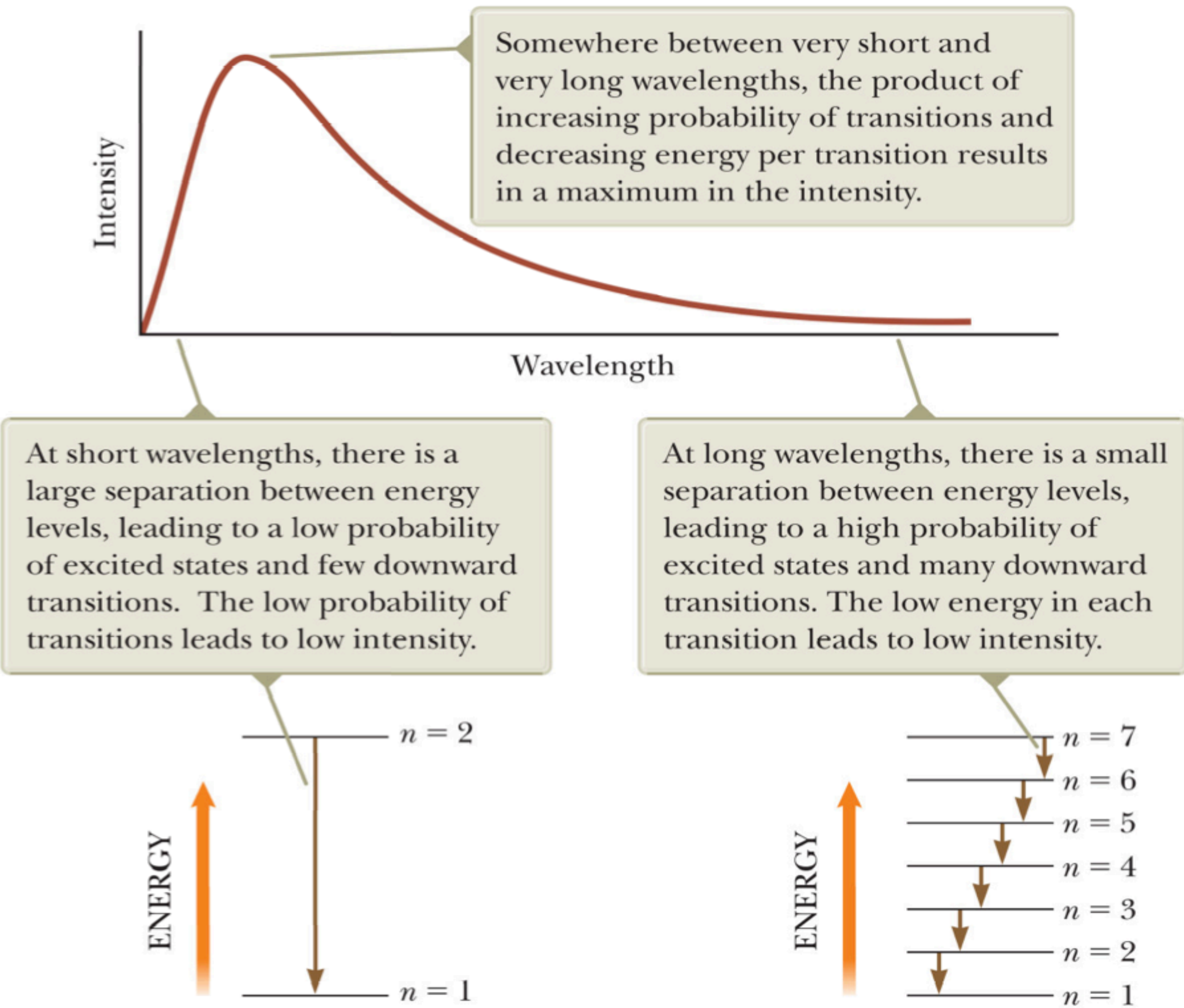
Figure 40.6 Allowed energy levels for an oscillator with frequency f.

Planck's wavelength distribution function

Planck generated a theoretical expression for the wavelength distribution that agreed remarkably well with the experimental curves

$$I(\lambda, T) = \frac{2\pi hc^2}{\lambda^5\left(e^{hc/\lambda k_B T} - 1\right)}$$

- Planck's constant $h = 6.626 \times 10^{-34}\,\mathrm{J \cdot s}$


Somewhere between very short and very long wavelengths, the product of increasing probability of transitions and decreasing energy per transition results in a maximum in the intensity.
Intensity
Wavelength
At short wavelengths, there is a large separation between energy levels, leading to a low probability of excited states and few downward transitions. The low probability of transitions leads to low intensity.
At long wavelengths, there is a small separation between energy levels, leading to a high probability of excited states and many downward transitions. The low energy in each transition leads to low intensity.
ENERGY
n = 2
n = 1
ENERGY
n = 7
n = 6
n = 5
n = 4
n = 3
n = 2
n = 1

Example 40.1 Thermal Radiation from Different Objects

(A) Find the peak wavelength of the blackbody radiation emitted by the human body when the skin temperature is 35°C.

Solve Equation 40.2 for λ_{max}:

$$(1) \quad \lambda_{\text{max}} = \frac{2.898 \times 10^{-3}\ \text{m} \cdot \text{K}}{T}$$

Substitute the surface temperature:

$$\lambda_{\text{max}} = \frac{2.898 \times 10^{-3}\ \text{m} \cdot \text{K}}{308\ \text{K}} = \boxed{9.41\ \mu\text{m}}$$

This radiation is in the infrared region of the spectrum and is invisible to the human eye. Some animals (pit vipers, for instance) are able to detect radiation of this wavelength and therefore can locate warm-blooded prey even in the dark.

(B) Find the peak wavelength of the blackbody radiation emitted by the tungsten filament of a lightbulb, which operates at 2 000 K.

SOLUTION

Substitute the filament temperature into Equation (1):

$$\lambda_{\text{max}} = \frac{2.898 \times 10^{-3}\ \text{m} \cdot \text{K}}{2\ 000\ \text{K}} = \boxed{1.45\ \mu\text{m}}$$

This radiation is also in the infrared, meaning that most of the energy emitted by a lightbulb is not visible to us.

Example 40.2 The Quantized Oscillator AM

A 2.00-kg block is attached to a massless spring that has a force constant of k = 25.0 N/m. The spring is stretched 0.400 m from its equilibrium position and released from rest.

(A) Find the total energy of the system and the frequency of oscillation according to classical calculations.

SOLUTION

Conceptualize We understand the details of the block's motion from our study of simple harmonic motion in Chapter 15. Review that material if you need to.

Categorize The phrase "according to classical calculations" tells us to categorize this part of the problem as a classical analysis of the oscillator. We model the block as a *particle in simple harmonic motion.*

Analyze Based on the way the block is set into motion, its amplitude is 0.400 m.

Evaluate the total energy of the block–spring system using Equation 15.21:

$$E = \tfrac{1}{2}kA^2 = \tfrac{1}{2}(25.0\ \mathrm{N/m})(0.400\ \mathrm{m})^2 = \boxed{2.00\ \mathrm{J}}$$

Evaluate the frequency of oscillation from Equation 15.14:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{25.0\ \mathrm{N/m}}{2.00\ \mathrm{kg}}} = \boxed{0.563\ \mathrm{Hz}}$$

(B) Assuming the energy of the oscillator is quantized, find the quantum number n for the system oscillating with this amplitude.

SOLUTION

Categorize This part of the problem is categorized as a quantum analysis of the oscillator. We model the block–spring system as a Planck oscillator.

Analyze Solve Equation 40.4 for the quantum number n:

$$n = \frac{E_n}{hf}$$

Substitute numerical values:

$$n = \frac{2.00\ \mathrm{J}}{(6.626 \times 10^{-34}\ \mathrm{J \cdot s})(0.563\ \mathrm{Hz})} = \boxed{5.36 \times 10^{33}}$$